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ON STRATEGIC R&D BEHAVIOR

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Tariffs and quotas are compared to assess their effects on firm behavior in a two-stage Cournot duopoly game, where R&D (or capital) is chosen initially and output is selected subsequently. In this quantity-setting game, the imposition of a quota may remove the possibility of a pure-strategy equilibrium, leaving only a mixed strategy equilibrium. Under either potential equilibria, a quota leads to higher profits than those obtained under a comparably restrictive tariff. However, both domestic R&D and output are relatively lower in a pure-strategy, cum-quota equilibrium. Two potential pure-strategy equilibria may result under apparently nonbinding quotas.

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1. Introduction

Governments often defend industry-promoting policies by claiming that these measures allow domestic producers to achieve long-term gains in profitability through the adjustment of underlying strategic variables, such as capacity, advertising, and R&D input. Spencer and Brander (1983) have approached this concept within a two-stage duopoly model, where R&D is chosen initially and output is selected subsequently. They assume that an incremental increase in R&D lowers marginal production costs. Consequently, each firm's optimizing strategy requires excessive investment in R&D from a cost-minimizing standpoint. This behavior is profit-maximizing because a firm can credibly commit to higher output through increased R&D usage. In this manner, rival output is discouraged and profits are enhanced. R&D therefore possesses a strategic value in addition to its cost-reducing value.

Spencer and Brander consider optimal industrial policy when a single domestic firm competes against its foreign rival in an overseas market. The analysis focuses on the welfare effects of an R&D subsidy and an export subsidy. Using a similar duopoly model, this paper addresses the effects of protective policy on firm behavior when the foreign firm competes in the domestic market. The nonequivalence of tariffs and quotas becomes immediately apparent through their starkly different impacts on R&D. For a potential range of R&D choices, the imposition of a quota severs the connection between domestic R&D and foreign output. The domestic firm no longer needs to use R&D

strategically at a cost of productive inefficiency. Under a tariff, strategic behavior is still necessary since the domestic firm's choice of R&D invariably affects the equilibrium level of foreign output.

Either a pure-strategy or mixed-strategy equilibrium results from the imposition of a quota. In a pure-strategy, cum-quota equilibrium, the elimination of strategic R&D behavior causes the domestic firm to choose a cost-minimizing level of R&D and act as a constrained monopolist. Relative to the equilibrium for an equally restrictive tariff, less R&D is used by both firms. In fact, quotas and tariffs can induce qualitatively different changes in domestic R&D. A mild quota reduces domestic R&D in a pure-strategy equilibrium, while the associated tariff increases R&D usage. Since the imposition of a quota improves productive efficiency, domestic profits are always relatively higher than under a comparable tariff. Unfortunately, consumers suffer because domestic production is lower in the quota case.

If policymakers impose a quota near the free-trade import level, then the domestic firm reacts by reducing its R&D level, its output, and its market share in a pure-strategy equilibrium. Furthermore, marginal production costs are above their original level.

These results cast serious doubts on the viability of quotas in an infant-industry case. We assume that, in the the first stage of the game, each firm selects a variable which can lower marginal costs in the subsequent output stage. This cost assumption is sufficiently general that it could refer to variables other than R&D, such as capacity or managerial talent. It is apparent that, in a pure-strategy equilibrium, the imposition of a mild quota reduces the scale of output and lowers domestic investment in cost-reducing variables. This behavior apparently violates the rationale for infant-industry protection, where firms enjoy long-term benefits from a learning process related positively to prior output and investment levels. On the other hand, tariffs are consistent with this rationale because they raise both domestic output and R&D.

In a pure-strategy equilibrium, a quota does prove relatively useful in increasing domestic profits and stimulating the movement of resources away from a particular industry. This objective may be desirable when policy attempts to lessen structural adjustment costs by delaying exit from a declining industry.

The possibility of a mixed-strategy equilibrium reveals another potentially adverse aspect of a quota; this policy may create unsteady behavior in a Cournot model. In the mixed-strategy equilibrium, the domestic firm either selects a cost-minimizing R&D level, or it chooses to act strategically and reduce foreign output below the quota level. In response, the foreign firm commits to a single R&D level. We will show that, if firms continue to pursue pure strategies in the absence of a pure-strategy, cum-quota equilibrium, then the imposition of a quota leads to a four-period orbit in R&D (and output) space. This pattern stands in marked contrast to the dynamic behavior exhibited in a tariff case, where convergence to a pure-strategy equilibrium necessarily occurs under the same initial assumptions.

Our mixed-strategy, cum-quota equilibrium occurs for reasons similar to those discovered by Krishna (1985) in a one-stage Bertrand game. It results from the change in domestic conjectures induced by the import quota. The presence of the import constraint implies that the domestic firm receives no strategic benefits until its decision variable reaches a specific threshhold level. At this threshhold, a discontinuity occurs in the domestic firm's conjecture which may present the possibility of two optimizing responses. In our model, the conjecture concerns the anticipated foreign response to a domestic R&D change.

As Edgeworth has shown, the presence of a mixed-strategy equilibrium is not unusual in constrained Bertrand games.
By constrast, our mixed-strategy equilibrium occurs in a Cournot output game. Quotas and tariffs have generally been thought to yield equivalent solutions in one-stage Cournot output models. We show definitively that this result does not extend to models where strategic behavior occurs in earlier stages. Moreover, mere evidence of Cournot behavior in the output stage is no longer sufficient to preclude a quota from causing unsteady behavior.

2. The Model

Consider a market serviced by a single domestic firm and its foreign rival. These firms compete in a one-period game, where R&D is selected in the first stage and output in the second stage.
The level of R&D reveals the cost of production, which essentially determines the output equilibrium.

The game is solved recursively. An output equilibrium can be determined for any given R&D combination. For a specific R&D level chosen by its rival, each firm responds by selecting an R&D level which maximizes its profits in the ensuing output stage. In this manner, each firm determines its own R&D reaction function. Under appropriate assumptions, a stable, pure-strategy R&D equilibrium exists prior to the imposition of any specific trade policy. Each firm is assumed to use Cournot conjectures in both the output and R&D stages.[1]

Turning to the output decision, let x represent domestic production and y denote foreign production. In all other notation, an asterisk distinguishes the foreign firm from the domestic firm. As described by the following equations, the profit of each firm equals its revenue less the sum of its direct production cost and its R&D cost:

$$\pi = R(x,y) - C(x,v) - av \tag{1}$$

$$\pi^* = (1 - t)R^*(x,y) - C^*(y,v^*) - a^*v^*, \tag{1*}$$

where $\pi(\pi^*)$ = profits of domestic (foreign) firm

$R(R^*)$ = revenue function of domestic (foreign) firm

$C(C^*)$ = direct production cost function of domestic (foreign) firm

[1] When R&D can be used to credibly commit to higher output levels, it would be unlikely that firms would attempt to act strategically in the output stage. Furthermore, the presence of a nonzero conjectural variation in a noncooperative game inherently assumes that each firm anticipates that some dynamic response pattern results from deviations in its output. The existence of this response pattern causes a firm to choose a reply which is nonoptimal, if the rival is willing to maintain its current output level. This result has been criticized as irrational by both Daugherty (1985) and Makowski (1987).

$a(a^*)$ = domestic (foreign) cost of R&D

$v(v^*)$ = R&D level of domestic (foreign) firm

t = import tariff rate

All functional relationships are assumed continuous. Notice that the foreign firm produces only for the home country's market.

Equations (2) and (2*) show first-order conditions for optimal output choice, where subscripts denote partial derivatives:

$$\pi_x = R_x(x,y) - C_x(x,v) = 0 \tag{2}$$

$$\pi^*_y = (1 - t)R^*_y(x,y) - C^*_y(y,v^*) = 0 \tag{2*}$$

Marginal production costs are considered positive and nondecreasing, as expressed below:

$$C_x > 0; \ C_{xx} \geq 0 \tag{3}$$

$$C^*_y > 0; \ C^*_{yy} \geq 0 \tag{3*}$$

We assume that, for a given firm, an increase in rival output causes a decline in both total revenue and marginal revenue. The following inequalities express these restrictions:

$$R_y < 0 \tag{4}$$

$$R^*_x < 0 \tag{4*}$$

$$\pi_{xy} = R_{xy} < 0 \tag{5}$$

$$\pi^*_{yx} = (1 - t)R^*_{yx} < 0 \tag{5*}$$

Furthermore, an increase in a firm's own output should cause a decline in its marginal revenue. Along with the prior marginal cost assumption, this restriction is sufficient to ensure that second-order conditions are satisfied for profit maximization. We represent these conditions in the following equations:

$$\pi_{xx} = R_{xx} - C_{xx} < 0 \tag{6}$$

$$\pi^*_{yy} = (1 - t)R^*_{yy} - C^*_{yy} < 0 \tag{6*}$$

If a stable output equilibrium exists, the following condition must also hold:

$$\pi_{xx}\pi^*_{yy} - \pi_{xy}\pi^*_{yx} > 0 \tag{7}$$

Given our prior assumptions, the stability condition is necessarily satisfied in the case of perfect substitutes. For imperfect substitutes, this restriction is valid under a variety of demand specifications based on reasonable utility assumptions.

We next examine the R&D decision of each firm. From equations (2) and (2*), we see that a Cournot-Nash output equilibrium can be derived whenever marginal production costs are established. However, our model assumes that both total and marginal production costs depend on R&D. As the following equations denote, these costs are assumed to decline if R&D increases.[2]

$$C_v < 0; \; C_{xv} < 0 \tag{8}$$

$$C^*_{v^*} < 0; \; C^*_{yv^*} < 0 \tag{8*}$$

Since each R&D choice determines a unique marginal cost function, any given R&D combination is associated with a unique Nash equilibrium. This equilibrium, expressed as $[x^o(v,v^*,t), y^o(v,v^*,t)]$, can be inserted into the profit functions represented in equations (1) and (1*). Differentiation of these equations yields the following first-order conditions for optimal R&D choice:[3]

$$\pi_v(v,v^*,t) = R_y(x^o(v,v^*,t),y^o(v,v^*,t))y^o{}_v - C_v(x^o(v,v^*,t),v) - a = 0 \tag{9}$$

$$\pi^*_{v^*}(v,v^*,t) = (1 - t)R^*_x(x^o(v,v^*,t),y^o(v,v^*,t))x^o{}_{v^*} - C^*_{v^*}(y^o(v,v^*,t),v^*) - a^* = 0 \tag{9*}$$

The terms, $R_y y^o{}_v$ and $R^*_x x^o{}_{v^*}$, indicate that the marginal value of R&D is negatively related to its effect on rival output. By differentiating equations (2) and (2*), we can determine

[2] Notice that, as previously mentioned, the same cost assumptions may be used to describe the effects of incremental capital changes on variable production costs under factor substitutability. So, R&D serves as a proxy for those variables which reduce direct production costs and can be committed prior to the output decision. Such factors as physical capital and managerial talent may fall into this categorization.

[3] Hereafter, we will often omit the parenthetical expression showing the dependence of equilibrium output on R&D.

the effects of an R&D change on equilibrium output:

$$x^{o}_{v} = C_{xv}\pi^{*}_{yy}/A > 0 \tag{10}$$

$$x^{o}_{v*} = -C^{*}_{yv*}\pi_{xy}/A < 0 \tag{10*}$$

$$y^{o}_{v} = -C_{xv}\pi^{*}_{yx}/A < 0 \tag{11}$$

$$y^{o}_{v*} = C^{*}_{yv*}\pi_{xx}/A > 0, \tag{11*}$$

where $A = \pi_{xx}\pi^{*}_{yy} - \pi_{xy}\pi^{*}_{yx} > 0$ (by equation (7))

For a given firm, an increase in R&D raises its own output and lowers that of its rival. Each firm accordingly derives revenue benefits from the drop in rival output associated with increased R&D usage. This effect represents the strategic value of R&D.

We assume that the effectiveness of R&D in reducing direct production costs declines with increased input, even though R&D is spread over larger output levels as input increases. This assumption is expressed in equations (12) and (12*):

$$C_{vv} + C_{xv}x^{o}_{v} > 0 \tag{12}$$

$$C^{*}_{v*v*} + C^{*}_{yv*}y^{o}_{v*} > 0 \tag{12*}$$

To satisfy second-order conditions and meet the stability condition for an internal R&D equilibrium, we make the following assumptions:

$$\pi_{vv} < 0 \tag{13}$$

$$\pi^{*}_{v*v*} < 0 \tag{13*}$$

$$\pi_{vv}\pi^{*}_{v*v*} - \pi_{vv*}\pi^{*}_{v*v} > 0 \tag{14}$$

Equations (12) and (12*) aid in satisfying the second-order conditions.[4]

We further assume that R&D reaction functions are downward-sloping, as expresed below:

$$\pi_{vv^*} < 0 \qquad (15)$$

$$\pi^*_{v^*v} < 0 \qquad (15^*)$$

It has been previously established that an increase in rival R&D reduces the output of a given firm. This effect diminishes the cost-reducing benefits derived from a firm's own R&D.[5]

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[4] Equations (13) and (13*) depend on the following restrictions:

$$C_{vv} + C_{xv}x^{o}_{v} > R_{yx}y^{o}_{v}x^{o}_{v} + R_{yy}y^{o}_{v}y^{o}_{v} + R_{y}(dy^{o}_{v}/dv)$$

$$C^*_{v^*v^*} + C_{*yv^*}y^{o}_{v^*} > (1 - t)\ [R^*_{xx}x^{o}_{v^*}x^{o}_{v^*} + R^*_{xy}x^{o}_{v^*}y^{o}_{v^*} + R^*_{x}(dx^{o}_{v^*}/dv^*)]$$

These restrictions require that a marginal increase in R&D exert a stronger impact on the marginal cost-reducing value of R&D than on its strategic value, if these two effects work in opposite directions. While we have previously assumed that the marginal cost-reducing value of R&D declines with increased usage, the impact on in its strategic value may be either positive or negative based on our prior assumptions. So, many specific cost and demand functions will satisfy the above constraints.

[5] By differentiation, it can be shown that:

$$\pi_{vv^*} = -C_{xv}x^{o}_{v^*} + (R_{yx}x^{o}_{v^*} + R_{yy}y^{o}_{v^*})y^{o}_{v} + R_{y}(y^{o}_{vx}x^{o}_{v^*} + y^{o}_{vy}y^{o}_{v^*})$$

The term, $-C_{xv}x^{o}_{v^*}$, is necessarily negative by equations (8) and (10*). Using equations (5), (6), (10*), and (11*), it can be shown that $(R_{yx}x^{o}_{v^*} + R_{yy}y^{o}_{v^*})y^{o}_{v} < 0$ under perfect substitutes, unless C_{xx} is large. The sign of the term, $R_{y}(y^{o}_{vx}x^{o}_{v^*} + y^{o}_{vy}y^{o}_{v^*})$, depends on the third derivatives of the cost and revenue functions. It can be

3. Policy Effects

Let our prior assumptions hold when $t = 0$. Consequently, a stable free-trade R&D equilibrium can be obtained where each firm pursues a pure strategy.[6] This R&D combination must be optimizing for each firm; thus, equations (9) and (9*) are both satisfied. Given that $R_y y^o_v > 0$ and $R^*_x x^o_{v^*} > 0$, each firm uses more than the cost-minimizing R&D level.[7]

Since our assumptions hold at $t = 0$, it is necessarily true that a change in R&D produces a stronger impact on the marginal cost-reducing value of a given firm's R&D than on its strategic value. Alternatively, these effects may work in the same direction.
For appropriately small tariff rates, our assumptions must continue to hold. Thus, a pure-strategy equilibrium would remain viable at low tariff rates. Each firm would continue to overinvest in R&D, since

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negative under a variety of cost and demand assumptions; such a result increases in likelihood as C_{xvx} acquires a larger negative value. If $C_{xvx} < 0$, then domestic R&D is less effective in reducing marginal costs at lower levels of domestic output. By reducing the equilibrium level of domestic output, increases in foreign R&D can reduce the strategic value of domestic R&D as expressed through its associated output effects. For the case of linear demand, $\pi_{vv^*} < 0$ whenever $C_{xvx} \leq 0$. The sign of $\pi^*_{v^*v}$ can be analyzed in an analagous manner.

[6] See Brander and Spencer (1983).

[7] This result is demonstrated by Brander and Spencer, noting that cost minimization would require the following conditions:

$$-C_v(x,v) - a = 0$$
$$-C^*_{v^*}(y,v^*) - a^* = 0.$$

Since $R_y y^o_v > 0$ and $R^*_x x^o_{v^*} > 0$, each firm uses more than the cost-minimizing R&D level whenever $\pi_{vv} < 0$ and $\pi^*_{v^*v^*} < 0$.•

the terms, $R_y y^o_v$ and $R^*_x x^o_{v*}$, remain in the first-order conditions. Strategic R&D behavior persists under a tariff because an increase in R&D still causes a decline in rival output. Of course, the validity of our initial assumptions may also persist at large tariff rates.[8]

We now consider the imposition of a quota, q, at the free-trade import level (referred to as y_0). From this example, we can make general inferences concerning the differences in firm behavior under any quota and a comparably restrictive tariff. Figure 1 describes the change in R&D behavior associated with the quota. The original free-trade equilibrium is represented by Point 0; domestic R&D equals v_0 and foreign R&D equals v^*_0. As previously demonstrated, each firm chooses an R&D level in excess of that needed for cost-minimization. Initially, D_0 represents the domestic R&D reaction function while F_0 represents the foreign counterpart.

Consider the quota's effect on the foreign R&D reaction function. Figure 2, which shows the output equilibrium resulting from the free-trade R&D choices, is helpful to our analysis. The foreign output reaction function (FF_0) intersects the domestic output reaction function (DD_0) at Point 0, where foreign production equals the free-trade level. With an imposed quota at the free-trade level, the foreign reaction function becomes vertical at y_0. Further increases in foreign R&D lower marginal costs and shift the unconstrained portion of the reaction function outward. As evidenced in

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[8] For the foreign firm, the terms expressing the strategic value of R&D are always multiplied by (1 - t). As t assumes larger values, these terms would be expected to decrease in significance. Such behavior tends to reinforce the assumed second-order conditions concerning R&D.

FIGURE I

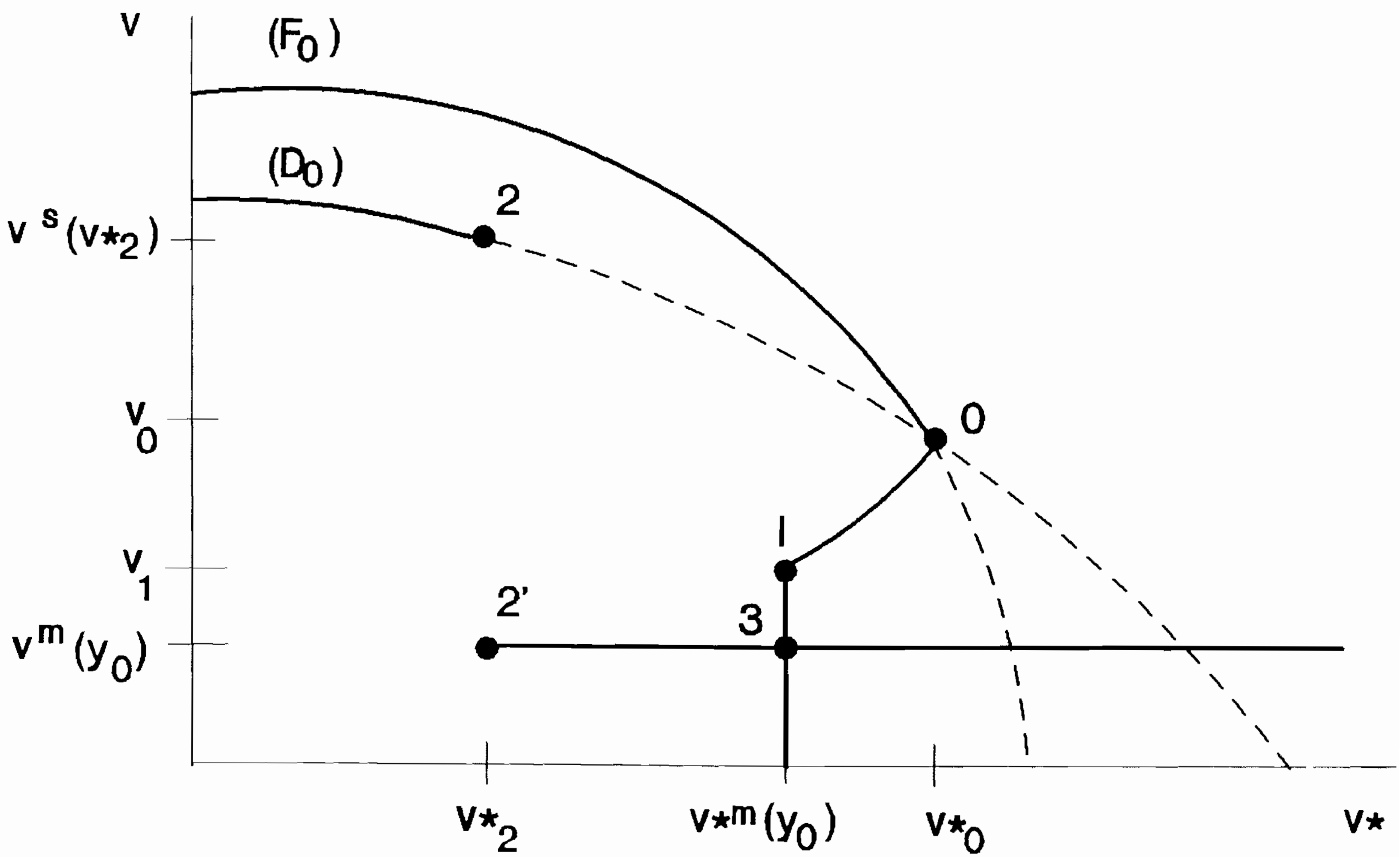

FIGURE 2

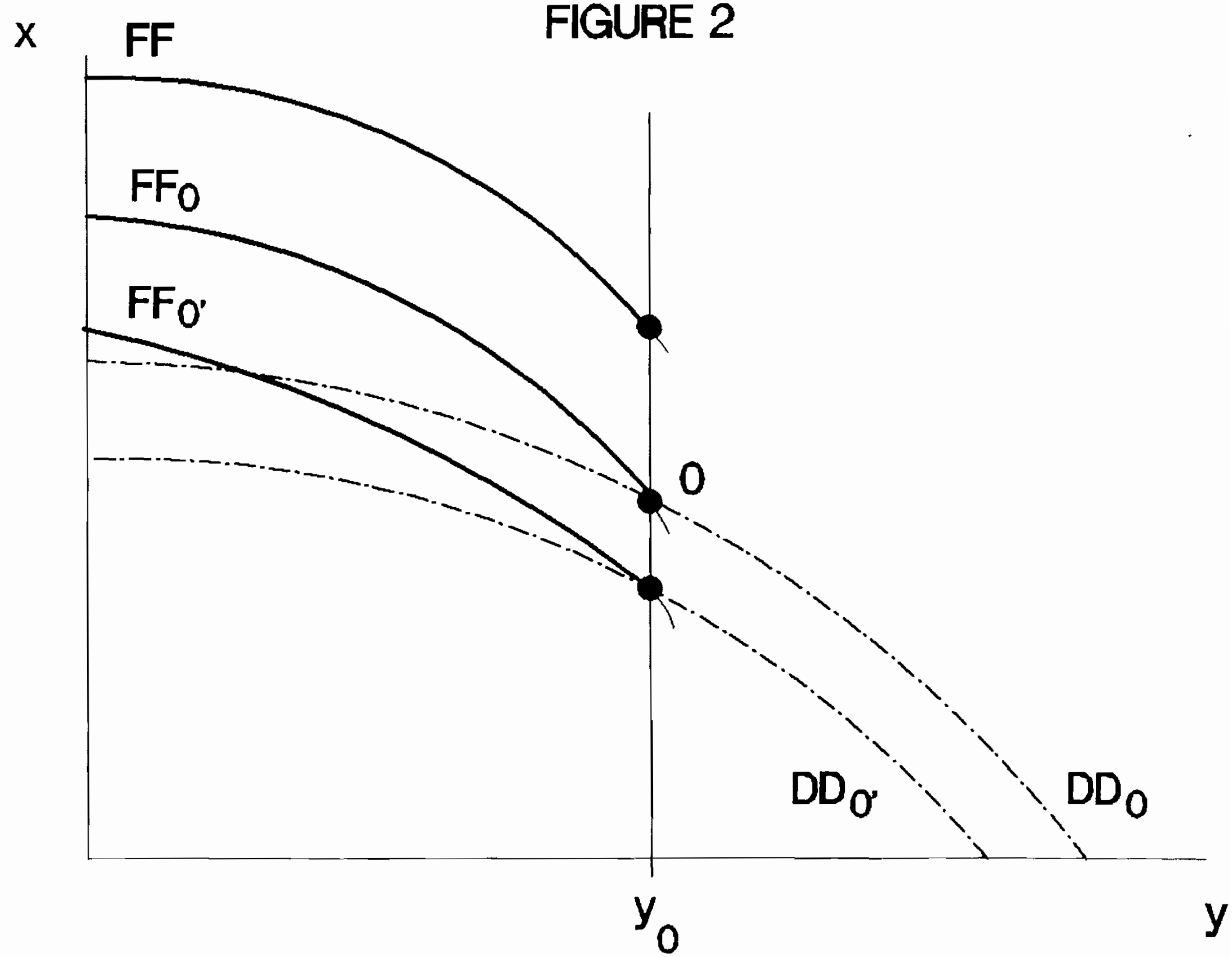

Figure 2, such a shift exerts no apparent influence on the cum-quota output equilibrium. Once the quota constraint is reached in the corresponding output equilibrium, foreign R&D ceases to possess any strategic value. This behavior creates a drop in π^*_{v*}, which can generally be described as follows:

$$
\begin{aligned}
\pi^*_{v*}(q,v,v^*) = \; & R^*_x(x^o(v,v^*);y^o(v,v^*))x^o_{v*} \\
& - C^*_{v*}(y^o(v,v^*),v^*) - a^*, \quad \text{if } v^* < v^{*q}(q,v) \\
= \; & -C^*_{v*}(q,v^*) - a^*, \quad \text{if } v^* > v^{*q}(q,v), \quad (16)
\end{aligned}
$$

where $v^{*q}(q,v)$ solves $y^o(v,v^*) = q$.

Notice that $v^{*q}(q,v)$ represents the level of foreign R&D needed to attain the quota level of output in an unconstrained equilibrium. If $q = y_0$ and $v = v_0$, then $v^{*q}(y_0,v_0) = v^*_0$. For $v^* < v^*_0$, $\pi^*_{v*}(y_0,v_0,v^*)$ assumes the same value as under free trade. Given that v^*_0 solves the foreign firm's first-order R&D condition in an unconstrained equilibrium, it is necessarily true that $\lim_{v^* \Rightarrow v^*_0{}^-} \pi^*_{v*}(y_0,v_0,v^*) = 0$. At the threshhold level of R&D represented by v^*_0, $\pi^*_{v*}(y_0,v_0,v^*)$ drops because strategic behavior is eliminated. Therefore, $\lim_{v^* \Rightarrow v^*_0{}^+} \pi^*_{v*}(y_0,v_0,v^*) < 0$. Our prior assumptions ensure that $\pi^*_{v*v*} < 0$, where defined.[9] If the domestic firm selects an R&D

[9] Under a quota, π^*, can be considered a combination of two continuous functions. One represents an unconstrained equilibrium; the other represents a constrained equilibrium where the foreign firm must produce at the quota level. In an unconstrained equilibrium, our original assumptions apply. Therefore, $\pi^*_{v*v*} < 0$. This result necessarily holds when $v^* < v^{*q}(q,v)$.

A constrained equilibrium applies when $v^* > v^{*q}(q,v)$. Since the foreign firm must always produce at the quota, foreign R&D

level of v_0, the optimal foreign response would remain at $v*_0$ in the quota **case**.

From equation (16), it is apparent that the behavior of

$\lim_{v* \Rightarrow v*^q(y_0,v)^-} \pi*(y_0,v,v*)$ is affected by our original assumptions

concerning $\pi*_{v*v}$ and $\pi*_{v*v*}$. Now assume that the domestic firm marginally reduces its R&D fom v_0 to $v_{0'}$. Referring to Figure 2, this change should shift the domestic output reaction function downward from DD_0 to $DD_{0'}$. In an unconstrained equilibrium, the foreign firm finds that the quota now becomes binding at a lower level of foreign R&D. We refer to this level as $v*_{0'}$, where $v*_{0'} = v*^q(y_0,v_{0'}) < v*_0$.[10] Under our original assumptions, $\pi_{vv} < 0$ and $\pi_{vv}* < 0$ for an unconstrained equilibrium. The reductions in both foreign and domestic R&D from their original levels imply that

$\lim_{v* \Rightarrow v*_{0'}^-} \pi*_{v*}(y_0,v_{0'},v*) > 0$.[11] By equation (16), $\pi*_{v*}(y_0,v_{0'},v*)$

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cannot influence either firm's output. As shown in equation (16), the value of R&D depends solely on its ability to reduce the cost of producing q units of output. Therefore, $\pi*_{v*v*} < 0$ if $C*_{v*v*} > 0$. This cost condition necessarily holds because it is less restrictive than the assumption expressed in equation (12*). In equation (12*), $C*_{v*v*} + C*_{yv*}y^o_{v*} > 0$, where $C*_{yv*}y^o_{v*} < 0$.

[10] More formally, equations (11) and (11*) can be used to express this change in foreign R&D:

$$dv*^q/dv = -y^o_v(v_0,v_{0*})/y^o_{v*}(v_0,v_{0*}) > 0$$

As domestic R&D declines, the amount of foreign R&D needed to reach the quota falls monotonically.

[11] For $v* < v*^q(q,v)$, our original assumptions imply that $\pi*_{v*v} < 0$. For $v* > v*^q(q,v)$, a constrained output equilibrium exists. The marginal value of foreign R&D depends solely on its cost-reducing ability, which is sensitive only to the level of foreign production. Since domestic R&D does not affect foreign output in a constrained equilibrium, we can assert that $\pi*_{v*v} = 0$.

must drop downward at $v^*_{0'}$. If $v_{0'}$ is sufficiently close to v_0, then $\lim_{v^* \Rightarrow v^*_{0'}{}^+} \pi^*_{v^*}(y_0, v_{0'}, v^*) < 0$.[12] Given that $\pi_{vv} < 0$, where defined, the foreign firm acts optimally by choosing $v^*_{0'}$ in response to the domestic R&D choice of $v_{0'}$. Thus, the domestic R&D reduction causes a decline in foreign R&D.

Further reductions in domestic R&D would result in continuing declines in foreign R&D, which creates the positively-sloped portion of the foreign R&D reaction curve in Figure 1. If the domestic firm's usage of R&D declines sufficiently, the foreign firm may find that the quota becomes binding at the R&D level needed for cost minimization.[13] Such behavior implies that $\lim_{v^* \Rightarrow v^{*q}(y_0, v)^+} \pi^*_{v^*}(y_0, v, v^*) = 0$. Let this condition be satisfied when $v = v_1$. For $v \le v_1$, the optimal foreign R&D choice must necessarily satisfy the following general condition:

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[12] The expression, $\lim_{v^* \Rightarrow v^{*q}(q, v)^+} \pi^*_{v^*}(q, v, v^*)$, applies to a a constrained output equilibrium. Equation (16) indicates that the marginal value of foreign R&D in a constrained equilibrium is represented by $-C^*_{v^*}(q, v^*) - a$, which is continuous in v^*. Since $v^{*q}(q, v)$ is continuous in v (footnote 10), the expression, $-C^*_{v^*}(q, v^{*q}(q, v)) - a$, is continuous in v. We can therefore assert that:

$$\lim_{v_{0'} \Rightarrow v_0^-} -C^*_{v^*}(y_0, v^{*q}(y_0, v_{0'})) - a = -C^*_{v^*}(y_0, v^*_0) - a < 0.$$

[13] The likelihood of this result depends on the assumptions concerning $C_x(x, v)$ as $v \Rightarrow 0^+$. As domestic R&D decreases, marginal production costs rise. The domestic output reaction function shifts inward. A sufficiently large inward shift implies that the quota becomes binding when foreign R&D equals (or is less than) its cost-minimizing level.

$$\pi^*_{v^*} = -C^*_{v^*}(q, v^*) - a^* = 0. \tag{17}$$

The foreign firm merely chooses the cost-minimizing R&D level for producing at the quota. In Figure 1, we represent this R&D choice by $v^{*m}(y_0)$. Since this choice is optimal for $v \le v_1$, the foreign R&D reaction curve becomes vertical (as shown in Figure 1).

We consider next the impact of the quota on the domestic R&D reaction function. Refer again to Figure 2, where Point 0 represents the output equilibrium corresponding to the free-trade R&D equilibrium. If a quota is imposed at the free-trade import level, foreign output is constrained until the domestic reaction function reaches DD_0. Further increases in domestic R&D shift the output reaction function upward, which renders the quota nonbinding. Domestic R&D therefore possesses strategic value, once it reaches a level sufficient to keep foreign output at q units in an unconstrained equilibrium. For a given choice of foreign R&D, let $v^q(q,v^*)$ represent this threshhold domestic R&D level. Based on our previous discussion, the imposition of a quota must alter π_v in the following manner:

$$\begin{aligned} \pi_v(q,v,v^*) &= -C_v(x^q(q,v),v) - a, && \text{if } v < v^q(q,v^*) \\ &= R_y(x^o(v,v^*), y^o(v,v^*))y^o_v \\ &\qquad - C_v(x^o(v,v^*),v) - a, && \text{if } v > v^q(q,v^*), \end{aligned} \tag{18}$$

where $x^q(q,v)$ solves $\pi_x = R_x(x,q) - C_x(x,v) = 0$.

$v^q(q,v^*)$ solves $y^o(v,v^*) = q$.

The emergence of strategic behavior implies that π_v takes an upward jump at the threshhold R&D level. If $q = y_0$ and $v^* = v^*_0$, then $v^q(y_0, v^*_0) = v_0$. For $v > v_0$, $\pi_v(y_0, v, v^*_0)$ assumes the same value as in an unconstrained equilibrium. Since v_0 represents an optimizing response to v^*_0 in an unconstrained equilibrium, we can assert that $\lim_{v \Rightarrow v_0^+} \pi_v(y_0, v, v^*_0) = 0$. However, the imposed quota causes π_v to jump upward at v_0, which implies that $\lim_{v \Rightarrow v_0^-} \pi_v(y_0, v, v^*_0) < 0$. Given that $\pi_{vv} < 0$ elsewhere, v_0 must now exceed the optimal domestic R&D choice under the quota.[14] This optimizing R&D response must satisfy the following general condition:

$$\pi_v(q,v) = -C_v(x^q(q,v),v) - a = 0 \tag{19}$$

The domestic firm now prefers to use its R&D solely for cost minimization, while the quota acts to restrain foreign output. In Figure 1, $v^m(y_0)$ represents the cost-minimizing R&D choice,

[14] To evaluate second-order conditions under a quota, consider that domestic profits essentially depend on two separate continuous functions:

$$\pi^1 = R(x^q(q,v),q) - C(x^q(q,v),v) - av,$$
$$\pi^2 = R(x^o(v,v^*),y^o(v,v^*)) - C(x^o(v,v^*),v) - av.$$

The first function represents a quota-constrained equilibrium, which applies if $v < v^q(q,v^*)$. The second function represents an unconstrained equilibrium, which applies if $v > v^q(q,v^*)$. For the unconstrained equilibrium, $\pi_{vv} < 0$ by our original assumptions. For the quota-constrained equilibrium, $\pi_{vv} < 0$ if the following condition is satisfied:

$$-[C_{vv}(x^q(q,v),v) + C_{xv}(x^q(q,v),v)x^q_v] < 0.$$

The above condition holds because it represents a less restrictive modification of equation (12).

where $v^m(y_0) < v_0$.

From equation (18), it is apparent that the behavior of

$\lim_{v \Rightarrow v^q(y_0, v^*)^+} \pi_v(y_0, v, v^*)$ is affected by our original assumptions concerning π_{vv^*} and π_{vv}. Assume that foreign R&D declines marginally from v^*_0 to $v^*_{0'}$. In an unconstrained equilibrium, less domestic R&D is now needed to keep foreign output at y_0. This new threshhold R&D level must necessarily equal $v_{0'}$, where $v_{0'} = v^q(y_0, v^*_{0'}) < v_0$.[15] Under our original assumptions, $\pi_{vv} < 0$ and $\pi_{vv}^* < 0$ for an unconstrained equilibrium. Due to the reduction in both foreign and domestic R&D from their original levels, we can assert that $\lim_{v \Rightarrow v_{0'}^+} \pi_v(y_0, v, v^*_{0'}) > 0$.[16] It must also be true that $\lim_{v \Rightarrow v_{0'}^-} \pi_v(y_0, v, v^*_{0'}) < 0$, if a sufficiently small drop in foreign R&D has occurred. Since $\pi_{vv} < 0$, two domestic R&D choices can potentially satisfy the first-order condition associated with equation (18).

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[15] Consider the equation, $y^o(v, v^{*qq}) = q$. For a given quota, this equation cannot be satisfied unless v^{*qq} rises monotonically as v increases (see footnote 9). Thus, the above equation can be inverted to yield the function, $v^{*qq} = v^{*q}(q, v)$. By another inversion, the following equation is derived:

$$v = v^q(q, v^{*qq}) = v^q(q, v^{*q}(q, v)).$$

For a domestic R&D choice of $v_{0'}$ and a quota of y_0, we can assert that $v_{0'} = v^q(y_0, v^*_{0'}) = v^q(y_0, v^{*q}(y_0, v_{0'}))$.

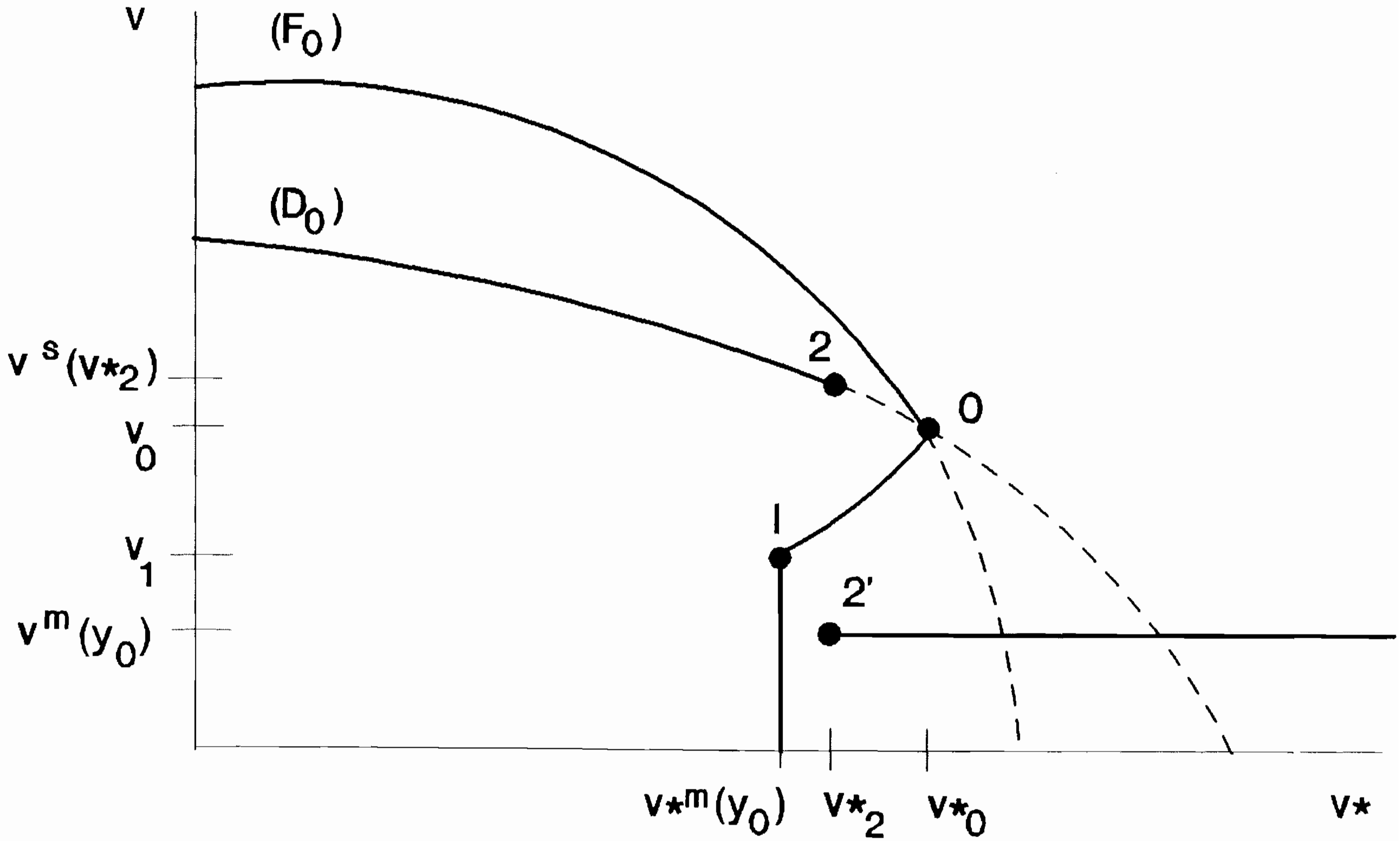
[16] For $v > v^q(q, v^*)$, our original assumptions ensure that $\pi_{vv^*} < 0$. For $v < v^q(q, v^*)$, the output equilibrium is quota-constrained, and the marginal value of domestic R&D depends solely on its ability to reduce direct production costs. The cost effectiveness of domestic R&D is influenced by the level of domestic output, but foreign R&D exerts no impact on domestic output in a constrained equilibrium. Consequently, $\pi_{vv^*} = 0$.

One choice is the cost-minimizing level, $v^m(y_0)$. The other choice represents the optimal R&D level when acting strategically. We shall refer to this latter response, which solves equation (9), as $v^s(v^*)$. This response is identical to that from the original domestic R&D reaction function.

If foreign R&D declines only slightly from v^*_0, then the domestic firm still prefers the cost-minimizing R&D choice instead of the strategic R&D choice. As foreign R&D continues to decline, the profits associated with the strategic R&D choice increase. However, the profits associated with the cost-minimizing R&D choice remain unchanged. Strategic behavior cannot occur unless foreign output is below the quota. When foreign R&D falls, foreign output continues to drop in an unconstrained equilibrium. This effect raises the profits associated with the domestic firm's strategic R&D choice. On the other hand, the output equilibrium arising from a cost-minimizing domestic R&D choice is always quota-constrained. Since changes in foreign R&D do not affect the output equilibrium arising from a binding quota, the use of a cost-minimizing strategy implies that domestic profits are unaffected by foreign R&D changes.

The previous discussion indicates that, at a sufficiently small level of foreign R&D, domestic profits may be the same from using either R&D choice. In Figure 1, this situation occurs when foreign R&D equals v^*_2. Any further drop in foreign R&D raises the profits

FIGURE 3



associated with a strategic domestic R&D choice. Thus, the domestic firm returns to its original R&D reaction function.[17]

The above analysis can easily be applied to any designated quota level, without changing the essential effects of this policy on each firm's R&D reaction curve. There are two potential equilibria which result from the imposition of a quota. One is the pure-strategy equilibrium shown at point 3 in Figure 1. Another is the mixed-strategy equilibrium which results from the situation depicted in Figure 3. From that diagram, an equilibrium occurs where foreign R&D equals v^*_2. In response, the domestic firm chooses $v^m(q)$ with probability p, and $v^s(v^*_2)$ with probability (1 - p).

We first consider the conditions associated with a pure-strategy equilibrium.

Proposition 1

Consider an import quota of q units. Let $v^{*m}(q)$ satisfy the following equation:

$$-C^*_{v^*}(q, v^*) - a^* = 0. \tag{17}$$

[17] The likelihood of this occurrence depends on the assumption concerning $C^*_y(y, v^*)$ as $v^* \Rightarrow 0^+$. Consider a situation where, at low levels of foreign R&D, the domestic firm can use less than a cost-minimizing R&D level and still restrict foreign output to q in an unconstrained equilibrium. If this situation exists, then the domestic firm necessarily reverts to its original R&D reaction function. When the variables, v^* and v, represent physical capital instead of R&D, the possibility of this outcome depends on the potential factor substitutability in production.

A pure strategy equilibrium occurs under the following condition:

$$\pi(q,v^m(q),v^{*m}(q)) \geq \pi(q,v^s(v^{*m}(q)),v^{*m}(q)) \quad (20)$$

where $v^m(q)$ satisfies $-C_v(x^q(q,v),v) - a = 0$ (19)

and $v^s(v^{*m}(q))$ satisfies $R_y[x^o(v,v^{*m}(q)),y^o(v,v^{*m}(q))]y^o{}_v$

$- C_v(x^o(v,v^{*m}(q)),v) - a = 0,$ (9)

This equilibrium is denoted by the R&D combination, $(v^m(q),v^{*m}(q))$.

Proof: Referring to Figure 1, a pure-strategy equilibrium can be attained if (and only if), the horizontal portion of the domestic reaction curve intersects the vertical portion of the foreign R&D reaction curve.[18] The foreign R&D choice necessarily satisfies equation (17) along the vertical portion of its reaction curve. If $v^{*m}(q)$ represents the solution to this equation, equation (18) establishes that either $v^m(q)$ or $v^s(v^{*m}(q))$ maximizes $\pi(q,v,v^{*m}(q))$.

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[18] Refer to Figure 1. The domestic reaction curve can never intersect the portion of the foreign reaction curve which lies between points 0 and 1. Along this portion of the foreign reaction curve, the foreign firm chooses $v^{*q}(q,v)$ in response to v. Let v′ refer to any domestic R&D level on this part of the curve. We will demonstrate that the combination, $(v',v^{*q}(q,v'))$, cannot lie on the domestic R&D reaction curve.

Footnote 14 establishes that $v^q(q,v^{*q}(q,v')) = v'$. The function, π_v, takes an upward jump at v′ when the foreign firm chooses an R&D level of $v^{*q}(q,v')$. Referring to equation (18), we can assert that the following condition holds:

$$\lim_{v \Rightarrow v'^-} \pi_v(q,v,v^{*q}(q,v')) < \lim_{v \Rightarrow v'^+} \pi_v(q,v,v^{*q}(q,v')).$$

Since $\pi_{vv} < 0$ (for $v \neq v'$), v′ can never represent an optimizing response to a foreign R&D choice of $v^{*q}(q,v')$.

The R&D choice, $v^m(q)$, lies along the horizontal portion of the domestic reaction curve. It cannot represent a maximum unless it corresponds to a quota-constrained equilibrium.[19] Thus, $v^m(q) < v^q(q, v^{*m}(q))$. Since $v^{*q}(q,v)$ is monotonically increasing in v, we can assert that the following condition holds:

$$v^{*q}(q, v^m(q)) < v^{*q}(q, v^q(q, v^{*m}(q)) = v^{*m}(q).$$

If the domestic firm chooses $v^m(q)$, then the foreign firm finds that the quota becomes binding before its R&D input reaches a cost-minimizing level. Given the properties of π_{v*} described in equation (16), $v^{*m}(q)$ must represent a best response to $v^m(q)$. Q.E.D.

Allow the foreign firm to choose an R&D level which minimizes the cost of producing q units of output. A pure-strategy equilibrium is attainable if the domestic firm's profit-maximizing response is to remain under the quota. This strategy would require that the domestic firm choose a cost-minimizing level of R&D, which leads to the following result:

Proposition 2

Assume that a quota is imposed at the free-trade import level.

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[19] If the R&D combination, $(v^m(q), v^{*m}(q))$, does not correspond to a quota-constrained equilibrium, then domestic R&D possesses a strategic value at $v^m(q)$. The optimal choice of domestic R&D must therefore consider its strategic value, implying that $v^s(v^{*m}(q))$ is necessarily the profit-maximizing response.

In a pure-strategy, cum-quota equilibrium, both domestic R&D and domestic output are lower than in the free-trade equilibrium. Also, the marginal cost curve for the domestic firm lies above that attained under free trade. Consumer surplus is relatively lower under the quota, because prices for both domestic and imported goods are relatively higher.

Proof: (See the appendix for a more formal proof.) Let y_0 represent the free-trade foreign output level. Our prior discussion has indicated that $v^m(y_0) < v_0$, where v_0 represents the free-trade domestic R&D level. In a pure-strategy, cum-quota equilibrium, the quota is necessarily binding. So, foreign output equals y_0 under both free trade and the quota. Given that $v^m(y_0) < v_0$, $C_{xv} < 0$, and $\pi_{xx} < 0$, the first-order condition expressed by equation (2) cannot be satisfied unless domestic output is relatively lower under the quota. Since both goods are substitutes, prices must necessarily be higher in the quota case. This result occurs because, in comparison to free trade, foreign output remains the same and domestic output falls.

The proof for Proposition 2 can be easily modified to demonstrate the following:

<u>Corollary 2.1</u>

Consider a quota and an equally restrictive tariff. In a pure-strategy, cum-quota equilibrium, both domestic R&D and domestic output are lower than in the cum-tariff equilibrium. Also, the marginal cost curve for the domestic firm lies above that attained under the tariff. Consumer surplus is relatively lower under the quota, because prices for both domestic and imported goods are relatively higher.

The use of a cost-minimizing strategy causes an improvement in productive efficiency while the quota is in effect. Such behavior leads to the following result:

Proposition 3

Assume that a quota is imposed at the free-trade import level. Domestic profits are higher in the pure-strategy, cum-quota equilibrium than in the free-trade equilibrium.

Proof: In equilibrium, foreign output is y_0 under both policies. The domestic firm would maximize its profits by solving the following problem:

$$\max_{x,v} \pi = R(x,y_0) - C(x,v) - av,$$

Recognizing that $C_x = C_x(x,v)$, the two first-order conditions are identical to those solved in equations (2) and (19). The domestic firm solves both of these equations in the pure-strategy, cum-quota equilibrium. In the free-trade case, the domestic firm chooses its R&D input by solving equation (9) instead of equation (19). Thus, domestic profits must be larger under the quota. Q.E.D.

The imposition of a quota raises domestic profits by eliminating strategic behavior in a pure-strategy equilibrium. The following corollary can be derived from a simple modification of the above proof:

Corollary 3.1

Domestic profits are relatively higher in a pure-strategy, cum-quota equilibrium than in a comparably restrictive cum-tariff equilibrium.

Now consider the case of a quota imposed above the free-trade level. Given that import restraints often allow for some growth, such an example may prove instructive. Our prior analysis of reaction function behavior still applies to this situation.

Proposition 4

Let condition (20) from Proposition 1 be satisfied with inequality. Consider the imposition of a quota, q, which exceeds the free-trade import level, y_0. A pure-strategy equilibrium arises as $q \Rightarrow y_0^+$. Therefore, a quota can be set above the free-trade level and still be binding. Compared to the free-trade equilibrium, the cum-quota equilibrium represents more foreign output and less domestic output and R&D. For a quota set marginally above the free-trade import level, domestic profits are necessarily higher than before.

Proof: See Appendix.

It is also easy to construct an example where a quota imposed above the free-trade level leads to two potential pure-strategy equilibria. One equilibrium occurs at the free-trade R&D combination. The other equilibrium occurs at the pure-strategy R&D combination associated with cost minimizing behavior under a binding quota constraint. Let (v_0, v^*_0) represent the free-trade R&D combination. A sufficiently large quota can be chosen to ensure that $v^{*m}(q) > v^*_0$. If $\pi(q, v^m(q), v^{*m}(q)) > \pi(q, v^s(v^{*m}), v^{*m}(q))$ and $v^m(q) > v_0$, then two potential pure-strategy R&D equilibria exist. We summarize this result below:

Proposition 5

For a quota imposed above the free-trade import level, two pure strategy equilibria may exist.

Proof: It is easy to show that $v^{*m}(q)$ rises with q, given that $C^*_{vq} = C^*_{qv} < 0$ and $C^*_{v^*v^*} > 0$ (by equations (8*) and (12*)). Since this relationship is monotonic, an appropriately large quota can be specified where $v^{*m}(q) > v^*_0$, the original foreign R&D level. By differentiation, we can also show the following:

$$d\pi(q, v^m(q), v^{*m}(q))/dq = R_y < 0$$

$$d\pi(q, v^s(v^{*m}(q)), v^{*m}(q))/dq = R_y y^o_{v^*}(dv^{*m}(q)/dq) < 0$$

Through an appropriate choice of technology and demand conditions, the latter equation can assume a larger negative value than the former equation. As q increases, a quota-constrained pure-strategy equilibrium must therefore become viable.

Furthermore, let v^*_2 represent the level of foreign R&D where the domestic firm earns equal profits from choosing $v^m(q)$ and $v^s(v^*_2)$. It can be shown that v^*_2 increases as q increases. Assume that foreign R&D is fixed at some given level. Since the choice of $v^m(q)$ always corresponds to a constrained output equilibrium, the profits from selecting this R&D level fall when the quota rises. However, $v^s(v^*)$ always corresponds to an unconstrained output equilibrium. The profits from this choice are unaffected by changes in the quota. As q increases, v^*_2 must eventually exceed v^*_0. When this situation arises, the domestic firm must prefer $v^s(v^*_0) = v_0$ in response to the foreign R&D choice, v^*_0. Thus, the free-trade R&D equilibrium also becomes viable. The possibility of two equilibria therefore exists at appropriately large values of q. Q.E.D.

As shown in the following proposition, qualitatively different R&D effects may result from imposing a quota instead of a tariff:

Proposition 6

The imposition of a quota slightly below the free-trade level reduces domestic R&D in a pure-strategy equilibrium while the imposition of a comparably restrictive tariff raises domestic R&D.

Proof: See Appendix.

We now turn to the possibility of a mixed-strategy equilibrium.

Proposition 7

For a quota imposed at the free-trade level, or below, a mixed-strategy R&D equilibrium occurs under the following condition:

$$\pi(q, v^m(q), v^{*m}(q)) < \pi(q, v^s(v^{*m}(q)), v^{*m}(q)) \quad (21)$$

where $v^{*m}(q)$ satisfies equation (17)

$v^m(q)$ satisfies equation (19)

$v^s(v^{*m}(q))$ satisfies equation (9)

Let v^*_0 satisfy $y^o(v^s(v^*), v^*) = q$. On the original domestic R&D reaction function, v^*_0 represents the level of foreign R&D where the optimal domestic R&D response implies that q is the foreign output level. In the mixed-strategy equilibrium, refer to the foreign firm's R&D choice as v^*_2. It must be true that $v^{*m}(q) < v^*_2 < v^*_0$. The domestic firm assigns a nonzero probability to two choices which represent the same profit level. One choice is $v^m(q)$, the cost-minimizing R&D choice in a quota-constrained equilibrium. The other choice, $v^s(v^*_2)$, forces output below the quota level.

Proof: See Appendix.

Since $v^m(q)$ is the same as the pure-strategy domestic R&D choice, some of our prior conclusions can be applied to the mixed-strategy equilibrium.

Proposition 8

Consider a quota imposed at the free-trade level, where the resulting equilibrium requires a mixed domestic strategy. When the domestic firm chooses $v^m(q)$ in the quota case, both domestic output and R&D are lower than in the free-trade equilibrium. When the domestic firm chooses $v^s(v^*_2)$, both domestic output and R&D are higher in comparison to free trade.

Proof: The proof to Proposition 2 establishes that, if the domestic firm chooses $v^m(q)$, both domestic output and R&D are lower in the cum-quota equilibrium than in the free-trade equilibrium. Since $v^s(v^*_2)$ solves equation (9), the R&D combination, $(v^s(v^*_2), v^*_2)$, lies on the domestic reaction function. Once again, let v^*_0 solve $y_o(v^s(v^*), v^*) = q$. From Proposition 7, we know that $v^*_2 < v^*_0$. Given that $\pi_{vv^*} < 0$ and $\pi_{vv} < 0$, we can assert that $v^s(v^*_2) > v^s(v^*_0)$. Thus, domestic R&D is higher and foreign R&D is lower than in the original equilibrium. Referring to equations (10) - (11*), these R&D changes must cause domestic(foreign) output to rise(drop) from its free-trade level. Q.E.D.

Proposition 8 indicates that, if the domestic firm chooses $v^s(v^*_2)$, domestic output is higher than in the free-trade equilibrium.

This situation adversely affects foreign profits. Since the foreign firm must consider a mixed domestic strategy, its R&D choice never represents an optimal response to any single domestic R&D choice. In contrast, the domestic firm uses a pure strategy in a free-trade equilibrium, and the foreign firm responds optimally. The following conclusion can be drawn from this behavior:

Proposition 9

Consider the imposition of a quota at the free-trade level. Even if the foreign firm can potentially keep any associated quota rents, foreign profits may be less in the mixed-strategy, cum-quota equilibrium than in the free-trade equilibrium.

Under a mixed-strategy equilibrium, the domestic firm earns the same profits from choosing either $v^m(q)$ or $v^s(v^*_2)$. The proof to Proposition 3 can be slightly modified to show the following:

Proposition 10

Consider a quota imposed at the free-trade level. Domestic profits are higher in the mixed-strategy, cum-quota equilibrium than in the free-trade equilibrium.

Corollary 10.1

Domestic profits are relatively higher in a mixed-strategy, cum-quota equilibrium than in a comparably restrictive cum-tariff equilibrium.

Refer to Figure 3, which correctly represents the mixed-strategy R&D equilibrium. From any starting point, let each firm use a pure strategy in responding optimally to its rival's R&D choice from the prior period. Inevitably, the dynamic process reaches the horizontal portion of the domestic R&D reaction function. From there, a four-period cycle commences. This behavior furnishes our final result:

Proposition 11

Consider a quota imposed at the free-trade level, or below. If equation (21) holds and each firm responds optimally to its rival's R&D choice from the prior period, then dynamic behavior shows convergence to a four-period orbit in R&D (and output) space.

4. Conclusion

Our results show that, by eliminating strategic behavior, a quota enhances productive efficiency. This effect implies that a domestic firm earns higher profits in a cum-quota equilibrium than in a comparable cum-tariff equilibrium. The gain in profits may be short-lived, however. When a pure-strategy equilibrium results, the imposition of a quota discourages domestic investment in capacity and R&D. Furthermore, the existence of a pure-strategy equilibrium may be eliminated by a quota, even in a multi-stage Cournot model.

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Appendix

Proof of Proposition 2

Let x_0 and y_0 represent the free-trade output levels for the domestic and foreign firm, respectively. In the associated free-trade R&D equilibrium, the domestic firm chooses v_0 and the foreign firm chooses $v*_0$. The domestic firm's output and R&D choices must satisfy the first-order conditions represented by equations (2) and (9):

$$R_x(x_0,y_0) - C_x(x_0,v_0) = 0 \qquad (2)$$

$$R_y(x_0,y_0)y^o{}_v - C_v(x_0,v_0) - a = 0 \qquad (9)$$

Now, consider a pure-strategy equilibrium for a quota imposed at the free-trade output level. Thus, $q = y_0$ and the quota is binding in equilibrium. The domestic firm must choose both output and R&D optimally, which implies that the following conditions are satisfied:

$$R_x(x',y_0) - C_x(x',v') = 0 \qquad (2')$$

$$-C_v(x',v') - a = 0, \qquad (19')$$

where $x'(v')$ = domestic output (R&D) choice under the quota.

We need to show that $v' < v_0$ and $x' < x_0$. Our assumptions require that $C_x = C_x(x,v)$, where $C_{xv} < 0$. Each R&D level determines a unique marginal cost function.

Assume that $v' = v_0$, which implies that $C_x(x,v') = C_x(x,v_0)$ for all values of x. Consequently, equations (2) and (2') cannot hold unless $x' = x_0$. Given this result, equations (9) and (19') cannot both be satisfied.

Now assume that $v' > v_0$. Since $C_{xv} < 0$, equations (2) and (2') cannot both be satisfied unless $x' > x_0$. Notice that v' represents the optimizing domestic R&D response under a quota-constrained output equilibrium. For any $v < v'$, a quota-constrained output equilibrium will also occur. We can therefore assert that the output combination, (x_0,y_0), is feasible under the quota. However, v' represents the profit-maximizing choice which satisfies equation (19'). Since $v' > v_0$ and $\pi_{vv} = -C_{vv}(x^q(q,v),v) - C_{vq}(x^q(q,v),v)x^q_v < 0$, the following result must hold:

$$-C_v(x_0,v_0) - a > -C_v(x',v') - a = 0.$$

The above condition implies that, if $v' > v_0$, then v_0 cannot represent the solution to equation (9).

Consequently, $v' < v_0$ and $C_x(x,v') > C_x(x,v_0)$. Equations (2) and (2') cannot hold simultaneously unless $x' < x_0$. Relative to the original equilibrium, domestic production drops while foreign production remains the same under the quota. If both goods are substitutes, the market cannot clear unless the prices of the foreign and domestic goods rise in the cum-quota equilibrium. Q.E.D.

Proof of Proposition 4

Consider a quota imposed at the free-trade level, $q = y_0$. If condition (20) from Proposition 1 holds with inequality, it must be true that:

$$\pi(y_0, v^m(y_0), v^{*m}(y_0)) > \pi(y_0, v^s(v^{*m}(y_0)), v^{*m}(y_0)) \qquad (20)$$

where $v^{*m}(y_0)$, $v^m(y_0)$, $v^s(v^{*m}(y_0))$ satisfy equations (17), (19), and (9), respectively.

By referring to equation (1) and our prior assumptions, we can assert that the domestic profit function is continuous in output and R&D. The Nash output equilibrium depends on the chosen R&D combination, which is a continuous relationship as described in equations (10) - (11*). After the quota is imposed, this functional relationship remains continuous everywhere but $v^q(q,v^*)$. So, $\pi(q,v,v^*)$ is locally continuous everywhere except $v = v^q(q,v^*)$. In Proposition 1, we established that $v^m(y_0)$ cannot be a global maximum unless $v^m(y_0) < v^q(y_0, v^{*m}(y_0))$. Since $v^s(v^{*m}(y_0))$ represents an optimal R&D response under strategic behavior, it must necessarily be true that $v^s(v^{*m}(y_0)) > v^q(y_0, v^{*m}(y_0))$. Thus, $\pi(y_0, v, v^{*m}(y_0))$ is locally continuous at both $v^m(y_0)$ and $v^s(v^{*m}(y_0))$.

Our prior assumptions can be used to show that $v^m(q)$, $v^q(q, v^{*m}(q))$, and $v^s(v^{*m}(q))$ are continuous in q. Due to continuity, we can assert the following:

$$\lim_{q \Rightarrow y_0^+} \pi(q, v^m(q), v^{*m}(q)) > \lim_{q \Rightarrow y_0 +} \pi(q, v^s(v^{*m}(q)), v^{*m}(q))$$

Thus, a pure-strategy equilibrium still exists for a quota imposed slightly above the free-trade level. The results in the proposition concerning output, R&D, and profits follow immediately from convergence properties and prior proofs. Q.E.D.

Proof of Proposition 6

Consider a quota imposed at the level, $q < y_0$. In a pure-strategy, cum-quota equilibrium, the domestic R&D choice must equal $v^m(q)$, where $v^m(q)$ satisfies equation (19). Since $v^m(q)$ is continuous in q, $\lim_{q \to y_0^-} v^m(q) = v^m(y_0) < v_0$.

To prove that a small, positive tariff increases domestic R&D, let equations (9) and (9*) can be expressed generally as $\pi_v(v,v^*,t) = 0$ and $\pi^*_{v^*}(v,v^*,t) = 0$. By total differentiation, we derive the following:

$$dv/dt = (-\pi^*_{v^*v^*}\pi_{vt} + \pi_{vv^*}\pi^*_{v^*t})/B,$$

where $B = \pi_{vv}\pi^*_{v^*v^*} - \pi_{vv^*}\pi^*_{v^*v} > 0$ (by equation (14)).

Total differentiation of equations (2) and (2*) leads to the following equations:

$$x^o_{v^*} = (C^*_{yv^*}/R^*_y)x^o_t$$

$$y^o_{v^*} = (C^*_{yv^*}/R^*_y)y^o_t$$

Using these results, we can differentiate equations (9) and (9*) to obtain the following:

$$\pi^*_{v^*v^*} = (C^*_{yv^*}/R^*_y)\pi^*_{v^*t} - C^*_{v^*v^*}$$

$$\pi_{vv^*} = (C^*_{yv^*}/R^*_y)\pi_{vt}$$

By substituting these equations into the expression for dv/dt, the following result is obtained:

$$dv/dt = C^*_{v^*v^*}R^*_y\pi_{vv^*}/C^*_{yv^*} > 0.$$

The assumption previously expressed in equation (12*) is sufficient to ensure that $C^*_{v^*v^*} > 0$. Therefore, dv/dt is positive in sign.

Proof of Proposition 7

Since $v^s(v^{*m}(q))$ satisfies equation (9), it only represents an optimal domestic R&D choice if the quota is not a binding constraint in the output equilibrium associated with the R&D combination, $(v^s(v^{*m}(q)),v^{*m}(q))$. By satisfying equation (17), the foreign R&D choice, $v^{*m}(q)$, represents an optimal foreign R&D choice if the quota is binding. Both of these requirements cannot be satisfied, so no pure-strategy equilibrium occurs at $(v^s(v^{*m}(q)),v^{*m}(q))$.

Let $(v_1,v^{*m}(q))$ represent an R&D combination on the cum-quota foreign R&D reaction function. Since this R&D combination must represent a quota-constrained equilibrium, it is necessarily true that $v_1 < v^s(v^{*m}(q))$. At $v^{*m}(q)$, the foreign R&D reaction curve lies below the domestic R&D reaction curve. This result is represented accordingly in Figure 3.

Let the foreign firm produce q units of output at the R&D combination, (v_0,v^*_0), on its original reaction curve. Prior discussion has indicated that the cum-quota foreign R&D reaction curve must be upward-sloping between v_1 and v_0. Furthermore, the optimal foreign R&D choice must be $v^{*q}(q,v')$, if $v_1 < v' < v_0$. As mentioned in footnote 17, no pure strategy equilibrium can occur at the R&D combination, $(v', v^{*q}(q,v'))$. Since π_v takes an upward jump at $v^q(q,v^*)$, the domestic firm would never choose $v^q(q,v^{*q}(q,v')) = v'$ in response to $v^{*q}(v')$. As shown in Figure 3, no pure-strategy equilibrium occurs at any point along the foreign R&D reaction curve between $(v_1, v^{*m}(q))$ and (v_0, v^*_0).

Next, consider domestic R&D behavior. At any given foreign R&D level, equation (18) establishes that the domestic firm necessarily maximizes its profits by choosing either $v^m(q)$ or $v^s(v^*)$. Without losing generality, we can assume that the R&D combination, (v_0, v^*_0), also lies on the original domestic reaction curve.[20] It has been previously confirmed that, after the imposition of a quota, $v^m(q)$ represents an optimal domestic response if $v^* \geq v^*_0$. Between $v^{*m}(q)$ and v^*_0, the optimal domestic R&D choice switches from $v^s(v^*)$ to $v^m(q)$. The profits from choosing $v^m(q)$ are unaffected by foreign R&D since a cost-minimizing R&D choice is only viable when it corresponds to a quota-constrained output equilibrium. If the profits from choosing $v^s(v^*)$ continuously decline as foreign R&D increases, then the domestic firm must be indifferent between choosing $v^m(q)$ and $v^s(v^*)$ at some level of foreign R&D, v^*_2. Given that $v^s(v^*)$ only represents an

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[20] The R&D combination, (v_0, v^*_0), lies on the original reaction functions of both firms only if the quota is set at the free-trade level. Let (v_{11}, v^*_{11}) represent the R&D combination on the original domestic reaction curve where the foreign firm produces q in equilibrium. Let (v_{22}, v^*_{22}) represent the analagous R&D combination on the original foreign reaction curve. If both reaction curves slope downward and the free-trade equilibrium is stable, then $v^*_{11} \leq v^*_{22}$ for any quota set at or below the free-trade level. No pure-strategy equilibrium can therefore occur between $v^{*m}(q)$ and v^*_{22}, because the cum-quota foreign reaction function is upward-sloping within this range.

optimal R&D response for an unconstrained output equilibrium, we can substitute $x^o(v^s(v^*),v^*)$ and $y^o(v^s(v^*),v^*)$ into equation (1) and differentiate with respect to v^*:

$$\pi_{v^*}(v,v^*) = R_y y^o_{v^*} < 0.$$

The above expression is negative in sign, and continuous by prior assumptions. So, the domestic profits from choosing $v^s(v^*)$ decline continuously as v^* rises. This result guarantees the existence of a foreign R&D level, v^*_2, where $\pi(q,v^m(q),v^*_2) = \pi(q,v^s(v^*_2),v^*_2)$. Notice that $v^{*m}(q) < v^*_2 < v^*_0$, as shown in Figure 3.

Consider a mixed domestic strategy. The domestic firm assigns probability p to the choice, $v^s(v^*_2)$, where $0 < p < 1$. It assigns probability $(1 - p)$ to the choice, $v^m(q)$. The foreign firm maximizes its expected profits, which can be expressed as follows:

$$\max_{v^*} E(\pi^*) = p\pi^*(v^s(v^*_2),v^*) + (1 - p)\pi^*(v^m(q),v^*).$$

Let v^*_3 represent the optimal foreign response to the domestic R&D choice, $v^m(q)$. Let v^*_4 represent its best response to $v^s(v^*_2)$. As evident in Figure 3, we can assert that $v^*_3 < v^*_2 < v^*_4$. Since $\pi^*_{v^*v^*} < 0$, where defined, it can also be stated that $d^2E(\pi^*)/dv^*dv^* < 0$ (where defined). If $v^*_3 < v^* < v^*_4$, then $d^2E(\pi^*)/dv^*dv^*$ and $dE(\pi^*)/dv^*$ are both continuous. Furthermore, $dE^2(\pi^*)/dv^*dp > 0$ and continuous. From these results, we can conclude that the optimal foreign R&D choice moves continuously from v^*_3 to v^*_4 as p increases. For some value of p, the foreign firm will choose v^*_2

and a mixed-strategy equilibrium thereby occurs.

Since $v^s(v^*_2)$ represents an optimal response under strategic behavior, the R&D combination, $(v^s(v^*_2), v^*_2)$, must represent an unconstrained output equilibrium. On the other hand, the R&D combination, $(v^m(q), v^*_2)$, must correspond to a quota-constrained equilibrium.