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STACKELBERG INCENTIVE CONTRACT EQUILIBRIUM

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ABSTRACT

This analysis derives the optimal incentive contracts owners offer managers who engage in Stackelberg-quantity competition. In contrast to the Cournot case, the owner of the leading firm motivates his manager to strictly maximize profits and thereby gives no incentives for increased production. This results in a reversal of the usual Stackelberg outcome; output and profits for the leading firm are less than those of the follower's. In another reversal of the standard Stackelberg result, the leader's output and profits are lower compared to when outputs are chosen simultaneously whereas the follower's are greater. While the owner of the leading firm then wants his manager to engage in simultaneous quantity competition, the manager always chooses to be a leader irrespective of his incentive contract.

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INTRODUCTION

The separation of ownership and control commonly observed in modern corporations naturally arises from the comparative advantage managers have in the daily running of the firm relative to owners (shareholders). Owners being unspecialized in the management of the firm hire managers to do this for them. Because it is difficult for owners to closely monitor their manager's behavior, they resort to using compensation schemes in order to motivate the managers to pursue their objectives.

Fershtman and Judd [1987a] (hereafter F & J), Sklivas [1987], and Fershtman [1985] have shown the owner/manager relationship may also arise for strategic reasons. In a duopolistic context where firms simultaneously compete in output, they show that each owner motivates his manager toward high production to induce competing managers, who are aware of the other manager's incentives, to reduce their output. Thus, owners can increase their profits at the other firm's expense by strategically manipulating their own manager's incentive contract. In equilibrium both firms increase output and neither gains an advantage.

A firm may also increase its profits relative to another firm by producing first. Preemptive expansion of output enables the leading firm to affect the output choice of the firm which produces second (the follower). In the Stackelberg game (Stackelberg, [1934]), the leadership position confers an advantage upon the leader resulting in higher profits relative to the follower. Leadership naturally results when firms have a choice in their timing of production. Dowrick [1986] shows that the equilibrium outcome of an endogenous move game where two firms choose when to produce is for one firm to emerge as the leader.[1] The Cournot solution (that is firms producing simultaneously) is not a Nash equilibrium when firms can choose their period of production. Consequently, Stackelberg leaders may be more likely to arise and simultaneous quantity competition may not occur in a duopoly setting.

Stackelberg equilibria compares favorably with Cournot equilibria conceptually. With a general downward-sloping demand function, Robson [1990] proves under mild conditions that Stackelberg equilibria exists in pure strategies whereas Cournot equilibria need not exist (see McManus [1964] or Roberts and Sonnenschein [1977]).

Although incentive contracts and leadership individually confer advantages upon the firm, it is not clear whether in combination such an advantage would be sustained. For example, incentive contracts enable an owner to strategically act like a Stackelberg leader relative to his rival's manager. When his manager is the de-facto leader in a Stackelberg setting, the owners ability to strategically use incentive contracts to increase sales may become quite limited. Thus, we may expect significant differences in a model with sequential output and incentive contracts in comparison with the incentive contract equilibrium for the Cournot model.

This paper examines the equilibrium incentive contracts when Stackelberg-quantity competition takes place. It is shown that, in contrast to the Cournot-quantity game, the leader's owner will motivate his manager to strictly profit maximize, thereby providing no incentives for increased production. As a result, the manager of the leading firm acts like the usual Stackelberg leader. In contrast, the follower's owner provides greater incentives for increased sales. By motivating his manager to produce more output, he strategically induces the leader 's manager, who is aware of the rival manager's incentives, to produce less. The equilibrium results in a reversal of the usual Stackelberg outcome. When the firms' cost functions are comparable, the follower's output <u>exceeds</u> that of the leader's. Consequently, the leading firm's profits are strictly <u>less</u> than the follower's. The leader is worse off relative to the follower. Rather than confering an advantage, the ability to produce first has proven detrimental to the leader's owner. The leading firm's output and profits are also strictly <u>less</u> than its output and profits under Cournot competition with optimal incentive contracts, while the follower's are strictly greater.

Since the profits of the leading firm's owner would increase if his firm produced simultaneously instead of first, he has an incentive to induce his manager not to be a leader. Without exercising direct control over the firm he can only accomplish this through a properly structured incentive contract. By endogenizing the manager's choice of producing first (playing a Stackelberg game) or simultaneously (playing a Cournot game), we show that no incentive contract exists which can deter the manager from being a leader. In a fundamental way, the owner's and the manager's interests are in conflict.

THE MODEL

Our model is identical to F & J's [1987a] except for the sequencing of output. We adopt their model in order to assess the effect of leadership on their findings. Each of two firms, denoted 1 and 2, have an owner and manager. Firms produce homogeneous goods. The owner's objective is to maximize the profits of the firm. They observe only profits and sales and, not being specialized in the management of the firm, do not concern themselves with its day-to-day operation. Instead they hire managers to observe demand and cost conditions and make production decisions. Owners and managers are both assumed to be risk neutral. Manager i's compensation is determined by a contract whose incentive portion, O_i, takes the particular form of a linear combination of profits, π_i, and sales, S_i.[2] In general the manager's compensation is $M_i = A_i + B_i O_i$, where A_i, B_i are constants with $B_i > 0$. Being risk neutral, the manager maximizes O_i and the values of A_i and B_i become irrelevant. The relative weight owners induce managers to give to profits is represented by $\alpha_i \in \Re$, i = 1, 2. Thus firm i's manager will seek, by the appropriate choice of output, q_i, to maximize

$$O_i = \alpha_i \pi_i + (1 - \alpha_i) S_i, \quad i = 1, 2. \tag{1}$$

This is the same class of contracts specified in F & J.

The cost function of each firm, $C(q_i) = c_i q_i$, i = 1, 2, has constant marginal cost where c_i is the unit cost of firm i. Both firms' costs are common knowledge. We assume, for ease of exposition, that $c_1 = c_2 = c$. However, our results are robust to small cost differentials between the two firms. Like F&J, the inverse market demand, P(Q), is specified to be linear and uncertain.[3] It is represented by

$$P(Q) = a - bQ, \qquad b > 0, \ a > c \tag{2}$$

where P is market price and Q is industry output, b is stochastic with mean $\bar{b}$, and a is common knowledge. Using (2), the profit function of firm i is then

$$\begin{aligned} \pi_i &= P(Q)q_i - C(q_i) \\ &= (a - b(q_1 + q_2) - c)q_i \end{aligned} \tag{3}$$

and O_i becomes

$$O_i = (a - bQ - \alpha_i c)q_i. \tag{4}$$

In stage one both owners face unrealized demand (as b is unknown) and simultaneouly determine, by choosing α_i, the incentive structure of their manager's contracts that will maximize the owner's expected profits.[4] By choosing $\alpha < 1$ an owner gives his manager an incentive to increase sales and his manager will respond by raising output since this increases his income.

After stage one, both managers have common knowledge of their own and their rival's incentive contracts, α_1 and α_2, and the realized value of the parameter b.[5] Then each manager makes an output decision to maximize his income, O_i. In the Cournot case, as examined by F & J, q_1 and q_2 are chosen simultaneously. In the Stackelberg case, as analyzed here, q_1 is chosen by firm

1's manager afterwhich firm 2's manager chooses q_2. Price then adjusts to clear the market, the managers are paid and the owners receive thier profits. We will use F & J's terminology and call the stage one subgame-perfect equilibrium choice of α_i and the resulting outputs an incentive equilibrium.

INCENTIVE EQUILIBRIUM WITH STACKELBERG COMPETITION

In the Stackelberg game, one firm's manager is designated the leader and produces first. We arbitrarily pick firm 1 as the leader. Manager 2, knowing both managers' incentive contracts, (α_1, α_2), and q_1, chooses an output rate. It is found by maximizing O_2 with respect to q_2 to be:

$$\Psi_2^s(q_1,\alpha_1,\alpha_2) = \max\{(a - bq_1 - \alpha_2 c)/2b, 0\} \tag{5}$$

where $\psi_i(\cdot)$ is manager i's best response function and the superscript s designates Stackelberg. At the subgame perfect equilibrium firm 2 will then set $q_2 = \Psi_2^s(q_1,\alpha_1,\alpha_2)$. Given this fact and working backward to stage two, manager 1's optimal strategy is that output rate which solves

$$\max_{q_1}\left[a - b\left(q_1 + \Psi_2^s(q_1,\alpha_1,\alpha_2)\right) - \alpha_1 c\right]q_1.$$

This yields

$$\Psi_1^s(\alpha_1,\alpha_2) = \max\{(a - 2\alpha_1 c + \alpha_2 c)/2b, 0\}. \tag{6}$$

For values of α_1 and α_2 inherited from stage one, the resulting subgame perfect equilibrium is

$$\hat{q}_1^s(\alpha_1, \alpha_2) = \begin{cases} (a - 2\alpha_1 c + \alpha_2 c)/2b & \text{if } \alpha_1 < \Delta_1 \text{ and } \alpha_2 < \Delta_2 \\ (a - \alpha_1 c)/2b & \text{if } \alpha_1 < \Delta_1 \text{ and } \alpha_2 \geq \Delta_2 \\ 0 & \text{if } \alpha_1 \geq \Delta_1 \text{ all } \alpha_2 \end{cases} \tag{7}$$

$$\hat{q}_2^s(\alpha_1, \alpha_2) = \begin{cases} (a + 2\alpha_1 c - 3\alpha_2 c)/4b & \text{if } \alpha_2 < \Delta_2 \text{ and } \alpha_1 < \Delta_1 \\ (a - \alpha_2 c)/2b & \text{if } \alpha_2 < \Delta_2 \text{ and } \alpha_1 \geq \Delta_1 \\ 0 & \text{if } \alpha_2 \geq \Delta_2 \text{ all } \alpha_1 \end{cases}$$

$$\hat{\pi}_1^s(\alpha_1, \alpha_2) = \begin{cases} [a - c(4 - 2\alpha_1) + \alpha_2 c][a - 2\alpha_1 c + \alpha_2 c]/8b & \text{if } \alpha_1 < \Delta_1 \text{ and } \alpha_2 < \Delta_2 \\ [a - \alpha_1 c]^2/4b & \text{if } \alpha_1 < \Delta_1 \text{ and } \alpha_2 \geq \Delta_2 \\ 0 & \text{if } \alpha_1 \geq \Delta_1 \text{ all } \alpha_2 \end{cases} \tag{8}$$

$$\hat{\pi}_2^s(\alpha_1, \alpha_2) = \begin{cases} [a - c(4 - \alpha_2) + 2\alpha_1 c][a - 3\alpha_2 c + 2\alpha_1 c]/16b & \text{if } \alpha_2 < \Delta_2 \text{ and } \alpha_1 < \Delta_1 \\ [a - \alpha_2 c]^2/4b & \text{if } \alpha_2 < \Delta_2 \text{ and } \alpha_1 \geq \Delta_1 \\ 0 & \text{if } \alpha_2 \geq \Delta_2 \text{ all } \alpha_1 \end{cases}$$

The output of both firms is strictly positive when $\alpha_1 < \Delta_1 \equiv (a + \alpha_2 c)/2c$ and $\alpha_2 < \Delta_2 \equiv (a + 2\alpha_1 c)/3c$. The equilibrium incentive contracts will indeed satisfy these conditions.

Knowing the outcomes of stage two and three, firm i's owner chooses α_i to maximize his expected profit. The strategy for firm i's owner then solves

$$\underset{\alpha_i}{\text{Max}}\, E\left[\left(a - b\left(\hat{q}_1^s(\alpha_1,\alpha_2) + \hat{q}_2^s(\alpha_1,\alpha_2)\right) - c\right)\hat{q}_i^s(\alpha_1,\alpha_2)\right].$$

Using (7) and solving the first order condition of firm 1's owner for α_1, one obtains

$$\phi_1^s(\alpha_2) \equiv \hat{\alpha}_1^s = 1. \tag{9}$$

This says that the owner of the leading firm offers his manager the same contract regardless of which incentive contract is expected to be chosen by the other owner. In contrast, the following firm's owner's best response function does depend upon the other manager's contract and is given by

$$\phi_2^s(\alpha_1) = (6c - a - 2\alpha_1 c)/3c.$$

Using (9) we find owner 2's equilibrium contract to be:

$$\hat{\alpha}_2^s = (4c - a)/3c. \tag{10}$$

Our first result is that the leading firm will have its manager always maximize profits, giving no incentive for increased sales. This is irrespective of manager 2's contract. This contrasts sharply with the Cournot game of F&J where manager 1 was given an incentive to boost production, i.e. $\alpha_1 < 1$. In the Cournot game, owner 1 motivated his manager towards higher production in order to induce the competing manager, who is aware of these incentives, to reduce his output. Firm 2's owner pursued the same strategy resulting in a dual leadership where both owners encouraged their managers to sell more. Manager 2, in our model, is still given incentives to increase sales since $\hat{\alpha}_2^s < 1$. In fact, he is given even greater incentives to increase output compared to the Cournot game (superscript c) since $\hat{\alpha}_2^s < \hat{\alpha}_2^c = (6c - a)/5c$.

Substituting (9) and (10) into (7) and (8), the Stackelberg incentive equilibrium quantities, gross profits, and price are:

$$\hat{q}_1^{si} = (a - c)/3b \qquad \hat{q}_2^{si} = (a - c)/2b \tag{11}$$
$$\hat{\pi}_1^{si} = (a - c)^2/18b \qquad \hat{\pi}_2^{si} = (a - c)^2/12b$$
$$\hat{p}^{si} = (a + 5c)/6.$$

It is readily apparent from (11) that Theorem 1 is true.

THEOREM 1: The output and profits of the follower exceed those of the leader at the Stackelberg incentive equilibrium.[6]

This is a reversal of the usual Stackelberg outcome; the leader earns lower profits than the follower. We can explain this result as follows. When manager 1 moves first, he takes into account the strategic impact of his output on manager 2. Consequently, the ability to strategically influence manager 2's output decision has been fully exploited and the owner of firm 1 is unable to improve upon this. As a result, the owner finds he cannot use incentive contracts for strategic purposes and offers the profit maximizing contract: $\hat{\alpha}_1^s = 1$. Since the ability to choose a sales weighted incentive contract has become superfluous for the owner of the leading firm, the Stackelberg incentive game is then equivalent to a game in which only owner 2 chooses an incentive contract in stage one after which manager 1 produces in stage two and manager 2 produces in stage three. By communicating to manager 1 that his manager is going to emphasize sales, owner 2 induces manager 1 to cut back on production. This results in decreased profits for the leading firm and increased profits for the following firm. Thus, owner 2 acts as a leader relative to manager 1 and consequently acts as the overall leader of this equivalent game despite the fact that manager 1 produces first. Therefore, the true first-mover advantage lies with the owner of the following firm.[7]

COMPARISON OF STACKELBERG AND COURNOT INCENTIVE EQUILIBRIA

In the usual Stackelberg setting without incentive contracts, the increased output of the leader forces the follower to reduce his production below his Cournot output. This causes the leader's profits to exceed those he would receive in a Cournot game while the follower's profits fall below their Cournot level.

This result does not extend to Stackelberg and Cournot incentive equilibria. The firm output at the Cournot equilibrium is $(a - c)/2.5b$. At the Stackelberg incentive equilibrium, the leader's output, $(a - c)/3b$, is strictly less than this whereas the follower's equilibrium output, $(a - c)/2b$, strictly exceeds it. Thus the leader's profits, $(a - c)^2/18b$, fall below the Cournot incentive equilibrium profit of $(a - c)^2/12.5b$, while the follower's rises above it to $(a - c)^2/12b$.

THEOREM 2: The output and profits of the leader (follower) at the Stackelberg incentive equilibrium are strictly less (more) than at the Cournot incentive equilibrium.[8]

This again is a reversal of the usual Stackelberg result when incentive contracts are not used. A graphical explanation of this finding is given in Figure 1. With no incentive contracts the best response functions of firms 1 and 2 are ψ_1^c and ψ_2^c, respectively. In the Cournot incentive contract game, both owners choose an α_i less than unity which shifts their best response functions out to ψ_1^{ci} and ψ_2^{ci}. This causes the equilibrium to shift from A to B. Note that both firm's outputs have increased since managers act as if $\alpha_i c_i$ is the marginal cost of production.

In the standard Stackelberg game without incentive contracts, firm 1, the leader, preemptively expands its production beyond the Cournot level. This causes the follower, firm 2, to produce less than its Cournot equilibrium output resulting in the Stackelberg equilibrium C. When incentive contracts are introduced into the Stackelberg setting, the leader's owner offers his manager a profit maximizing contract so that there is no change in the best response function. Thus the leader's best response function in this game, ψ_1^{si}, is the same as his Cournot best response function, ψ_1^{c}. Since firm 2's owner chooses a lower value for α_2 in the Stackelberg game compared to the Cournot game, his manager has increased incentives to expand output causing the follower's best response function to shift out past ψ_2^{ci} to ψ_2^{si}. By virtue of his being a leader, firm 1 again produces at a point off his best response function ψ_1^{si} and Stackelberg incentive equilibrium occurs at D where firm 1's profits are lower than at the other equilibria. Firm 2 has more than completely offset firm 1's leadership advantage. Followers do better with incentive contracts than do leaders.

A comparison of the equilibria for different games is provided in Table 1. It is still true that price at the Stackelberg incentive equilibrium is below that at the Cournot incentive equilibrium, as is the case without incentive contracts. The Cournot incentive equilibrium price is $(a + 4c)/5$, whereas the Stackelberg incentive equilibrium price is lower at $(a + 5c)/6$. Efficiency is improved. Since Cournot incentive equilibrium quantities are higher and price and profits are lower relative to the usual Cournot equilibrium without incentive contracts, the Stackelberg incentive equilibrium represents a further increase in welfare relative to standard Cournot competition.

TABLE 1

Type of Competition	q_1		q_2	π_1		π_2	P
Cournot	(a-c)/3b	=	(a-c)/3b	$(a-c)^2/9b$	=	$(a-c)^2/9b$	(a+2c)/3
Stackelberg	(a-c)/2b	>	(a-c)/4b	$(a-c)^2/8b$	>	$(a-c)^2/16b$	(a+3c)/4
Cournot Incentive	(a-c)/2.5b	=	(a-c)/2.5b	$(a-c)^2/12.5b$	=	$(a-c)^2/12.5b$	(a+4c)/5
Stackelberg Incentive	(a-c)/3b	<	(a-c)/2b	$(a-c)^2/18b$	<	$(a-c)^2/12b$	(a+5c)/6

MANAGER'S EQUILIBRIUM BEHAVIOR

Far from being beneficial, the ability of the leading firm's manager to produce first has actually reduced the owner's profits. The owner, therefore, wants to induce his manager not to be the leader and instead produce simultaneously, i.e. engage in Cournot competition.

In Stackelberg competition, one firm is assumed to have the ability to lead. It is natural to expect, however, that the firm could choose to forfeit its leadership by electing to produce simultaneously in the period of the follower rather than lead. In this section, we model manager 1's choice of Cournot or Stackelberg competition by allowing him to produce first in stage two or delay his quantity decision to when the following firm produces in stage three (so that outputs are produced simultaneously).[9]

In our framework, an owner cannot force his manager to produce sequentially or simultaneously, but he can influence his behavior through the manager's incentive contract. The problem for the owner of firm 1 is then to construct an incentive structure that will motivate his manager to pursue Cournot competiton. Manager 1 will only produce simultaneously in stage three if his compensation in the Cournot game equals or exceeds that in the Stackelberg game. Unfortunately for the owner, he is incapable of achieving this objective.

Given (α_1, α_2) from stage one, let $\left(\hat{q}_1^c(\alpha_1,\alpha_2), \hat{q}_2^c(\alpha_1,\alpha_2)\right)$ be the equilibrium quantities when managers produce simultaneously. For our demand and cost assumptions, these quantities are solved for in F & J (see (13)). If manager 1 produces first, he can always choose to produce $\hat{q}_1^c(\alpha_1,\alpha_2)$. Since manager 2 will respond by producing $\hat{q}_2^c(\alpha_1,\alpha_2)$, manager 1's income from moving first is at least as high as that he obtains by waiting and moving simultaneously with manager 2. Thus, manger 1 always weakly prefers moving first. Furthermore, Theorem 3 shows that equilibrium must have manager 1 moving first.

THEOREM 3: In a duopoly where one manager can choose to either produce first or wait and produce simultaneously with the other firm, the incentive contract equilibrium always results in the manager producing first.

PROOF: Given (α_1, α_2), manager 1 will produce simultaneously if and only if

$$O_1^c(\alpha_1,\alpha_2) \geq O_1^s(\alpha_1,\alpha_2). \tag{12}$$

Using (4), the manager's compensation in the two games is defined as

$$O_1^c(\alpha_1,\alpha_2) = \left[a - b\left(\hat{q}_1^c(\alpha_1,\alpha_2) + \hat{q}_2^c(\alpha_1,\alpha_2)\right) - \alpha_1 c\right]\hat{q}_1^c(\alpha_1,\alpha_2)$$

$$O_1^s(\alpha_1,\alpha_2) = \left[a - b\left(\hat{q}_1^s(\alpha_1,\alpha_2) + \hat{q}_2^s(\alpha_1,\alpha_2)\right) - \alpha_1 c\right]\hat{q}_1^s(\alpha_1,\alpha_2).$$

From F & J we know

$$\hat{q}_1^c(\alpha_1,\alpha_2) = \begin{cases} (a - 2\alpha_1 c + \alpha_2 c)/3b & \text{if } \alpha_1 < \Delta_1 \text{ and } \alpha_2 < \Delta_2^c \\ (a - \alpha_1 c)/2b & \text{if } \alpha_1 < \Delta_1 \text{ and } \alpha_2 \geq \Delta_2^c \\ 0 & \text{if } \alpha_1 \geq \Delta_1 \text{ all } \alpha_2 \end{cases} \tag{13}$$

$$\hat{q}_2^c(\alpha_1,\alpha_2) = \begin{cases} (a - 2\alpha_2 c + \alpha_1 c)/3b & \text{if } \alpha_2 < \Delta_2^c \text{ and } \alpha_1 < \Delta_1 \\ (a - \alpha_2 c)/2b & \text{if } \alpha_2 < \Delta_2^c \text{ and } \alpha_1 \geq \Delta_1 \\ 0 & \text{if } \alpha_2 \geq \Delta_2^c \text{ all } \alpha_1 \end{cases}.$$

The Cournot incentive equilibrium output of both firms is strictly positive if $\alpha_1 < \Delta_1$ and $\alpha_2 < \Delta_2^c = (a + \alpha_1 c)/2c$. Five cases must be considered.

(i) $\alpha_1 \geq \Delta_1$ all α_2. Manager 1's Cournot and Stackelberg outputs are zero. Owner 1 can guarantee himself strictly positive profits in either game by choosing α_1 such that $q_1 > 0$. Therefore case (i) is not an equilibrium.

(ii) $\alpha_1 < \Delta_1$; $\Delta_2 < \alpha_2 < \Delta_2^c$. Here $\hat{q}_2^s = 0$ but $\hat{q}_2^c > 0$ and $\hat{q}_1^s, \hat{q}_1^c > 0$. Using (7) and (13), (12) becomes $[a - 2\alpha_1 c + \alpha_2 c]^2/9b \geq [a - \alpha_1 c]^2/4b$ which, for α_1, α_2 of this case, is never satisfied.

(iii) $\alpha_1 < \Delta_1$; $\Delta_2^c < \alpha_2 < \Delta_2$. In this case $\hat{q}_2^c = 0$ but $\hat{q}_2^s > 0$ and $\hat{q}_1^s, \hat{q}_1^c > 0$. The constraint (12) is then $[a - \alpha_1 c]^2/4b \geq [a - 2\alpha_1 c + \alpha_2 c]^2/8b$ and never holds here.

(iv) $\alpha_1 < \Delta_1$; $\Delta_2, \Delta_2^c \leq \alpha_2$. Then $\hat{q}_2^s = \hat{q}_2^c = 0$ and $\hat{q}_1^s, \hat{q}_1^c > 0$.. Since firm 1 is a monopolist in this case, the owner has no preference for Stage II or Stage III production.

(v) $\alpha_1 < \Delta_1$; $\Delta_2, \Delta_2^c > \alpha_2$. When the incentive parameters take on these values, there is an interior solution for both firms in the Stackelberg and Cournot games. In this instance (12) is $[a - 2\alpha_1 c + \alpha_2 c]^2/9b \geq [a - 2\alpha_1 c + \alpha_2 c]^2/8b$ and is never satisfied when the output of both firms is positive. Q.E.D.

While an owner prefers his manager not be a Stackelberg leader, he cannot induce his manager to delay output since there is a first-mover advantage in the manager subgame. Given any linear incentive contract, manager 1 will always produce first to the disadvantage of his owner. In principle, an owner could prevent this from occurring by modifying his manager's contract to include a penalty for producing first. The order of output could be ascertained from direct observation of each firm's production date or inferentially from sales.[10] Both cases require the owner to have extensive knowledge of the market. This familiarity is probably unlikely since it can only be attained by actively managing the firm, a task the owner hires a manager to do for him. In addition, it is not clear that one could specify such a penalty in a contract or that the contract would be enforceable if such a clause were specified.

CONCLUDING REMARKS

When one manager can preemptively commit to a rate of production, the follower will resort to incentive contracts to strategically offset the inherent disadvantage from producing second. This reverses the outcome of the standard Stackelberg setting. In Stackelberg incentive equilibrium the follower's production and profit exceed the leader's. In addition, the leader's output and profit's are less than at the Cournot incentive equilibrium. Industry output is greater and price and industry profits are lower compared to the Cournot incentive equilibrium of F & J. The combination of leadership and incentive contracts produces a first-mover disadvantage.

This raises questions concerning the motivations for having a firm be a leader. Our analysis, however, has focused only on the output decision of the firm. A firm may also compete with other firms through research and development, advertising, product differentiation, product quality, store location and other instruments. Strategic use of these decision variables may make it advantageous for a firm to produce first. Alternatively, it may simply be an institutional feature of an industry that one firm is the leader. Since expectations are fulfilled in equilibrium, this will be an industry equilibrium if all firms expect the firm to lead . Our model has revealed that being a leader may be disadvantageous. The leading firm, therefore, may want to change the institutional arrangement of the industry.

Others papers have also put into question the advantage of being a leader when firms compete in output. The follower is better off compared to the leader when a Stackelberg leader has private information about demand (Gal-Or [1987]). Mailath [1988] shows that the only equilibrium outcome for this game, where the leader can choose to lead or produce simultaneously, involves the leader moving first even when this results in profits lower than those of the follower. The disadvantage to leadership identified by these papers and our study suggest that leadership in an industry may be less likely to occur than preceding work would suggest.

REFERENCES

Dowrick, S., "von Stackelberg and Cournot Duopoly: Choosing Roles," *RAND Journal of Economics*, 1986, 17, 251-260.

Fershtman, Chaim, "Internal Organizations and Managerial Incentives as Strategic Variables in Competitive Environment," *International Journal of Industrial Organization*, 1985, 3, 245-53.

Fershtman, Chaim and Judd, Kenneth L., "Equilibrium Incentives in Oligopoly," *American Economic Review*, 1987a, 77, 927-940.

___________, "Strategic Incentive Manipulation in Rivalrous Agency," Hoover Institution Working Papers in Economics E-87-11, 1987b.

Gal-Or, Esther, "First Mover and Second Mover Advantages," *International Economic Review*, 1985, 26, 649-653.

___________, "First Mover Disadvantages with Private Information," *Review of Economic Studies*, 1987, 54, 279-292.

Mailath, George J., "Endogenous Sequencing of Firm Decisions," University of Pennsylvania mimeo, 1988.

McManus, Maurice, "Equilibrium, Number and Size in Cournot Oligopoloy," *Yorkshire Bulletin of Social and Economic Research*, 1964, 16, 68-75.

Roberts, John and Sonnenschein, Hugo, "On the Foundations of the Theory of Monopolistic Competition," *Econometrica*, 1977, 45, 101-113.

Robson, Arthur J., "Stackelberg and Marhsall," *American Economic Review*, 1990, 80, 69-82.

Sklivas, Steven D., "The Strategic Choice of Managerial Incentives," *RAND Journal of Economics*, 1987, 18, 452-458.

Stackelberg, H. von, "*Marktform und Gleichgewicht*," Berlin and Vienna: Springer, 1934.

FOOTNOTES

1. If both managers try to produce the Stackelberg leader's output simultaneously, Stackelberg warfare rather than Cournot competition results. A manager's Stackelberg warfare compensation is less than either Cournot, leader or follower compensation, but leader compensation strictly exceeds Cournot or follower compensation. The only equilibrium that can emerge, therefore, is one in which one manager is the leader. Given that one manager produces first, the other manager prefers to follow rather than engage in Stackelberg warfare. Given that one manager follows, the other manager will prefer to lead rather than produce simultaneously (Dowrick [1986]). An alternative derivation of this result involving costs to holding inventories is given by Robson [1990].

2. As F&J indicate, this formulation is moderately general in that it is equivalent to maximizing linear combinations of profits and costs or sales and costs.

3. Without uncertainty, there would be no justification for ruling out fixed quantity contracts that would force the usual Cournot and Stackelberg outcomes. Contracts linear in profits and sales are superior to contracts which yield the usual oligopoly outcomes when each owner wants his manager to react to the realization of an uncertain environment [F&J (1987a)]. Choosing b, a scale of market parameter, as the uncertain element simplifies the analysis since b does not enter into the owner's choice of α_1 or α_2 as is later seen in (8). If instead the parameter a were uncertain, our main results would not be altered.

4. The owners actually maximize expected profits net of their manager's opportunity costs. Managers have a common reservation wage, $\overline{M}$. Since managers are risk-neutral, hiring one requires that the owners only guarantee them $E[A_i + B_iO_i] = \overline{M}$, where $E[\cdot]$ is the expectation operator. Thus the cost of hiring a manager is fixed making the problem of maximizing expected profits net

of manager's opportunity costs equivalent to maximizing only expected profits. The O_i result from the owners optimal choice of α's and then the A_i's and B_i's are choosen to guarantee $\overline{M}$. For more discussion of this point see F & J.

5. Like F&J, we assume these contracts are common knowledge. Third party verification enables owners to credibly signal the contents of their manager's contract. The recently proposed SEC requirement that compensation packages be disclosed would enhance this verification. In addition, as we will show, an owner has an incentive to truthfully reveal his manager's incentive contract. For these reasons, we believe the assumption that α_1 and α_2 are commonly known is reasonable.

6. Even when the leader's costs are lower, the leader's profit can still be less than the follower's if the leader's costs are not too much lower. The analysis of the case when costs differ is available upon request.

7. If contract signals are cheap talk, owner 2 may have an incentive not to reveal α_2 truthfully. By contracting $\alpha_2 = 1$ but announcing the optimal Stackelberg follower's contract of $\alpha_2 = (4c - a)/3c$, owner 2 signals manager 1 to produce the Stackelberg incentive output of $(a - c)/3b$ while manager 2 is induced to choose $q_2 = (a - c)/3b$. The standard Cournot outcome results. Both firms are better off compared to both Stackelberg and Cournot incentive equilibrium and the first-mover disadvantage vanishes. This game of lying is not really implementable, however, since it requires manager 1 to believe the announced value of α_2 is truthful. If a mechanism of verification does not exist, manager 1 will in general suspect the reliability of owner 2's signal. Just as owner 2 knows owner 1 will choose $\alpha_1 = 1$ no matter what the value of α_2 is, so manager 1 knows that firm 2 benefits from deception. Thus manager 1, unless presented credible evidence to the contrary, will presume owner 2 is lying and infer $\alpha_2 = 1$. Based on $(\alpha_1, \alpha_2) = (1,1)$, manager 1 will produce $q_1 = (a - c)/2b$ and the standard Stackelberg

outcome will result. Since firm 2's profits will be less than those at the Stackelberg incentive equilibrium, it is in owner 2's interest to truthfully and credibly reveal α_2.

8. This result holds when firms have different costs so long as each firm's equilibrium output is positive.

9. George Mailath used the same extensive form to analyze the endogenous sequencing of firm decisions (Mailath,G. [1988]).

10. If the owner knew the parameter b when the market cleared, he could determine whether his manager had produced Cournot or Stackelberg output and punish accordingly.

Figure 1

Stackelberg, Stackelberg Incentive Contract, Cournot, and Cournot Incentive Contract Equilibria