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The 1984 DOJ Merger Guidelines define geographic and product markets as an area and a group of products such that a cartel of suppliers in that area would find it profitable to raise the price by a small amount. Since the profitability of a price rise depends on the own-price elasticity of demand, the Guidelines implicitly define relevant markets in terms of this elasticity.[1] The precise value of the critical elasticity for market definition depends on several parameters, including the size of the price rise and its duration. It also depends on the underlying cost function. This paper shows how the critical demand elasticity can be expressed as a function of the supply elasticity, provides estimates of the critical demand elasticity for several price rules, and discusses statistical tests of market definition under this approach.

The Merger Guidelines define a market as a

> product or group of products and a geographic area in which it is sold such that a hypothetical, profit maximizing firm... would impose a "small but significant and non-transitory" increase in price...
>
> [1984 DOJ Merger Guidelines, section 2.0]

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[1] This point is developed by David T. Scheffman and Pablo T. Spiller, "Geographic Market Definition Under the Merger Guidelines," FTC Working Paper No. 129, August 1985.

Subsequently, the Guidelines stress that the market is the smallest area and group of products for which a price rise could be sustained:[2]

> In defining the geographic market or markets affected by a merger, the Department will begin with the location of each merging firm... and ask what would happen if a hypothetical monopolist of the relevant product at that point imposed a "small but significant and nontransitory" increase in price. If this increase in price would cause so many buyers to shift to products produced in other areas that a hypothetical monopolist producing or selling the relevant products at the merging firm's location would not find it profitable to impose such an increase in price, then the department will add the location from which production is the next-best substitute for production at the merging firm's location and ask the same question again. This process will be repeated until the department identifies an area in which a hypothetical monopolist could profitably impose a "small but significant and nontransitory" increase in price.
> [DOJ Guidelines Section 2.31; see Section 2.11 for product market definition]

The 1982 Guidelines proposed that the "small but significant and nontransitory increase in price" be interpreted as a five percent increase for a period of one year. The 1984 Guidelines abandoned this interpretation and now are silent on both the size and the duration of the hypothetical price rise. In the absence of a new standard, 1 year and 5% presumably remain as the benchmark for the price rise, although other threshold values might also be considered.

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[2] These two definitions are not fully consistent. The first asks whether a firm <u>would raise price by x%</u>. The subsequent definition asks whether an <u>x% price rise would be profitable</u>. The first definition implies a lower critical demand elasticity than does the second. For example, we note below that, if costs are constant and demand is linear with elasticity of 10, a firm would raise its price by 5%. However, as long as the elasticity is less than 20, the 5% price rise would be profitable (since the firm would be selling a positive output at price above its constant average costs). The "could" standard is clearly less likely to reject a proposed market than the "would" standard. In this analysis I focus on the "would" standard as it is the one which is consistent with profit maximization, and because we are generally interested in what firms will do, not what they can do. Note that the two definitions can be made to merge (approximately) by the choice of the price rule.

The profitability of a given price increase depends on the price elasticity of demand for goods in the specified market. If costs are constant, demand is linear, and the price initially is at the competitive level, a firm would raise price by 5% if the demand elasticity were equal to 10, and would raise price by 10% if elasticity were 5. Estimation of the elasticity of demand therefore provides a test of whether the hypothesized market is indeed a market for antitrust purposes.

This simple relation between critical elasticities and the price rule presupposes constant short run marginal costs. (I stress "short run" since, if the benchmark period is one year, the firm would presumably not make any significant changes in its fixed capital plant.) What is the critical elasticity of demand for market definition under the Merger Guidelines if cost is not constant? The answer is transparently simple if we know the parameters of the cost function. Even with limited information we can narrow the range on the critical demand elasticity. In particular, the critical demand elasticity can be approximated as a function of the supply elasticity.

Consider the usual model of costs, in which marginal costs decline over some region and then subsequently rise. The variable cost function is represented by $V(Q)$ and units are defined so that the competitive output Q_0 equals 1. (I ignore fixed costs as they do not affect the firm's decisions in the short run.) Competitive equilibrium is then $V'(1) = P_0$, and monopoly equilibrium is $V'(Q_1) = MR(Q_1)$, where $MR(Q)$ is the marginal revenue function.[3] We examine the monopoly equilibrium for two specifications of demand, linear and constant elasticity.

If we knew the parameters of the function $V(Q)$ we could solve for the monopoly output directly. Otherwise, as is the case here, we can approximate $V'(Q_1)$ by a linear extrapolation from $V'(1)$:

$$(1) \qquad V'(Q_1) \approx V''(1)\,(Q_1 - 1) + V'(1)$$
$$= V'(1)\,[(Q_1 - 1)/e_s + 1]$$

where e_s is the elasticity of the supply curve evaluated at $Q = 1$ and is equal to $V'(1)/V''(1)$.

If demand is linear,

$$(2) \qquad MR(Q_1) = P_1 + Q_1\, dP/dQ$$
$$= P_0\,(1 + u) + P_0\, Q_1 / e_d$$

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[3] The analysis envisions a market which initially is in competitive equilibrium and then, perhaps because of a merger, is reorganized to become a perfect cartel. However, there is a conceptual difficulty which always arises in analyzing a market as it moves from competition to collusion. Under competition, each firm has its own cost curve. Under perfect collusion, the firms act as if they have a single common industry cost curve. Here I will assume that all of the firms have identical cost curves. If this is true, then the industry cost curve corresponds to the horizontal summation of the individual cost curves and the relations between average variable cost and marginal cost are unaltered. That is, we can take a sample set of cost curves from one firm, and simply change the units on the horizontal axis so that now (nQ) -- where n is the number of colluding firms -- corresponds to an output which had been labeled Q. Equivalently, the elasticity of the industry supply curve at (nQ) equals the elasticity of any individual firm's marginal cost curve at Q. For our purposes, the assumption of identical firms means that we can take the cost function for an individual firm, change the measurement of units of output, and derive a market cost curve which, at marginal revenue = marginal cost, generates the same market output as does an analysis of the individual firms.

where u is the fraction by which the monopoly price exceeds the competitive price ($P_1 = P_0(1 + u)$) and e_d is the price elasticity of demand, evaluated at $Q = 1$. Equating $MR(Q_1)$ and marginal cost $V'(Q_1)$, and dividing both terms by $P_0 = V'(1)$, obtains

(3) $$1 + u + Q_1/e_d = (Q_1 - 1)/e_s + 1$$

Since

(4) $$\begin{aligned} e_d &= [(Q_1 - Q_0)/(P_1 - P_0)]\,[P_0/Q_0] \\ &= (Q_1 - 1)/u \end{aligned}$$

the above expression can be restated as:

(5) $$e_d^2 - 2e_s e_d - e_s/u = 0$$

(6) $$e_d = e_s - \sqrt{e_s^2 + e_s/u}$$

Alternatively, if demand is characterized by constant elasticity,

(7) $$MR(Q_1) = P_0\,(1 + u)\,(1 + 1/e_d)$$

Equating marginal revenue and marginal cost, and collecting terms, we can derive an indirect expression for e_d :

(8) $$e_s = (Q_1 - 1) \,/\, [(1 + u)(1 + 1/e_d) - 1]$$

$$(9) \qquad e_s = [(1 + u)^{e_d} - 1] / [(1 + u)(1 + 1/e_d) - 1]$$

since $Q_1 = (1 + u)^{e_d}$ if elasticity is constant.

Three points should be noted about the elasticities in equations (6) and (9). First, the demand elasticity is a negative number. To simplify the following discussion, we look at its absolute value $|e_d|$ when discussing its magnitude. (Strictly speaking, discussing the absolute value could cause some confusion when looking at the distribution of e_d since there may be a tail of the distribution such that $e_d > 0$.) Second, the supply elasticity is the supply elasticity for all the firms in the proposed market. (This analysis is not to be confused with, say, a dominant firm analysis, where we look at the supply elasticity for only a portion of the market, the fringe firms.) And third, as noted earlier, the supply elasticity is a short-run elasticity, reflecting the ability of firms to expand output within the one year time frame implicit in the Guidelines for market definition.

Table 1 lists (absolute) values of critical demand elasticities $|e_d^*|$ corresponding to a range of supply elasticities for three price rules. These values are computed from equations (6) and (9), with u set equal to the price rule. These values are not precise but rather tend to be smaller than the true value of the elasticity at which the cartel would raise its price by the percentage shown. That is, for a given combination of supply and demand elasticities, the cartel would raise its price by more than the percentage shown. This is because the linear approximation of $V'(Q_1)$ lies below the true value of $V'(Q_1)$ if $V''(Q_1) > 0$ (marginal cost is not linear but rather curves upwards), so the true intersection of marginal cost and marginal revenue corresponds to a lower output than that implied by the approximation.

Table 1 should be read as follows:

(i) First, select the appropriate supply elasticity, demand specification, and price rule.

(ii) If demand is less elastic than the value given by the table (that is, if $|e_d| < |e_d^*|$), the cartel would raise the price by at least the percentage shown. Hence, the proposed market is an antitrust market.[4]

(iii) If demand is more elastic than the value given by the table, the cartel probably would not raise the price by the full percentage shown. Hence, the proposed market is probably not an antitrust market.

Unfortunately, what seems to be a simple test in concept -- compare the actual demand elasticity e_d with the critical demand elasticity e_d^* -- is somewhat more complex in empirical applications. First, as described above, there is no single value of e_d^* for a given price rule; rather, the value of e_d^* varies with the corresponding supply elasticity e_s. Therefore, we must estimate supply as well as demand. Since we would normally estimate demand in the context of a simultaneous system, it may not be much extra work to estimate supply too.

Second, econometric testing of a hypothetical market will require a joint test on the estimated critical demand elasticity $|\hat{e}_d^*|$ (derived from $\hat{e}_s$)

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[4] If point (ii) is satisfied -- that is, the cartel would raise the price -- the relevant market may actually be narrower than the one we are looking at. For example, we might establish that a cartel of U.S. producers of widgets would raise price. What about west coast widget makers? The Guidelines indicate that we should next try narrowing the market, either by excluding products or firms as may be appropriate. Such a refinement may, however, be unnecessary. If concentration in the broad market (U.S. widgets) raises competitive concerns, then often concentration in the narrower market (west coast widgets) will be higher and raise even greater concerns.

and $|\hat{e}_d|$ to determine whether $|e_d| < |e_d^*|$ (or vice versa). If $\hat{e}_s$ is drawn from a normal distribution, $|\hat{e}_d^*|$ does not follow any nicely defined statistical distribution, so it is difficult to derive an analytic solution. The alternative approach, which is followed here, is numerical simulation.[5]

Suppose that our underlying model has constant elasticity supply and demand curves. How can we tell, with 95 percent confidence, whether $|e_d| < |e_d^*|$ (that is, whether the proposed market is an antitrust market)? Let x_d represent the 90th percentile of the distribution of $|\hat{e}_d|$, and let x_s represent the 10th percentile of $\hat{e}_s$, and then compute the value of e_d^* which corresponds to x_s and denote it by x_d^*.[6] (See Figure 1.) If $x_d \leq x_d^*$ then, based on our simulations, the proposed market is an antitrust market at the desired confidence level. Alternatively, if we define y_d as the 10th percentile of $|\hat{e}_d|$ and y_s as the 90th percentile of $\hat{e}_s$, and define y_d^* as the value of e_d^* which corresponds to y_s, then $y_d \geq y_d^*$ implies that the

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[5] The simulation assumes that $\hat{e}_d$ and $\hat{e}_s$ are drawn from a bivariate normal distribution, and compares $\hat{e}_d$ and $\hat{e}_d^*$ for each pair of sampled observations. The simulation results reported in the text appear to be robust over the range examined, which included all three price rules (5%, 10%, and 20%) and t-statistics from 2 to 8. A simulation program for an IBM-PC or compatible is available to those who are interested: given the supply and demand coefficents for a proposed market, their standard errors, and their correlation, this program computes the probability that the proposed market is an antitrust market. The program works for either a linear supply-demand specification or for a constant elasticity specification.

[6] Since $|e_d^*|$ is a positive montonic function of e_s over the relevant region, the 10th percentile of e_s corresponds to the 10th percentile of $|e_d^*|$.

proposed market is not an antitrust market at the 95 percent confidence level.[7]

These results are depicted in Figure 2, which plots $|\hat{e}_d|$ against $\hat{e}_s$ for a 10% price rule. The center line, labelled "no error," reflects the relationship between the supply elasticity and the critical demand elasticity when the value of these elasticities is known with certainty (the coordinates of points on this line correspond to numbers shown in Table 1, second column from the right). A combination of supply and demand elasticities with coordinates below this line indicates that a proposed market is an antitrust market; coordinates above this line indicates it is not a market.[8] The remaining lines in the figure delineate regions where one can say whether or not a proposed market is an antitrust market at a 95% confidence level when there is some error in the estimates of the elasticities. For example, the lowest and the uppermost lines correspond to t-statistics of 3 for both supply and demand elasticities. If the coordinates of the elasticities are below the lower line then it is a market, if they are above the upper line it is not,

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[7] The values of the critical percentiles were generated under the assumption that $\hat{e}_s$ and $\hat{e}_d$ are uncorrelated, so $\hat{e}_d$ and $\hat{e}_d^*$ are uncorrelated. The critical percentiles change slightly if the elasticities are correlated. For example, if the correlation coefficient between $\hat{e}_s$ and $\hat{e}_d$ is -0.5 (since $\hat{e}_d < 0$ this means that $|\hat{e}_d|$ and $\hat{e}_s$ are positively related) then the critical percentiles would be approximately the 87th and 13th for x_d and x_s (or y_s and y_d) instead of the 90th and the 10th. If the correlation coefficient is +0.5, then the critical percentiles are approximately the 92nd and the 8th.

[8] For a given value of e_s, the center "no error" line gives the corresponding critical demand elasticity $|e_d^*|$. For a point below this line, $|e_d| < |e_d^*|$, that is, the point represents an antitrust market. For a point above the line, the opposite holds.

and if they lie between the lines we cannot conclude whether it is or is not a market at the desired level of confidence.[9]

Overall, the procedure set out in this paper provides a fairly simple and workable test for an antitrust market (especially if one uses the simulation program described in n.5 above). However, it is hardly surprising that one generally needs good estimates of supply and demand elasticities in order to draw an inference about market definition with a high degree of confidence. In many situations, even good estimates will fall in the grey zone where one can neither accept nor reject the proposed market.

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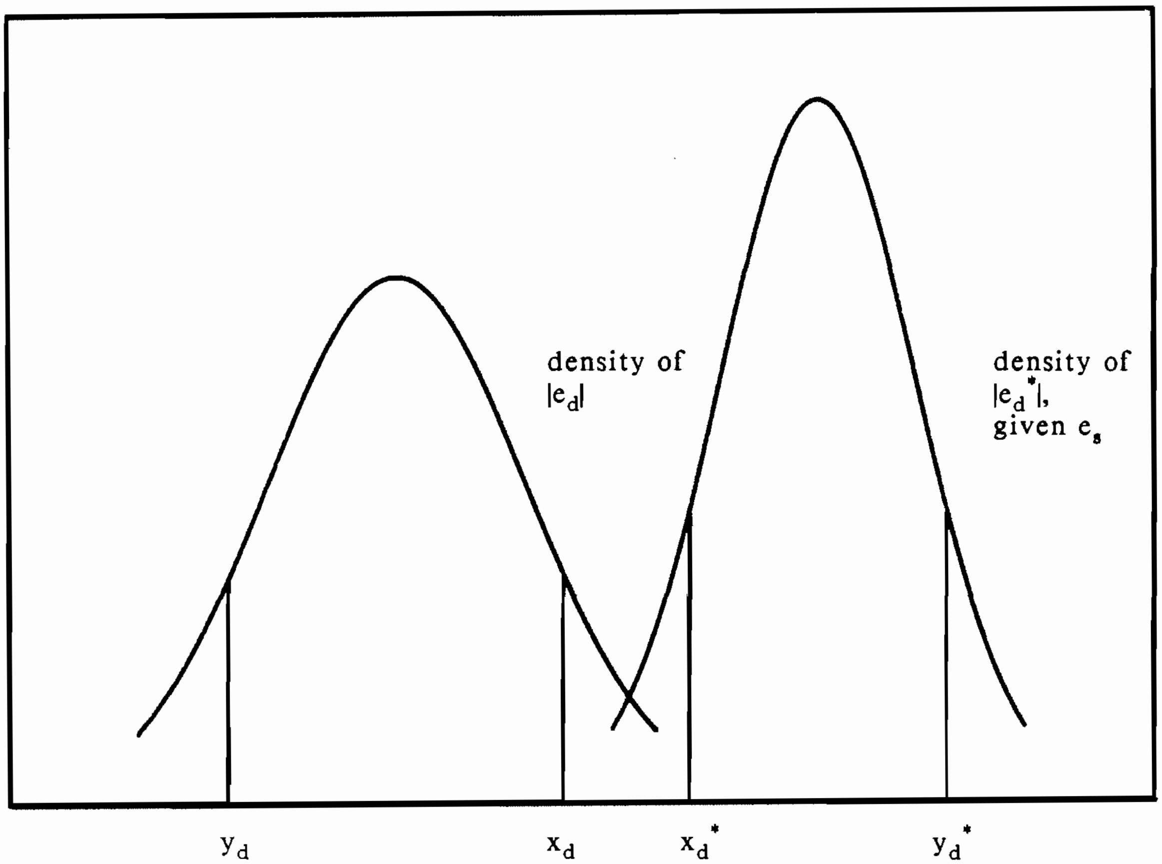
[9] The lines representing the 95% confidence bounds were computed in the following manner. Consider, for example, a point on the line labelled "lower bound, t=3." This point represents a hypothetical estimate of e_s and $|e_d|$, with standard errors of $e_s/3$ and $|e_d|/3$ respectively (so that the t-statistic of each is equal to 3 if the number of degrees of freedom is large). The 10th percentile of e_s (which is 1.28 standard deviations below its mean) and the 90th percentile of $|e_d|$ (1.28 standard devations above its mean) correspond to a point on the "no error" line. This implies that the 90th percentile of $|e_d|$ is below the 10th percentile of $|e_d^*|$ ($x_d < x_d^*$ in our earlier notation). Consequently, for a point on the "lower bound, t=3" line, there is 95% probability that $|e_d| < |e_d^*|$, and for a point below the line, there is an even greater probability. Points on or below this line, therefore, represent antitrust markets with at least 95% confidence.

TABLE 1: Critical Demand Elasticities at which a Cartel Would Raise Price, for Various Supply Elasticities

Supply Elasticity	Critical Demand Elasticity					
	linear demand			constant elasticity		
	5% rule	10% rule	20% rule	5% rule	10% rule	20% rule
1.0	3.6	2.3	1.4	4.4	3.1	2.2
2.0	4.6	2.9	1.7	6.0	4.2	3.0
3.0	5.3	3.2	1.9	7.1	4.9	3.4
4.0	5.8	3.5	2.0	8.0	5.4	3.8
5.0	6.2	3.7	2.1	8.8	5.9	4.0
∞	10.0	5.0	2.5	21.0	11.0	6.0

FIGURE 1

Comparison of Density Functions of Actual Demand Elasticity $|e_d|$ and Critical Demand Elasticity $|e_d^*|$ for Market Definition



Market: $|e_d| < |e_d^*|$

Not a market: $|e_d| > |e_d^*|$

x_d: 90th percentile of $|e_d|$

x_d^*: $|e_d^*|$, evaluated at 10th percentile of e_s

y_d: 10th percentile of $|e_d|$

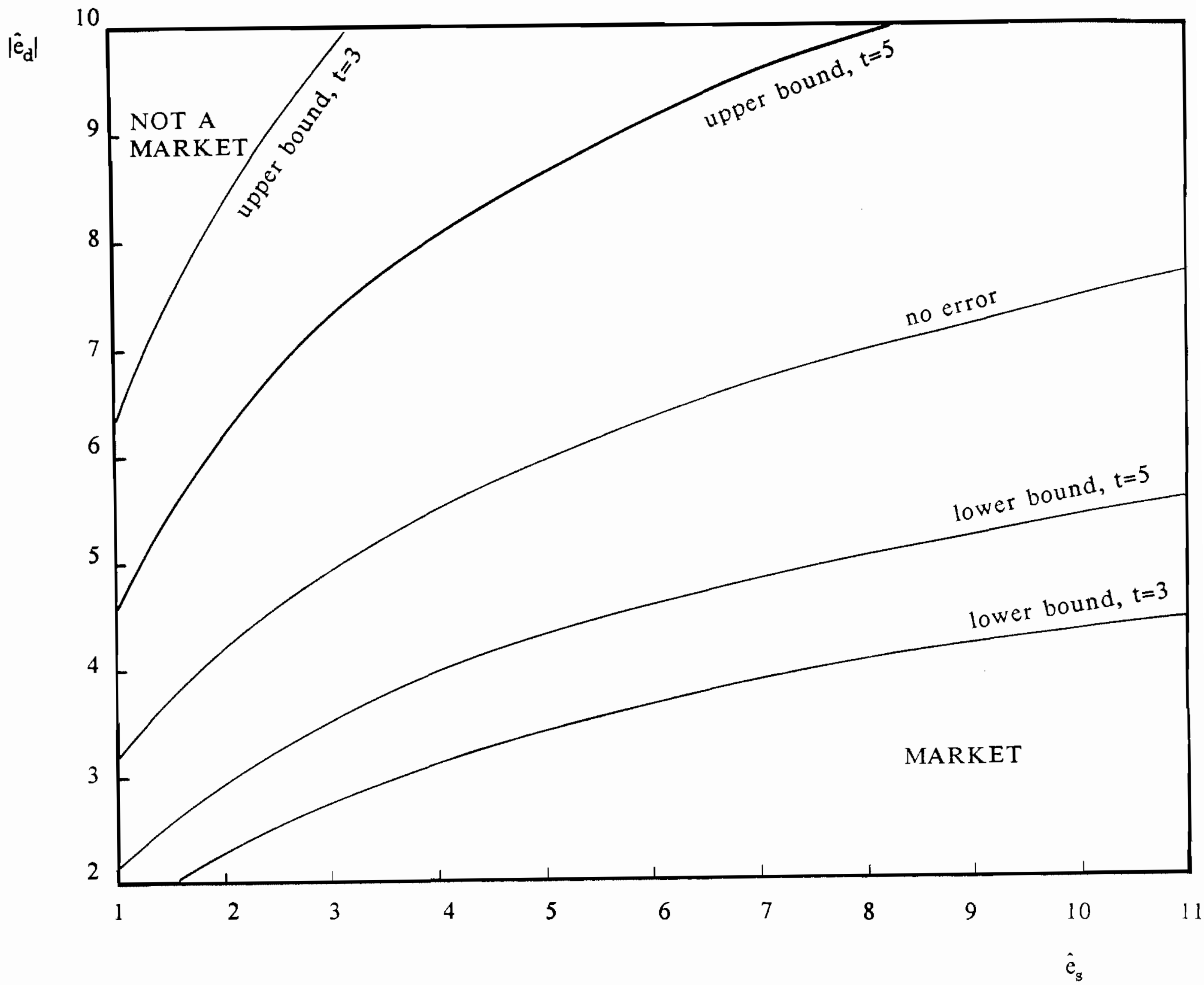
y_d^*: $|e_d^*|$, evaluated at 90th percentile of e_s

if $x_d < x_d^*$ (as shown), then $|e_d| < |e_d^*|$ with 95% confidence

if $y_d > y_d^*$, then $|e_d| > |e_d^*|$ with 95% confidence

FIGURE 2

Determining Whether a Proposed Market is an Antitrust Market at a 95% Confidence Level, Given Estimates of Demand Elasticity $|\hat{e}_d|$ and Supply Elasticity $\hat{e}_s$



Assumptions: 95% confidence level
10% price rule
Constant elasticity of demand
Elasticities are not correlated

It is a market if the coordinates of the elasticities lie below the lower line corresponding to the error in the estimates of the elasticities; it is not a market if the coordinates lie above the corresponding upper line.