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A Theory of Input Exchange Agreements

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In the U.S., substantial quantities of producer goods are often transacted via exchange agreements, rather than through spot markets, even when spot markets for the goods exist. An exchange is a simultaneous buy/sell arrangement generally involving equal amounts of each commodity. Joyce (1983) notes that markets in which exchanges are observed are characterized by the presence of vertical integration, high concentration in the input markets, and the presence of small non-integrated downstream producers. In this paper we consider the effects of input exchanges in models of markets with these characteristics.

Exchange agreements are common in markets for petroleum and many chemical and paper products.[1] For example, a typical agreement may involve an exchange of several thousand gallons of gasoline in one location for an equal quantity in another, with a monetary adjustment that compensates for transportation cost differences. Another example arises in the case of corrugated cardboard products, an industry that stimulated our interest in exchange agreements. The two primary inputs used to fabricate cardboard are corrugating medium and linerboard, which are used in approximately fixed proportions. Corrugating medium (or "medium") is the paperboard product that is exclusively used to construct the fluted middle layer of corrugated sheets of cardboard, and linerboard is the paperboard product used for the flat outer facings of such sheets. Although these inputs can be purchased in open markets, most of the large vertically integrated box producers are asymmetrically configured with respect to their production of linerboard and medium in regional markets, and use common exchange agreements to align their input production and needs.[2] The bilateral exchange agreements typically stipulate that inputs will be traded on an equal tonnage basis monthly for a specified period of time. The corresponding monetary adjustment that accounts for differences in monetary values of medium and linerboard is made at a rate that is based on independent assessments of prevailing (spot) prices reported in Official Board Markets, which is published by an independent price reporting service.

It is important here to distinguish between pure barter contracts and the exchange contracts that will be considered in this paper. In a barter arrangement, the parties trade commodities (or the same commodity in different locations), but they must negotiate the terms of the barter. Exchange agreements differ in that the ratio of physical units of the commodities is fixed (usually the same, e.g. tons or gallons, and firms engage in "exchanges" of equal quantities of the two commodities). The fixity of quantity ratios often requires the use of a monetary adjustment per unit of exchange to compensate for value differentials; this adjustment may be negotiated bilaterally, but typically it is based on representative transactions prices that are representative of market conditions. This paper contains a theoretical analysis of the competitive implications of exchange agreements.

A recent FTC trial concerned the acquisition of a West Coast medium plant by an integrated producer of boxes and other paper products.[3] The central issue in that case was whether the merger would increase the likelihood of collusion. The FTC Complaint Counsel argued that exchange agreements may facilitate collusion, i.e. the use of exchanges constitutes a "plus factor".[4] However, because exchanges affect vertical relationships between firms, it is possible that exchanges could also have procompetitive effects on industry performance (as is the case with many other vertical "restraints").

To see why an exchange agreement could enable box producers to restrict output, suppose that the inputs are used in fixed proportions (which closely approximates reality), and consider a local box market with two producers. Assume that one producer manufactures only medium and boxes; the other manufactures only linerboard and boxes. Suppose that neither producer is able to purchase inputs from producers outside the local market at a delivered price that makes such "outside" procurement profitable. Then, when the two producers agree on an exchange quantity they, in effect, agree on the quantity of boxes to be produced in that local market. Of course, a merger of the two producers would generate the same result without the need for exchange agreements. However, diseconomies of horizontal integration or antitrust regulation may preclude such action.

Exchanges could also have beneficial effects on market performance. Suppose that both vertically integrated producers specialize in the production of one input. If, in the absence of exchanges, non-produced inputs must be obtained at a price exceeding marginal cost, then firms' output production decisions will be based on marginal costs that exceed the marginal social cost of producing the output. Exchange arrangements may permit firms to consider and respond to the correct marginal social costs, if the monetary adjustment per unit of exchange accurately reflects the differential cost of producing the inputs being exchanged. Another potential efficiency-based motivation for exchange agreements is that they may economize on contracting and enforcement costs. The costs of negotiating contracts may be low for exchanges of equal physical quantities when the monetary adjustment is determined by a standard formula.[5] Exchanges may also reduce the costs associated with enforcing contracts. Oliver Williamson (1983) notes that an exchange is essentially a bilateral hostage-holding situation in which each party can punish the other for failure to deliver. But firms could also establish interdependence with two standard sales contracts, specifying, for example, quantities to be purchased per month at spot market prices.

This paper provides a rationale for the use of input exchange agreements and an analysis of the effects of such agreements on competition and economic efficiency. Section I presents the common structure of the models to be analyzed. Section II considers the situation in which all input transfers are made by exchange, and Section III contains an analysis of the equilibrium in the output and input markets in the absence of exchanges. In Section IV, the performance of markets with exchange agreements is compared with the performance of markets without exchange agreements. Finally, the analysis developed in Section V allows inputs to be acquired by negotiating bilateral exchanges and/or by engaging in unilateral market purchases of inputs. The concluding section contains a discussion of the antitrust implications of our analysis.

I. The Setting

To provide motivation for our analysis of exchanges, we present a simple example that illustrates Augustine Cournot's (1838) classic analysis of a duopoly, without exchanges, in which two inputs are used in fixed proportions to produce a homogeneous final product. Each input is produced by one firm. These two firms sell to firms in a competitive downstream industry that produces Q units of the final product. Since the two inputs are different products, Cournot assumed that the input producers choose prices independently. Cournot's main result can be illustrated graphically for the special case in which the inverse demand function, denoted f(Q), is linear, inputs are used in equal proportions, and all production costs (upstream and downstream) are zero. Units can be defined so that one unit of each input is required to produce one unit of the final product. Thus the competitive equilibrium price of the final product will equal the sum of the input prices charged by the upstream firms, and the sales quantity of each upstream firm will equal the output Q of the downstream industry.

In Figure 1 it is easily seen that when one firm selects a price of p_3, the other faces a demand function represented by the line segment DE in the graph with the point (0, p_3) as the origin. Obviously, the best response to a price of p_3 is for the other firm to select a price of p_1-p_3, which we have constructed to equal p_3. This is the unique, noncooperative equilibrium. Notice that the price that maximizes joint profits is p_2 which is <u>below</u> the noncooperative equilibrium price p_3, i.e. monopoly output is <u>above</u> noncooperative output.[6] This relationship provides the upstream firms with incentives to find a way to increase industry output. As we will see, the use of exchange agreements can have that effect.

As in the Cournot example, all firms in the models in this paper are assumed to behave noncooperatively.[7] We assume that all producers of inputs are vertically integrated into the production of output (boxes). There are two inputs that are used in fixed proportions to produce a homogeneous final product (e.g. boxes). The output quantity of firm i is denoted by q_i. The output-market inverse demand relationship is given by the function, p = f(Q), which is assumed to be strictly decreasing and twice continuously differentiable. We will also assume that industry total revenue, Qf(Q), is concave in Q. Thus industry marginal revenue in the market for the final product is a decreasing function of Q, which implies:

(1) $f''(Q)Q + 2f'(Q) \leq 0$.

The two inputs in our model are indicated by the letters b and m (e.g. liner<u>board</u> and <u>medium</u>), and the production of q_i units of the final product requires βq_i units of input b and μq_i units of output m. Inputs b and m are produced at constant marginal costs, c_b amd c_m, up to capacity levels that are denoted by $\bar{b}_i$ and $\bar{m}_i$ for firm i. Most firms in our models will be vertically integrated in the sense that each firm produces the final product and at least one of the inputs. Each firm that produces the final product (boxes) faces a variable cost associated with the fabrication of boxes, and the average fabrication cost is assumed to be constant and identical for all vertically integrated firms. The inclusion of such a cost is equivalent to the subtraction of a constant from the inverse demand function, f(Q), and therefore we will not introduce specific notation for fabrication costs.[8]

We assume that exchange agreements require that inputs be exchanged on an equal, unit-for-unit (e.g. ton-for-ton) basis, although units need not involve equal physical quantities.[9] The firm that provides input b pays $r to the other firm per unit exchanged, where r is the monetary adjustment rate which may be negative. The determination of r will be discussed later.

II. Input Specialization with Transfers Made by Exchange Agreements

In this section we will consider a simple market structure in which units of an input can only be acquired through bilateral exchange. This structure will be used to provide a benchmark for use in later sections when this assumption and others are relaxed.

Input Exchanges in Duopoly

It is convenient to begin with the special case of a duopoly in which both firms produce the final product but firm 1 only produces input b and firm 2 only produces input m.[10] Let e denote the exchange quantity; then firm 2 provides e units of input m to firm 1 in exchange for e units of input b and a payment of \$re, which can be negative. The production of inputs b and m by firms 1 and 2 respectively are denoted by b_1 and m_2. Since, by assumption, production of the output is characterized by fixed coefficients, the firms' outputs are determined: $q_1 = \min\{\beta(b_1 - e), \mu e)\}$ and $q_2 = \min\{\beta e, \mu(m_2 - e)\}$, if input capacity constraints are nonbinding. Productive efficiency requires that $b_1 = (\beta + \mu)e/\beta$, $m_2 = (\beta + \mu)e/\mu$, $q_1 = \mu e$, and $q_2 = \beta e$, and therefore, the total industry output of final product, Q, is $(\beta + \mu)e$. Profits (net of fixed costs) for each firm, denoted π_1 and π_2, can be expressed in terms of e:

$$\pi_1(e) = f(\beta e + \mu e)\mu e - \frac{(\beta + \mu)e}{\beta} c_b - re \ , \tag{2}$$

$$\pi_2(e) = f(\beta e + \mu e)\beta e - \frac{(\beta + \mu)e}{\mu} c_m + re \ , \tag{3}$$

where r represents the monetary adjustment rate.

Both firms must agree to any exchange, so we assume that the quantity exchanged will be the minimum of the exchange quantities preferred (i.e., that maximize a firm's profits) by the two firms. The assumed concavity of industry total revenue implies that both $\pi_1(\cdot)$ and $\pi_2(\cdot)$ are concave in e, and so if the resulting level of exchange, e*, is strictly positive, then it must be the case that $\pi_1'(e^*) \geq 0$ and $\pi_2'(e^*) \geq 0$, with equality holding for at least one.[11] The level of r will determine which firm's preferences are binding on the level of exchange.[12]

In this section we will consider exchanges with a monetary adjustment rate that exactly compensates for cost differences, i.e. $r = c_m - c_b$. The effects of deviations from this rate will be discussed later. When $r = c_m - c_b$, one can write the profit expressions in (2) and (3):

$$\pi_1(e) = \mu[f(\beta e + \mu e)e - \frac{ec_b}{\beta} - \frac{ec_m}{\mu}] \, , \tag{4}$$

$$\pi_2(e) = \beta[f(\beta e + \mu e)e - \frac{ec_b}{\beta} - \frac{ec_m}{\mu}] \, . \tag{5}$$

Because Q equals $(\beta + \mu)e$, the level of e that maximizes each firm's profit expression will result in a value of Q that satisfies:

$$f'(Q)Q + f(Q) - \frac{c_b}{\beta} - \frac{c_m}{\mu} = 0 \, , \quad \text{if } Q > 0. \tag{6}$$

Thus if $r = c_m - c_b$, firms' prepared levels of exchange are identical. This industry output that results from unanimity in bilateral exchange is the output for which industry marginal revenue, $f'(Q)Q + f(Q)$, is equal to $c_b/\beta + c_m/\mu$, which is the marginal cost of producing the final product. Thus bilateral exchange when r reflects cost differences will result in the output, denoted $\hat{Q}$, that maximizes industry profit. The use of exchanges in this case is equivalent to a merger of the two firms.

When $r = c_m - c_b$, industry output is maximized in the sense that any change in the level of r from this level will reduce the most preferred exchange quantity for at least one of the firms. To see this, note that the total sales revenues in the expressions for $\pi_1(e)$ in (2) and $\pi_2(e)$ in (3) are concave in e, and the coefficients of r in the costs are of opposite signs. Thus any change in r reduces either arg max $\pi_1(e)$ or arg max $\pi_2(e)$, and so any deviation in r from the level $r = c_m - c_b$ will reduce the minimum of the preferred exchange quantities.

Thus far we have assumed that the two firms specialize in different inputs. Input specialization may be required for production efficiency if there are diseconomies of horizontal integration. However, even if input specialization is not complete, i.e. each firm has some capacity to produce the "other" input, capacity constraints are likely to be binding on one of the inputs. Suppose that a firm has the capacity to produce only one input in desired quantities, and that it supplements its production of the other input through exchange. In this case, exchanges still control industry output at the margin, and the analysis of this section can be extended in a straightforward manner to show that neither firm would prefer a level of exchange that would raise the industry output above the collusive output determined by equation (6).[13]

Although exchanges usually involve equal physical units, this is not required in the previous analysis. A change in the units of measurement for one of the inputs would alter the ratio of physical quantities in our unit-for-unit analysis of exchanges, and it would alter the marginal cost and input coefficient for that input (since we have defined these to be the marginal costs and production coefficients for the unit of an input used in exchange). A change in one of the input coefficients would alter the firms' relative outputs and profits, since it is apparent from (4) and (5) that the ratio of firms' profits when $r = c_m - c_b$ depends on the relative magnitudes of μ and β. But regardless of the units of measurement, the use of exchanges with $r = c_m - c_b$ would yield the industry profit-maximizing output in this duopoly model. In other words, the division of profits depends on the ratio in which physical units of the two inputs are exchanged, but joint profits will be maintained if the monetary adjustment exactly compensates for differences between the costs of what each firm provides in its part of the exchange. Our analysis does not provide an explanation of how the profits are divided or of why exchanges typically involve equal physical quantities.

Criss-Crossed Bilateral Input Exchanges in an Oligopoly

Consider the n-firm case in which each firm only produces one of the inputs. The number of firms that only produce input b is denoted by k, and therefore we will index firms so that $\overline{m}_i = 0$, $i = 1,\ldots k$, and $\overline{b}_i = 0$, $i = k+1,\ldots n$. Each firm can maintain bilateral exchange agreements with any other firm. The notation e_{hj} indicates the level of exchange between firms h and j: firm h exchanges e_{hj} units of input b for an equal quantity of input m from firm j, where $h \in \{1,\ldots k\}$ and $j \in \{k+1,\ldots n\}$. As before, firm h pays an amount re_{hj} to firm j. It is convenient to let e_i denote the total quantity of exchange for firm i, $i = 1,\ldots,n$, and to let e denote the total quantity of each input that is exchanged by all firms in the aggregate, and in this sense e is the quantity of exchange. As in the previous section, it is straightforward to use production efficiency conditions to show that each firm's profits are identical functions of the industry output Q and the firm's own exchange level e_i when $r = c_m - c_b$. Let these functions be denoted by $\pi_i(Q,e_i)$, $i = 1,\ldots n$, where the i subscript on the $\pi(.)$ function is unnecessary.

A unit-for-unit exchange will allow the firm receiving input b to expand output by β units and it will allow the firm receiving input m to expand output by μ units. Therefore each firm recognizes that $\partial Q/\partial e_i = \beta + \mu$, and it follows from the symmetry of the $\pi_i(Q,e_i)$ functions that the total derivatives, $d\pi_i/de_i$, are identical functions of e_i. Hence, each firm prefers the same level of exchange between itself and its exchange partners. The actual level of e will be determined by the number of firms producing input b if $k \leq n/2$ and by the number of firms producing input m if $k \geq n/2$. Let $s \equiv \min\{k, n-k\}$. Then there are s firms for which $d\pi_i(Q,e_i)/de_i = 0$, and the actual exchange quantity, e, is equal to the sum of the exchange quantities, e_i, for these s firms. It is straightforward to show that the summation of the respective sides of the first-order conditions for these s firms can be expressed:

$$(7) \qquad f'(Q)Q + s[f(Q) - \frac{c_b}{\beta} - \frac{c_m}{\mu}] = 0 \ .$$

In the duopoly case, $n = 2$, $k = 1$, $s = 1$, and (7) reduces to the monopoly condition (6) that industry marginal revenue equal industry marginal cost, as was shown previously. If $s > 1$, the level of Q determined by (7) exceeds the monopoly level. If the numbers of firms producing the two inputs are equal, $s = n/2$, it is straightforward to show that (7) is an equation that characterizes the industry output in a Cournot equilibrium with n/2 firms that each produce output at a constant marginal cost of $c_b/\beta + c_m/m$.[14] Since producers of b (of m) prefer low (high) values of r, any change in r from the level $c_m - c_b$ will reduce the desired exchange quantities of one class of input producer, so the cost-compensating adjustment rate maximizes industry output. The obvious generalization, which summarizes the results of this section, is:

Proposition 1

If there are k firms specializing in the production of input b and n-k firms specializing in the production of input m, and if $r = c_m - c_b$, then the industry output of the final product that results from the exclusive use of exchange agreements is equal to the industry output in a Cournot equilibrium with $\min\{k, n - k\}$ firms, each with constant average costs of $c_b/\beta + c_m/\mu$.

This proposition has important implications for how concentration should be calculated in an output market in which inputs are traded only through exchanges.[15] For example, if $\mu = \beta$, so that firms' outputs are equal, the market with exchanges behaves as if there were $\min\{k, n - k\}$ identical firms rather than n identical firms in the output market, so the relevant Herfindahl measure of concentration would be $10000/[\min\{k, n - k\}]$ rather than 10000/n. Thus the effect of exchange agreements is to effectively change the level of concentration in the downstream market, and so exchanges should be viewed as having structural effects rather than as being simply plus factors in merger analysis. Another interesting implication of the proposition is that exchanges yield the perfectly collusive outcome if either input is monopolized.

III. Input Specialization With No Exchanges

In order to evaluate the effects of input exchange agreements, it is necessary to consider a model in which exchanges are not used. As in the previous section, we begin with the duopoly case and assume that firm 1 only produces input b and firm 2 only produces input m. Both firms also produce the final product. Without exchanges, each firm must purchase the input that it does not produce from the other firm. Input b is sold at a price p_b, which is set by firm 1, and input m is sold at a price p_m, which is set by firm 2. We assume that firms set their prices noncooperatively, given correct expectations of the effects of changes in the price of an input on the other firm's input purchase and output production quantities.

It is useful to consider the analysis of the (subgame-perfect) equilibrium in two stages. We will begin with the "second-stage" analysis of the equilibrium levels of input purchases for given values of the input prices, p_b and p_m. Afterwards, we will consider the "first-stage" determination of noncooperative (Nash) equilibrium input prices given that firms correctly anticipate the actions taken in the second stage.

Input Purchase Decisions

In order to produce q_1 units of output, firm 1 must purchase q_1/μ units of input m at a total cost of $p_m q_1/\mu$. Similarly, firm 2 purchases q_2/β units of input b at a total cost of $p_b q_2/\beta$. Each firm is the only supplier of its own input in the market so firm 1 incurs the cost of producing Q/β units of input b and firm 2 incurs the cost of producing Q/μ units of input m. Thus, firms' profits, net of fixed costs, can be expressed as functions of output quantities:

(8) $$\pi_1 = f(Q)q_1 - c_b Q/\beta - p_m q_1/\mu + p_b q_2/\beta ,$$

(9) $$\pi_2 = f(Q)q_2 - c_m Q/\mu - p_b q_2/\beta + p_m q_1/\mu .$$

Given the other firm's input price, each firm must choose its own input purchase quantity, which uniquely determines its own output quantity because of the fixed output/input coefficients. Since firms commit themselves to output levels when they simultaneously choose input purchase quantities, we consider a noncooperative (Cournot) equilibrium in the second stage game in which p_m and p_b are given and the decisions are quantities. It follows from the concavity of industry total revenue (1) and the convexity of costs that the firms' profit expressions are concave functions of their own output decisions. Thus a noncooperative equilibrium with positive outputs will satisfy:

(10) $$f'(Q)q_1 + f(Q) - c_b/\beta - p_m/\mu = 0 ,$$

(11) $$f'(Q)q_2 + f(Q) - c_m/\mu - p_b/\beta = 0 .$$

Equations (10) and (11) implicitly determine the input purchase (and output) quantities as functions of the input prices.

Input Price Decisions

The firms' first-order conditions (10) and (11) for the second stage simultaneously determine the effects of changes in either input price on both input purchases and output quantities, effects which we assume to be correctly anticipated in the first stage. A straightforward comparative-statics analysis of (10) and (11) yields the following "cross effects" of input price changes

$$(12) \qquad \frac{\partial q_1}{\partial p_m} = \frac{1}{\mu} \cdot \frac{f''(Q)q_2 + 2f'(Q)}{f'(Q)[f''(Q)Q + 3f'(Q)]} \, ,$$

$$(13) \qquad \frac{\partial q_2}{\partial p_b} = \frac{1}{\beta} \cdot \frac{f''(Q)q_1 + 2f'(Q)}{f'(Q)[f''(Q)Q + 3f'(Q)]} \, ,$$

where the partial derivative notation indicates that the other input price is held constant. It follows from (1) (the assumed concavity of industry total revenue) that the numerators in both comparative statics derivatives above are negative. It also follows from (1) and from the negativity of $f'(Q)$ that the denominators are both positive, and therefore, both derivatives are negative. Finally, the symmetric structure of the expressions for these derivatives implies that

$$(14) \qquad \mu \frac{\partial q_1}{\partial p_m} = \beta \frac{\partial q_2}{\partial p_b} \quad \text{for any } (p_b, p_m) \text{ if } q_1 = q_2 \, .$$

This symmetry result will be useful in the analysis of a symmetric equilibrium.

Input prices are determined noncooperatively, so that the Nash equilibrium input prices, if they are positive, must satisfy the conditions: $d\pi_1/dp_b = 0$ and $d\pi_2/dp_m = 0$. In (8) and (9), π_1 and π_2 are expressed as functions of q_1, q_2, p_b and p_m. Since the second stage maximization requires that $\partial\pi_i/\partial q_i = 0$, i = 1, 2, it follows that

$$\text{(15)} \qquad \frac{d\pi_1}{dp_b} = [f'(Q)q_1 - c_b/\beta + p_b/\beta]\frac{\partial q_2}{\partial p_b} + q_2/\beta = 0 \text{ , and}$$

$$\text{(16)} \qquad \frac{d\pi_2}{dp_m} = [f'(Q)q_2 - c_m/\mu + p_m/\mu]\frac{\partial q_1}{\partial p_m} + q_1/\mu = 0 \text{ .}$$

The term in square brackets in (15) is the profit margin on sales of $1/\beta$ units of b, generalized to include the output effect, $f'(Q)q_1$, on the final-product sales revenue for firm 1.[16] Recall that the cross derivative in (13), $\partial q_2/\partial p_b$, is negative. If $q_2 > 0$, equation (15) implies that the generalized profit margin for sales of input b is positive:

$$\text{(17)} \qquad f'(Q)q_1 - c_b/\beta + p_b/\beta > 0 \text{ ,}$$

so that $p_b > c_b$. Analogously, it is easily shown that the generalized profit margin for $1/\mu$ units of input m is positive:

$$\text{(18)} \qquad f'(Q)q_2 - c_m/\mu + p_m/\mu > 0 \text{ ,}$$

so that $p_m > c_m$.

The equilibrium levels of the four endogenous variables, p_b, p_m, q_1, and q_2, are determined by the two equilibrium conditions for second stage, (10) and (11), and by the two equilibrium conditions for the first stage, (15) and (16), after using (12) and (13) to eliminate the input price derivatives in (15) and (16).

We will first show that these equations must have a symmetric solution with $q_1 = q_2$. Subtracting the left side of (11) from the left side of (10), we obtain:

(19) $f'(Q)q_1 + (p_b - c_b)/\beta = f'(Q)q_2 + (p_m - c_m)/\mu$.

Thus, the generalized profit margin for $1/\beta$ units of input b must equal the generalized profit margin for $1/\mu$ units of input m. Since the production of a unit of output requires $1/\beta$ units of input b and $1/\mu$ units of input m, equation (19) requires that the output-equivalent profit margins be the same for each input. This result occurs because each firm faces identical demand conditions in the downstream market.

To complete the proof that $q_1 = q_2$, use (15), (16) and (19) to show that:

(20) $$\frac{\beta}{q_2}\frac{\partial q_2}{\partial p_b} = \frac{\mu}{q_1}\frac{\partial q_1}{\partial p_m} \quad .$$

Next substitute the comparative-statics derivatives, (12) and (13), into (20), multiply both sides of (20) by q_1q_2, and the resulting equation can be expressed: $f''(Q)(q_1)^2 = f''(Q)(q_2)^2$, which proves that $q_1 = q_2$.

It follows from (19) that $(p_b - c_b)/\beta = (p_m - c_m)/\mu$ when $q_1 = q_2$, so the ordinary (ungeneralized) output-equivalent profit margins are equal. In this unique, symmetric equilibrium, the four equilibrium conditions [(10), (11), (15), and (16)] that determine p_b, p_m, q_1, and q_2 will reduce to two equations that determine the input price markups and the common output per firm. The resulting industry output for the duopoly without exchanges will be denoted by Q^*.

Consider now the downstream profit margin of the two firms, $f(Q) - p_b/\beta - p_m/\mu$ (ignoring fabrication costs). This expression is a profit margin since the opportunity cost of input b (input m) for the downstream subsidiary of firm 1 (firm 2) would be the "transfer price" of p_b (p_m). To obtain an expression for this profit margin, sum (10) and (11) to obtain:

$$(21) \qquad f'(Q)Q + f(Q) - c_m/\mu - c_b/\beta = -[f(Q) - p_b/\beta - p_m/\mu] \ .$$

Let Q^* denote the equilibrium industry output that results, without exchanges, so equation (21) must hold at $Q = Q^*$. Since the left sides of (17) and (18) are positive at Q^*, their sum must also be positive, yielding

$$(22) \qquad f'(Q^*)Q^* - c_b/\beta - c_m/\mu + p_b/\beta + p_m/\mu > 0 \ .$$

By adding $f(Q^*) - p_b/\beta - p_m/\mu$ to both sides of (22), one obtains

$$(23) \qquad f'(Q^*)Q^* + f(Q^*) - c_b/\beta - c_m/\mu > f(Q^*) - p_b/\beta - p_m/\mu \ .$$

It follows from (21) and (23) that $[f(Q^*) - p_b/\beta - p_m/\mu] < 0$, so that the profit margin for the downstream subsidiary of each firm is <u>negative</u>. Although we have assumed that there are no non-integrated firms, the negative profit margin result shows that there could be no non-integrated firms, since their profits at equilibrium prices would be negative (unless they were to possess special cost advantages).[17] Thus the model predicts a <u>"vertical-squeeze"</u> of non-integrated producers, even though this is not the result of a deliberate strategy of the integrated firms. To summarize:

<u>Proposition 2</u>

If the two inputs are used in fixed proportions and one vertically integrated firm specializes in each unit, equilibrium input and output prices without exchanges will be such that: (i) no non-integrated firm is viable unless it has a cost advantage over integrated firms; and (ii) accounting profits for the downstream operations of the vertically integrated firms will be negative.

IV. The Effect of Exchanges

The next task is to compare the duopoly output Q^* that occurs in the absence of exchanges with the industry profit-maximizing output $\hat{Q}$ that occurs in a duopoly with exchanges and a monetary adjustment rate that accounts for cost differences. To do this, recall that equation (21) is satisfied at $Q = Q^*$, and that the right side of (21) is positive because the accounting profit margin in square brackets has been shown to be negative. The left side of (21), which is also positive at $Q = Q^*$, is simply the derivative of industry-wide total profit with respect to Q, and it follows from the concavity of industry profit in Q that Q^* is less than the output, $\hat{Q}$, that maximizes industry profit.[18]

Since Q^* is below the output $\hat{Q}$ that occurs with exchanges when $r = c_m - c_b$, it follows that exchanges in this duopoly model will increase Q, and hence welfare will increase, even though the use of exchanges enables the two firms to reach the industry-profit-maximizing output. The intuitive explanation for this result is apparent from a comparison of equation (6) that holds with exchanges and equations (10) and (11) that hold without exchanges. When the monetary adjustment rate is equal to the actual cost difference, $c_m - c_b$, exchanges in a duopoly effectively permit firms to horizontally integrate and produce at the correct marginal costs of c_b for input b and c_m for input m.

In contrast, it is straightforward to show that equations (10) and (11) characterize a Cournot equilibrium in which the firms' marginal costs are $c_b/\beta + p_m/\mu$ for firm 1 and $c_m/\mu + p_b/\beta$ for firm 2. Recall that $p_m > c_m$ and $p_b > c_b$, and therefore, these marginal costs exceed the marginal social costs. The output-contracting effect of this cost bias more than offsets the output-expanding effect of the Cournot equilibrium relative to collusion. Because the monopoly power in the monopolized input markets yields large potential gains from arrangements, such as exchanges, that eliminate the "successive markups" problem in vertical market relationships, this intuition is similar to the intuition for other situations in which vertical "restrictions" can increase welfare.[19]

Proposition 3

If the two inputs are produced in fixed proportions and one firm specializes in the production of each, then the industry output of the final product in the symmetric equilibrium for a duopoly without input exchanges is less than the (perfectly collusive) industry output that arises with exchange agreements and $r = c_m - c_b$.

Thus far we have only been able to compare exchange and no-exchange equilibria in a duopoly. Since the inputs are homogeneous, the analysis of the oligopoly case without exchanges is straightforward if there are enough firms selling each input for price competition to drive input prices down to marginal costs, c_b or c_m. When $p_b = c_b$ and $p_m = c_m$, each of the n firms will produce its output in the second stage with constant marginal costs of $c_b/\beta + c_m/\mu$. The resulting industry output, will be the Cournot output for an industry with n firms and constant marginal costs of $c_b/\beta + c_m/\mu$.[20]

Recall the result of proposition 1 that the industry output for an oligopoly with exchanges is the output for a Cournot equilibrium with $s = \min\{k, n-k\}$ firms that have constant marginal costs of $c_b/\beta + c_m/\mu$. Because $n > s$, price competition in the input markets can, in the absence of exchanges, result in a higher industry output than would be the case if all input transfers were made by exchange at a rate that fully compensates for production cost differences.

The intuition for the results in this section is that 1) exchanges permit output restriction because each party to a voluntary exchange has an effect on the sum of other firms' outputs at the margin, and 2) exchanges with a monetary adjustment rate that equals the input cost difference is equivalent to vertical integration that eliminates the distortionary effects of the monopoly power of input producers. This efficiency effect dominates the output-restriction effect when each firm is a monopolist for its own input, but the output-restriction effect of exchanges dominates when price competition in the input markets eliminates any market power in input markets. If there is market power less than complete monopoly in input markets, the effects of exchanges is ambiguous.

V. A Model with Simultaneous Exchanges and Market Sales of Inputs

In this section, we analyze the equilibrium for a market that may contain non-integrated downstream firms. We show that the integrated firms will prefer to obtain some input through exchange. We also show that, as in the model of the previous section, the downstream subsidiaries of these integrated firms will show losses when the "transfer prices" used to calculate input costs are the spot market input prices.

The analysis of exchanges in section I was based on the assumption that the monetary adjustment rate fairly compensates for cost differences: $r = c_m - c_b$. This monetary adjustment rate "clears the exchange market" in the sense that desired input exchange quantities are balanced. In practice, some intercorporate transfers of inputs are arranged by ordinary market sales, and the monetary adjustment rate may be determined by observed transactions prices of inputs. For example, integrated box producers typically sell some medium and linerboard in spot markets, and the practice is to set the monetary adjustment rate for exchanges to be equal to the difference between the average of reported transactions prices of medium and linerboard published in Official Board Markets, i.e.,[21]

(24) $$r = p_m - p_b \, .$$

Independent of the institutional arrangements in the corrugated box market, arbitrage will require (24) to hold in any market with both exchanges and sales. Suppose, for example, that $r > p_m - p_b$. Then a speculator could purchase a unit of input m at price p_m and offer to exchange it for a unit of input b and a monetary adjustment of $r - \varepsilon/2$. This offer would be accepted, and the speculator could sell the unit of b obtained at a price $p_b - \varepsilon/2$. The resulting earnings would be $p_b + r - p_m - \varepsilon$, which would be positive for a small value of epsilon when $r > p_m - p_b$. An analogous argument works for the case in which $r < p_m - p_b$.

In this section, we will consider a model in which two vertically integrated firms may engage in exchanges and may simultaneously arrange market sales of inputs. The model also includes a fringe of non-integrated producers (or "box shops") who purchase all of their requirements for each input. Thus transactions can occur via exchanges or in spot markets. Firm 1 receives e units of medium by exchange and an amount, denoted x_m, by direct purchase from firm 2. Firm 2 purchases an amount of input b, denoted x_b, from firm 1. Thus firm 1 obtains $e + x_m$ units of input m, so production efficiency implies that $q_1 = \mu(e + x_m)$. Similarly, $q_2 = \beta(e + x_b)$.

The combined output of the non-integrated fringe firms will be denoted by q_3. Regardless of the nature of the fabrication cost function for fringe firms, it is possible to derive some general properties of their purchase decisions for inputs b and m. Since the inputs are used in fixed proportions, firms that purchase inputs at prices p_b and p_m will incur a constant cost of $p_b/\beta + p_m/\mu$ for the inputs needed to produce one unit of output, so that

$$(25) \qquad \beta \frac{\partial q_3}{\partial p_b} = \mu \frac{\partial q_3}{\partial p_m} < 0 ,$$

regardless of the state of competition in the sector of the market that generates output q_3.

At this point, it is useful to summarize the relationships between the input and output variables:

$$q_1 = \mu(e + x_m) ,$$

$$(26) \qquad q_2 = \beta(e + x_b) ,$$

$$Q = (\beta + \mu)e + \mu x_m + \beta x_b + q_3 .$$

Using this notation and the fact that firms 1 and 2 produce all units of inputs b and m, respectively, one can express the profits of firms 1 and 2:

$$(27) \qquad \pi_1 = f(Q)q_1 - c_b Q/\beta - re - p_m x_m + p_b x_b + p_b q_3/\beta \ ,$$

$$(28) \qquad \pi_2 = f(Q)q_2 - c_m Q/\mu + re - p_b x_b + p_m x_m + p_m q_3/\mu \ .$$

As before, we assume that vertically integrated firms select input prices noncooperatively and correctly anticipate the effects of input prices on the quantities of inputs demanded, and hence on the industry output Q. We begin by describing the properties of the second stage equilibrium, given the optimal input prices selected in the first stage. Consider the firms' decisions regarding e, x_b, and x_m. At the second stage, firms face an exchange adjustment rate: $r = p_m - p_b$. The first order conditions for e, x_m, and x_b are:

$$(29) \qquad \frac{\partial \pi_1}{\partial e} = f'(Q)(\beta + \mu)q_1 + \mu f(Q) - (\beta + \mu)c_b/\beta - p_m + p_b \geqq 0$$

$$(30) \qquad \frac{\partial \pi_2}{\partial e} = f'(Q)(\beta + \mu)q_2 + \beta f(Q) - (\beta + \mu)c_m/\mu + p_m - p_b \geqq 0$$

$$(31) \qquad \frac{\partial \pi_1}{\partial x_m} = f'(Q)\mu q_1 + \mu f(Q) - \mu c_b/\beta - p_m \leqq 0$$

$$(32) \qquad \frac{\partial \pi_2}{\partial x_b} = f'(Q)\beta q_2 + \beta f(Q) - \beta c_m/\mu - p_b \leqq 0 \ .$$

Now consider the differential (marginal) profitiability of exchange versus input purchases:

$$(33) \qquad \left[\frac{\partial\pi_1}{\partial e} - \frac{\partial\pi_1}{\partial x_m}\right] = \beta f'(Q)q_1 + p_b - c_b \quad ,$$

$$(34) \qquad \left[\frac{\partial\pi_2}{\partial e} - \frac{\partial\pi_2}{\partial x_b}\right] = \mu f'(Q)q_2 + p_m - c_m \quad ,$$

The right-hand terms in (33) and (34) are the generalized profit margins for firms 1 and 2 on their <u>input</u> sales,[22] and these are positive, so an immediate consequence is that either x_m or x_b is zero. If this were not the case then it would have to be that $\partial\pi_1/\partial x_m$ and $\partial\pi_2/\partial x_b$ are both equal to zero, and the implication of (33) and (34) in this case is that <u>both</u> $\partial\pi_1/\partial e$ and $\partial\pi_2/\partial e$ are positive, which contradicts the voluntary nature of exchange.

The positive signs of the left sides of (33) and (34) imply that firms prefer exchanges to input purchases, and a firm with strictly positive purchases of the input that it does not produce will prefer an increase in the level of exchanges. Both firms cannot have positive input purchases because the equilibrium cannot be a situation in which both firms wish to increase the level of exchanges. Since at least one of the input purchase quantities is zero, any equilibrium involving positive levels of both q_1 and q_2 must have a positive level of e. Thus at least one of the exchange derivatives, $\partial\pi_1/\partial e$ or $\partial\pi_2/\partial e$, is zero in an equilibrium with positive outputs, i.e. at least one firm's preferences are binding on the level of exchanges.

Up to this point we have ignored the issue of whether fringe firms could operate profitably. (Recall that fringe firms would be unprofitable in the model in section III without exchanges, in the absence of special fabrication cost advantages.) If one divides both sides of the inequalities (30) and (31) by μ and β respectively, and if the two resulting inequalities are summed, the result is:

$$(35) \qquad f'(Q)[q_1 + q_2] + f(Q) - \frac{c_b}{\beta} - \frac{c_m}{\mu} \leq -[f(Q) - \frac{p_b}{\beta} - \frac{p_m}{\mu}] \ .$$

The profit margin for a fringe firm that purchases both inputs is the expression in square brackets on the right side of (35). To sign the left side of (35), sum the inequalities in (29) and (30) and divide by $\beta + \mu$ to obtain:

$$(36) \qquad f'(Q)[q_1 + q_2] + f(Q) - \frac{c_b}{\beta} - \frac{c_m}{\mu} \geq 0 \ .$$

It follows from (35) and (36) that $f(Q) - p_b/\beta - p_m/\mu$ is negative.

Recall that constant average fabrication costs for the integrated producers were built into the model by letting f(Q) represent price minus average fabrication cost. It follows from these observations that fringe firms would not earn positive profits in this model unless they enjoyed cost advantages, perhaps due to their geographic proximity to buyers. The second implication of these observations is that the downstream subsidiaries of the integrated producers will report losses when internal input transfers are priced at spot market prices. It is again interesting to note this apparent vertical squeeze is <u>not</u> the result of a deliberate predatory strategy on the part of the integrated firms.

To summarize, we have extended our analysis to permit inputs to be obtained by either direct purchase or by exchange with a monetary adjustment that depends on input prices. Exchanges will be used, and non-integrated fringe firms without fabrication cost advantages will not operate profitably in this model. If we assume symmetry, further results can be derived. Suppose that $\beta = \mu = 1$, or, equivalently, that inputs are exchanged in the ratio of quantities that result in efficient production (because units could be defined so that exchange is one to one and $\beta = \mu = 1$ in this case). Consider a symmetric equilibrium with equal outputs and equal input price markups for firms 1 and 2. When markups are equal, $p_m - p_b = c_m - c_b$. Using this observation, the symmetry condition that $q_1 = q_2$, and the assumption that $\beta = \mu = 1$, it is straightforward to show that $\partial\pi_1/\partial e$ in (29) is equal to $\partial\pi_2/\partial e$ in (30). At least one of these derivatives must be zero, and therefore, both are zero in the symmetric solution under consideration. This result, together with (33) and (34), implies that $\partial\pi_1/\partial x_m < 0$ and $\partial\pi_2/\partial x_b < 0$. Thus $x_m = x_b = 0$ and both firms prefer exchanges to input purchases in a symmetric equilibrium. Outputs are equal in such a symmetric equilibrium since $q_1 = q_2 = e$ under the assumption that $\beta = \mu$.

Next, we consider the relationship between r and $c_m - c_b$. To do so we must analyze the equilibrium of the first stage (price choice) of the model. As before, input prices are chosen noncooperatively in the first stage. For firm 1, the optimal level of p_b will satisfy:

$$(37) \qquad \left[\frac{\partial\pi_1}{\partial q_1}\frac{\partial q_1}{\partial e} + \frac{\partial\pi_1}{\partial q_2}\frac{\partial q_2}{\partial e}\right]\frac{\partial e}{\partial p_b} + \frac{\partial\pi_1}{\partial q_3}\frac{\partial q_3}{\partial p_b} + \frac{\partial\pi_1}{\partial p_b} = 0 .$$

In the symmetric equilibrium, $x_m = x_b = 0$, $q_1 = q_2 = e$, and therefore the term in square brackets in (37) is $\partial\pi_1/\partial e$, which is zero in the symmetric equilibrium, because the firms' most preferred levels of exchange are the same in this case. Using this result and the formula for π_1 in (27) with $\beta = \mu = 1$, one can express (37):

$$(38) \qquad [f'(Q)q_1 + p_b - c_b]\frac{\partial q_3}{\partial p_b} - e\frac{\partial r}{\partial p_b} + q_3 = 0 .$$

Similarly, the necessary condition for firm 2's optimal choice of p_m can be expressed:

$$(39) \qquad [f'(Q)q_2 + p_m - c_m]\frac{\partial q_3}{\partial p_m} + e\frac{\partial r}{\partial p_m} + q_3 = 0 .$$

If r is determined by (24), then $-\partial r/\partial p_b = \partial r/\partial p_m = 1$. Also, it follows from (25) with $\beta = \mu = 1$ that $\partial q_3/\partial p_b = \partial q_3/\partial p_m$. Therefore (38) or (39) are identical and either can be used to determine the common markup, $p_b - c_b = p_m - c_m$,[23] in the first stage of a symmetric equilibrium with $q_1 = q_2$.[24]

The equality of the markups in this symmetric case means that $c_m - c_b$ equals $p_m - p_b$, which equals r by (24), and so the monetary adjustment rate fully compensates for cost differentials in the symmetric case.

The results of this section can be summarized:

Proposition 4

In a market with two vertically integrated firms that can sell the inputs to each other or to other non-integrated firms, the two vertically integrated firms will use exchange agreements, and non-integrated firms without fabrication cost advantages will earn negative profits. In a symmetric model with $\beta = \mu = 1$, the symmetric equilibrium has the property that both vertically integrated firms choose to use exchanges exclusively to acquire the input produced by the other firm.

It is not uncommon for the downstream divisions of vertically integrated firms to experience periods of negative accounting profits and for non-integrated downstream producers to complain about a coincidental vertical squeeze. Our explanations of these observations and of the use of exchanges were derived in the context of a simple model with two integrated firms and two inputs that are perfect complements. It will be important to determine to what extent our analysis could be generalized, for example, to a market with more than one producer of each input.

VII. Conclusion

Although bilateral input exchange agreements between vertically integrated producers are very common in some industries, such agreements have been largely ignored in the economics literature. This paper contains an analysis of the incentives that firms have to engage in input exchange agreements and of the economic effects of such agreements. When the monetary adjustment rate per unit of exchange fully compensates for input cost differences, the use of exchanges is similar to vertical integration in the sense that both can reduce inefficiencies caused by monopoly power in the input markets. But exchanges can have anticompetitive effects because of the coordination exchanges can provide for output decisions. We have analyzed market performance for a simple model with fixed-coefficients production, and we have shown that firms have an incentive to negotiate exchange agreements. The net effect of such agreements is to expand industry output when input markets are monopolized. Price competition in the markets for each input, however, reduces the distortions that result from incomplete vertical integration, and as a consequence, procompetitive effects of exchanges can be dominated by the output-restriction effect.

The models in this paper are highly stylized in that spatial dimensions are largely ignored; the analysis of specific antitrust issues should be based on an elaboration of these models that is tailored for the market under consideration. Nevertheless, the general implication of the analysis in this paper is that input exchanges should only be of concern to antitrust authorities to the extent that there is market power in the input markets. For merger cases, our analysis suggests that rather than being simply a "plus factor," exchanges change the effective level of concentration of the market. Our analysis indicates a precise way in which the Herfindahl indices should be modified to account for exchanges.

Finally, our duopoly analysis indicates that the input prices selected by two vertically integrated producers, each of which specializes in the production of one input, will result in a vertical price squeeze that would prevent entry of non-integrated producers of the final product (unless such entrants possessed unique cost or marketing advantages). This vertical price squeeze would arise from noncooperative, nonstrategic input pricing behavior, even in the absence of threatened entry by non-integrated fringe producers.

Footnotes

* University of Virginia and Federal Trade Commission respectively. We would like to acknowledge helpful comments and criticisms that we received from participants in seminars at both places and from Eric Engen, James Walker and Steve Salop. Peggy Claytor assisted in the preparation of this manuscript.

1. Walter Measday (1982) discusses the prevalence of exchanges in petroleum products markets. Joyce (1983) reports that exchanges are also common in markets for inorganic chemicals, iron ore, gypsum, and aluminum. He suggests that exchanges permit vertically integrated input producers with market power to transfer inputs among themselves at below-market prices.

2. From 1977 to 1980, about three-fourths of domestic inter-corporate transfers of medium by West Coast medium producers were made through bilateral exchange agreements. See In the Matter of Weyerhaeuser Company and Weyerhaeuser West Coast, Inc. (hereafter Weyerhaeuser), Complaint Counsel's Proposed Findings of Fact, Conclusions of Law, and Order, before the Federal Trade Commission, June 1983, p. 154.

3. Weyerhauser (1983).

4. In the parlance of merger analysis, plus factors are non-structural factors that are likely to facilitate collusion, ceteris paribus. Frederick Scherer (1980) discusses a number of factors that facilitate collusion, but he does not specifically mention input exchange agreements.

5. Walter Measday (1982), however, argues that the costs of administering the elaborate network of exchanges observed in the petroleum products industry would exceed the costs of doing business in the market.

6. Cournot shows that this result holds in a more general setting with nonlinear demand and unequal input proportions. It is of some interest to note that Cournot, whose name is associated with a noncooperative equilibrium in output quantities, uses a noncooperative equilibrium in (input) prices in this context in which the two firms' products are different.

7. Tacit collusion may enable firms to raise prices above noncooperative levels.

8. There may be fixed costs associated with the production of each input, but these fixed costs do not affect the firms' optimal decisions (at the margin).

9. A change in the units of measure for an input would simply change the levels of the production coefficient and marginal cost for that input.

10. John Bryant (1983) considers a simple model of the macroeconomy in which each of n individuals specialize in the production of a distinct input, and production of the single final product requires fixed proportions for the n inputs. He discusses the possibility of multiple competitive equilibria. Thus the production structure is similar to that used in our paper; the main difference is that agents in Bryant's model do not use contracts to coordinate the production of the inputs.

11. For example, if $\pi_1'(e^*) = 0$ at some level e^* and $\pi_2'(e^*) > 0$, then firm 1 declines to increase the level of exchange even though firm 2 would prefer a higher level of exchange.

12. For example, it is easily seen that firm 1's preferred level of exchanges is a monotonically decreasing function of r and firm 2's preferred level of exchange is a monotonial increasing function of r.

13. To verify this result, one should begin by rewriting the profit expressions to allow for the effects of the other-input capacity, $\bar{m}_1$ for firm 1 and $\bar{b}_2$ for firm 2. Details are available from the authors on request.

14. Charles Holt and David Scheffman (1985) use (1) to show that if a Cournot equilibrium with positive outputs for each firm exists, then the equilibrium is unique.

15. The 1984 Department of Justice Merger Guidelines specify the way in which pre-merger and post-merger Herfindahl indices are used to determine whether a proposed merger is likely to be challenged.

16. Note that $1/\beta$ units of input b is needed to produce one unit of output. Each sale of $1/\beta$ units of input b yields a profit margin of $p_b/\beta - c_b/\beta$ for firm 1, but the sale will increase q_2 and reduce output price by approximately $f'(Q)$ at the margin, which reduces input sales revenue by an amount $f'(Q)q_1$.

17. Recall that fabrication costs have been assumed to be constant, and have been subsumed in $f(Q)$ by letting $f(Q)$ represent price minus average fabrication cost. For a fringe firm with a cost advantage, average revenue minus average fabrication cost would equal $f(Q) + c_f$, where c_f is its absolute cost advantage. Such a cost advantage might arise from geographic location.

18. For example, consider a market with linear demand: $f(Q) = A - BQ$, $A > 0$, $B > 0$. Also, let $\mu = \beta = 1$. With these parameters it is straightforward to show that $p_m = (5A - 5c_b + 6c_m)/11$ and $p_b = (5A - 5c_m + 6c_b)/11$. If $c_m \neq c_b$, the Nash equilibrium input prices will differ, but both markups will be equal: $p_b - c_b = p_m - c_m = (5/11)(A - c_b - c_m)$. Also, $Q^* = (4/11B)(A - c_m - c_b)$, and this industry output level is below $\hat{Q}$, which is $(1/2B)(A - c_m - c_b)$ in this example.

19. For example, see the discussion of the vertical integration of successive monopolies in Frederick Warren-Boulton (1978).

20. We have assumed that firms must make output production decisions independently and in advance, so Cournot is the relevant single-period equilibrium assumption. If, instead, firms selected output prices

independently and produced to meet orders, the outcome, in the absence of fixed costs, would be a competitive output price.

21. This is an independent trade publication that reports prices by product and region on a regular basis.

22. For example, a unit of input b costs c_b to produce, it sells for p_b, and the purchaser uses this input in the production of β units of output, which reduces price by $\beta f'(Q)$ at the margin. Thus the generalized profit margins on the right sides of (33) and (34) are calculated on a per-unit-of-input basis. The profit margins discussed earlier in footnote 16 differ from the margins in (33) and (34) by factors of β and μ respectively because the margins discussed in footnote 16 were calculated on a per-unit-of-final-product basis.

23. Analysis of (38) and (39) would not change if firms perceive the monetary adjustment rate to be exogenous: $\partial r/\partial p_b = \partial r/\partial p_m = 0$.

24. To complete the analysis of the symmetric case, note that there are five endogenous variables to be determined: q_1, q_2, q_3, p_b, and p_m. We have shown that, in the symmetric equilibrium, the weak inequalities in (29) and (30) hold with equality. The third and fourth equations are (38) and (39). Consideration of the fringe firms that purchase both inputs provides a fifth equation that represents the behavior of these fringe firms. For most commonly used behavioral assumptions, e.g. competitive and Cournot, the fringe can be represented by a functional relationship between the aggregate output of the fringe, q_3, and the variables that fringe firms would consider to be fixed parameters: p_b, p_m, q_1, q_2 (or $p = f(q_1 + q_2 + q_3)$ in the competitive case). This functional relationship provides third equation that can be used to determine simultaneously the common level of q_1 and q_2, the common level of $p_b - c_b$ and $p_m - c_m$, and the level of q_3 in a symmetric equilibrium.

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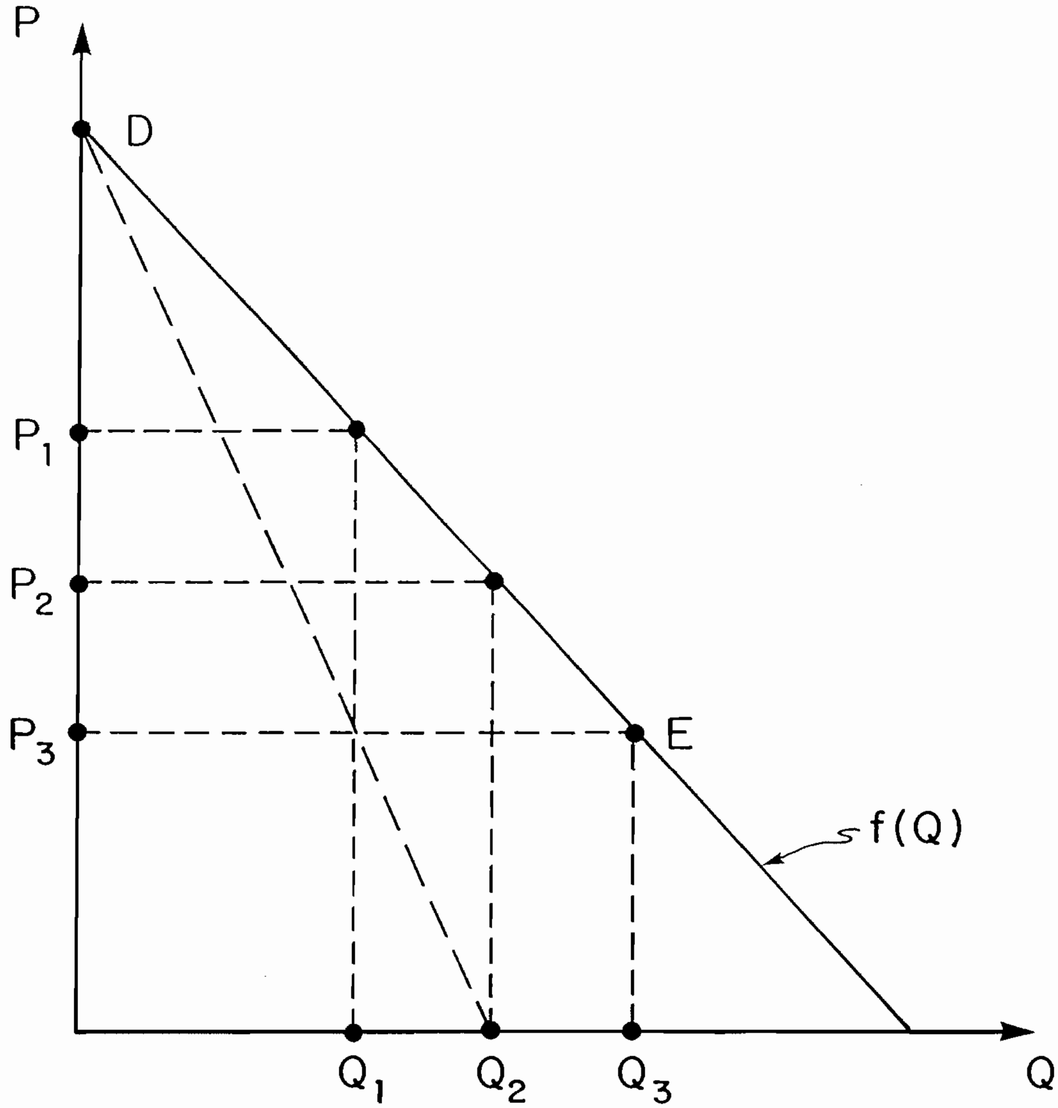
P
D
P_1
P_2
P_3
E
f(Q)
Q_1
Q_2
Q_3
Q