

Competition in Pricing Algorithms*

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September 14, 2020

Abstract

Increasingly, retailers have access to better pricing technology, especially in online markets. Firms employ automated pricing algorithms that allow for high-frequency price changes. What are the implications for price competition? We develop a model of price competition where firms can differ in pricing frequency and choose algorithms that autonomously react to rivals' prices. We demonstrate that pricing technology with these features can increase prices in competitive equilibrium, relative to the standard simultaneous price-setting model. Using high-frequency data from major online retailers, we document asymmetric pricing frequency and pricing patterns consistent with the model. A simple counterfactual simulation implies that pricing algorithms lead to meaningful increases in markups, especially for firms with superior pricing technology.

Keywords: Pricing Algorithms, Pricing Frequency, Online Competition
JEL Classification: L40, D43, L81, L13, L86

*We thank Scott Kominers, Matt Grennan, Marc Rysman, Mike Sinkinson, Giacomo Calzolari, George Hay, Ralph Winter, Emilio Calvano, and Nate Miller for helpful comments. We also thank seminar and conference participants at Harvard Business School, the IIOC, the ASSA Meeting (Econometric Society), the Toulouse Digital Economics Conference, the NYU Law/ABA Antitrust Scholars Conference, the Winter Business Economics Conference, the NBER Economics of Digitization meeting, and Brown University. We are grateful for the research assistance of Pratyush Tiwari and Alex Wu.

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1 Introduction

Increasingly, retailers have access to better pricing technology, especially in online markets. In particular, pricing algorithms are becoming more prevalent. Algorithms can change pricing behavior by enabling firms to update prices more frequently and automate pricing decisions. Thus, firms can commit to pricing strategies that react to price changes by competitors. This may have important implications for price competition relative to standard oligopoly models in which firms set prices simultaneously. Do pricing algorithms lead to higher prices?

In this paper, we present a new model of price competition that captures the features of increased pricing frequency and short-run commitment that are enabled by pricing algorithms. The model also allows for asymmetric technology among firms. Frequency, commitment, and asymmetry are important features of price competition among real-world firms that have pricing algorithms.¹ We show that asymmetry in pricing technology can fundamentally shift equilibrium behavior; if one firm adopts superior technology, both firms can obtain higher prices. Thus, our paper illustrates a novel way in which pricing algorithms can increase prices relative to the standard simultaneous price-setting (Bertrand) equilibrium.

Frequency, commitment, and asymmetry have not previously been considered in the context of pricing algorithms. The existing literature has almost exclusively assumed that firms set price simultaneously, focusing on whether algorithms facilitate collusion in this environment (e.g., Calvano et al., 2019; Miklós-Thal and Tucker, 2019; Salcedo, 2015).² The equilibria of the simultaneous price-setting model has been extensively studied and forms the basis of most applied work, including analysis by antitrust authorities.³ By contrast, we provide new results on equilibrium prices and strategies using our more general model. We focus on Markov perfect equilibrium (MPE), wherein firms can only condition on payoff-relevant variables. In the standard simultaneous price-setting model, the unique MPE is the Bertrand equilibrium. We demonstrate that algorithms expand the set of MPE, allowing firms to obtain higher prices and profits in competitive equilibrium. Overall, our results indicate that the potential of algorithms to raise prices goes beyond the possibility of facilitating collusion.

We also present new facts about pricing behavior that motivate the features of our model of price competition. We collect a novel dataset on high-frequency pricing behavior from five large online retailers. We document that these retailers have asymmetric pricing technology. Two retailers in our dataset have the ability to update prices once each week, one has the ability to update prices once each day, and two have the ability to update prices within each hour. The

¹For instance, we document large differences in the maximum frequency with which online firms can update prices. In addition, a key characteristic advertised by third-party pricing algorithm solutions is how frequently they can update prices.

²One exception is Klein (2019) who examines algorithmic collusion when firms take turns setting prices. The concern about collusion has also been the focus of the popular press. See, e.g., “When Bots Collude,” *The New Yorker*, April 25, 2015 and “Price-Bots Can Collude Against Consumers,” *The Economist*, May 6, 2017.

³See, for instance, “Commentary On The Horizontal Merger Guidelines” by the U.S. Department of Justice.

pricing patterns we observe are consistent with automated software programmed to run at set times. Though we focus on the collection of data for one product category (allergy products), each retailer we study, to our knowledge, employs the same pricing technology across hundreds (or thousands) of categories on its website.

We use our model to analyze the potential empirical implications of differences in pricing technology. Consistent with the model, we document that firms with faster pricing technology appear to react to the price changes of slower rivals, and firms that have higher-frequency pricing have lower prices than their competitors. Thus, while the previous literature has focused on the role of search frictions as an explanation for price dispersion, our model provides an alternative, complementary explanation: differences in prices for the same product across websites can be driven by pricing technology alone. We use a counterfactual simulation to quantify these impacts, finding that asymmetric pricing technology leads to higher prices for all retailers and exacerbates price differences among similar retailers.

First, we motivate our model with a discussion of the features of pricing algorithms in Section 2. We then begin our theoretical analysis by introducing a game in which firms may differ in pricing frequency, e.g., changing prices once each week versus once each day (Section 3). This asymmetry arises in many contexts due to variation in pricing technology. We show that the model generates prices that lie between the simultaneous and sequential equilibria and nests both as special cases. In typical settings, the faster firm has lower prices and higher profits than the slower firm. Moreover, when firms can choose their pricing frequency, each firm has a unilateral profit incentive to choose either more frequent or less frequent pricing than their rivals. Therefore, the simultaneous price-setting model is not an equilibrium outcome when pricing frequency is endogenous.

In Section 4, we develop a more general model where algorithms enable firms to differ in their pricing frequency and also to commit to a pricing strategy for future price updates. This model nests the pricing frequency game developed earlier. Further, we show that a model with asymmetric commitment—i.e., when only one firm can condition its algorithm on its rival’s price—closely parallels the model of asymmetric frequency. We then analyze the case where all firms can condition on rivals’ prices, deriving a one-shot competitive game in which firms submit pricing algorithms, rather than prices. We use the one-shot game to show that short-run commitments, in the form of automated pricing, can also generate higher prices.

We demonstrate that symmetric commitment enabled by algorithms can generate higher prices even when we eliminate clearly collusive strategies, such as cooperate-or-punish equilibria. Thus, we focus on equilibrium pricing strategies that, in some sense, “look competitive.” Even with these restrictions, we show that the use of pricing algorithms can increase prices relative to the Bertrand game. Supracompetitive prices, including the fully collusive prices, can be supported with algorithms that are simple linear functions of rivals’ prices.⁴ In this way, algo-

⁴In practice, it is typical for algorithms to have a linear adjustment based on the average price of a set of

rithms fundamentally change the pricing game and provide a means to increase prices without resorting to collusive behavior.

We also address the question of whether pricing algorithms can arrive at competitive prices. Our model provides a negative result, showing that algorithms that depend on rivals' prices do not generate Bertrand prices in equilibrium. In particular, it is not an equilibrium for all firms to choose algorithms that equal their price-setting best-response (Bertrand reaction) functions. Intuitively, our results are supported by the following logic: A superior-technology firm commits to "beat" (best respond to) whatever price is offered by its rivals, and its investments in frequency or automation makes this commitment credible. The rivals take this into account, softening price competition. Our model nests several different theoretical approaches that were developed prior to the advent of pricing algorithms and have largely been dismissed in the modern literature, including conjectural variations. We highlight these connections below.

In our empirical analysis, we study prices for over-the-counter allergy medications for the five largest online retailers for the category.⁵ Our novel dataset is described and analyzed in Section 5. By studying prices at the hourly level, we are able to document heterogeneity in pricing technology. We find that two firms have within-the-hour ("hourly") pricing technology, one firm has daily pricing technology, and the remaining two have weekly pricing technology, updating their prices early every Sunday morning. This high degree of asymmetry is associated with asymmetric prices. Relative to the firm with the fastest pricing technology, the firm with daily pricing technology sells the same products at prices that are 10 percent higher, whereas the firms with weekly pricing technology sell those products at prices that are approximately 30 percent higher. We also document that price changes by high-frequency retailers are more likely after a price change by a low-frequency retailer, suggesting that high-frequency retailers monitor the prices of rivals.

The empirical literature on price competition and firm markups has almost exclusively assumed that firms play a simultaneous pricing game. As a first step toward quantifying the role of heterogeneous pricing technology, we compare observed prices to a counterfactual equilibrium in which firms have simultaneous price-setting technology (Section 6). We introduce a generalized spatial differentiation model that allows for flexible substitution patterns among retailers and provides a tractable empirical approach for examining competition in algorithms. Using the observed pricing technology of the retailers as an input, we fit the model to average prices and market shares in our data. We then use the estimated demand parameters to simulate the counterfactual equilibrium for simultaneous Bertrand price competition. Relative to the Bertrand equilibrium, the calibrated model predicts that algorithmic competition increases

competitors. In one interesting example, a retailer on Amazon.com set its price for a book to be 0.9983 times its rival's price, and the rival set its price to be 1.270589 times the retailers' price. The price of the book rose to nearly \$24 million. This, we note, was not an equilibrium. See "How A Book About Flies Came To Be Priced \$24 Million On Amazon," *Wired*, April 27, 2011. <https://www.wired.com/2011/04/amazon-flies-24-million/>

⁵Based on search share and estimated revenue. See discussion in Section 5.1.

average prices by 5.2 percent across the five firms. This corresponds to a 9.6 percent increase in profits and a 4.1 percent decrease in consumer surplus. The effect on markups and profits is especially large for firms with superior pricing technology, i.e., those with the ability to quickly adjust prices.

Online markets have allowed retailers to easily monitor their rivals' prices and incorporate these prices into pricing algorithms. Evidence suggests that these algorithms are becoming more widespread as online retailing continues to grow (Cavallo, 2018). Indeed, there are even firms that specialize in providing retailers with information on competitors' prices for use in pricing algorithms.⁶ This growing prevalence of pricing algorithms has drawn significant attention from antitrust authorities.⁷

Overall, our results imply that pricing algorithms can support higher-price equilibria, even when firms act competitively. Our empirical analysis shows price patterns consistent with the model and suggests that pricing algorithms can have an economically meaningful effect on markups. Thus, if policymakers are concerned that algorithms will raise prices, then the concern is much more broad than that of collusion. Of course, algorithms may also have several benefits, such as the ability to more efficiently respond to time-varying demand. In light of these issues, we briefly discuss how policymakers can regulate pricing algorithms in Section 7. Though we focus on competitive equilibria, our study also has important implications for collusion. By increasing competitive prices and profits, algorithms may make punishment less severe in a collusive scheme, reducing the likelihood of collusion. Additionally, our model explicitly features a new dimension in the strategy space, allowing firms to change pricing technology as an either a substitute or a complement to the pursuit of collusion.

Related Literature

We contribute to a growing literature on the impacts of algorithms on prices. Our primary contribution is to study competitive equilibria when firms compete in pricing algorithms. Further, our counterfactual exercise presents the first empirical results on the potential impacts of pricing algorithms on prices. A recent paper by Assad et al. (2020) adds to the nascent empirical literature, providing reduced-form evidence that algorithms might increase prices in retail gasoline markets.⁸

For our analysis, we present a new model of price competition to capture features of algorithms—frequency and commitment—that have not been studied previously. The prior

⁶For instance, Intelligence Node allows retailers to “get your competitor’s pricing and positioning data from the world’s largest retail database.” ChannelAdvisor advertises its automated pricing product as “constantly monitoring top competitors on the market.”

⁷See, for instance, the U.K. Competition and Markets Authority’s 2018 report, “Pricing Algorithms” and Germany’s “Twenty-second Biennial Report by the Monopolies Commission.” Thus far, competition authorities have focused on the potential for algorithms to facilitate collusion.

⁸Assad et al. (2020) find price effects only when both firms in duopoly markets adopt superior pricing technology, which suggests that the mechanism in their setting may be collusion or symmetric commitment.

literature has focused on the price effects of *learning* algorithms (Salcedo, 2015; Calvano et al., 2019) or *prediction* algorithms (Miklós-Thal and Tucker, 2019; O’Connor and Wilson, 2019) in the context of a standard simultaneous price (or quantity) game. This literature focuses on how learning or prediction algorithms affect the sophistication of players and their ability to collude.⁹ The equilibria of the environments studied by these papers have been extensively studied. By contrast, we examine how pricing algorithms change the nature of pricing game, focusing on Markov perfect equilibria as in Maskin and Tirole (1988b).¹⁰ Our model generates a new set of equilibrium strategies and outcomes that can be supported by algorithms.

Next, we contribute to a broader empirical literature on the study of supracompetitive prices (e.g., Porter, 1983; Nevo, 2001; Miller and Weinberg, 2017; Byrne and de Roos, 2019). We provide a new model and empirical results that suggest that the mode of competition can lead to meaningful price increases without the need for collusion. Further, our data suggests that our model may be particularly relevant in online markets, where algorithms and asymmetries in pricing technology are prevalent. Previous empirical studies of supracompetitive prices have exclusively considered stage games with symmetric technology where firms choose actions (price or quantity) simultaneously. Our empirical framework takes a first step at incorporating heterogeneous pricing technology and quantifying its implications.

We also contribute to the empirical literature on online competition by showing how a new supply-side mechanism—asymmetric pricing technology—can generate price dispersion. Despite the fact that online competition is thought to reduce search costs and expand geographic markets, substantial price dispersion has been documented (e.g., Baye et al., 2004; Ellison and Ellison, 2005). A large empirical literature has focused on demand-side features such as search frictions, but little attention has been paid to firm conduct.¹¹ One exception is Ellison et al. (2018), who examine managerial inattention and price dispersion in an online marketplace in 2000 and 2001, prior to the widespread use of pricing algorithms.

We argue that a key feature of pricing algorithms is the ability to condition on the prices of rivals. This mechanism relates to a large class of models where firms internalize the reactions of their rivals, including conjectural variations (Bowley, 1924) and the classic Stackelberg model. The real-world applicability of these models has been subject to a long debate (e.g., Fellner, 1949). The conjectural variations model has fallen out of favor, likely because consistent conjectures other than Cournot are difficult to rationalize (Daughety, 1985; Lindh, 1992). Models with sequential behavior have been dismissed as unrealistic for empirical settings because it requires the assumption that one firm can honor a (sub-optimal) commitment to an action or

⁹Klein (2019) considers the same question but in the alternating-move setting of Maskin and Tirole (1988b).

¹⁰Maskin and Tirole (1988b) show that higher prices can result in a duopoly game where firms set prices in alternate periods using strategies that rely exclusively on payoff-relevant variables. Our analysis complements their work by showing how higher prices may be obtained in Markov perfect equilibrium in a different economic environment—one in which algorithms provide variation in pricing frequency and enable short-run commitment.

¹¹Work examining online search frictions includes Hong and Shum (2006), Brynjolfsson et al. (2010), and De los Santos et al. (2012).

strategy while the other reacts. For this reason, applied researchers and antitrust authorities have almost universally assumed that firms play a simultaneous Bertrand or Cournot game. We argue that such commitments are credible, made possible by investments in differential pricing technology. Algorithms provide a natural mechanism for the type of technological commitment discussed in Maskin and Tirole (1988a). Thus, one interpretation of our model is that it provides a new foundation for theoretical results arising in this older literature. By nesting these models under a common structure, we also provide a framework for firms to choose among different models of competition by changing their pricing technology.

The logic of how pricing algorithms leads to higher prices is similar to that of price-matching guarantees, which some have argued can be anticompetitive (Salop, 1986; Hay, 1981; Moorthy and Winter, 2006). Both are predicated on commitment, which software makes possible in online markets.¹² We show that price-matching guarantees are not chosen in equilibrium in our model. There are also parallels between our model and previous literature focused on commitment in other settings. Grossman (1981) and Klemperer and Meyer (1989) study supply function equilibrium in which firms simultaneously decide on quantities in response to a (endogenously-determined) market price in a setting with homogeneous products. Lazarev (2019) shows that higher prices can result when firms first commit to a restricted set of prices, then choose from among those prices in a second stage. Conlon and Rao (2019) find that wholesalers can set the collusive price when they can commit to a price schedule. The game-theoretic notion of commitment ties into a broader literature on strategic delegation that has been applied in diverse settings.¹³ We consider algorithms to be an economic mechanism to make such commitments credible.¹⁴ Moreover, we are the first to link pricing algorithms to models with these features.

2 Algorithms and Pricing Behavior

Broadly speaking, an algorithm is a set of instructions executed by a computer.¹⁵ The instructions map input to a desired set of output. In the context of price competition, algorithms have previously been studied as mechanisms to enable better forecasts (Miklós-Thal and Tucker,

¹²Hal Varian discussed the appeal of price matching in online markets in the August 24, 2000 *New York Times* article “When commerce moves online, competition can work in strange ways.” In a set of lab experiments, Deck and Wilson (2000, 2003) find that subjects that use automated price-matching strategies obtain higher profits than those that manually set prices.

¹³Fershtman and Judd (1987) and Sklivas (1987) show that, by giving managers a mixture of revenue-based and profit-based incentives, owners can commit to behavior that is not profit maximizing, leading to higher prices.

¹⁴A related strand of literature deals with one-shot games where players choose contracts (or commitment devices) that condition their actions on the strategies of the other players (Tennenholtz, 2004; Kalai et al., 2010; Peters and Szentes, 2012). In this literature, (equilibrium) contracts are functions of the other players’ contracts. Tennenholtz (2004) gives the example of submitting a computer program that reads the rivals’ computer program and chooses an action accordingly. Another related concept is the cartel punishment device of Osborne (1976).

¹⁵We follow the convention in the literature that associates algorithms with computing. The concept of an algorithm and indeed the word itself predates modern computers by many centuries.

2019; O’Connor and Wilson, 2019), reflecting the potential for machine learning algorithms to enable better predictions (Agrawal et al., 2018). In addition, learning algorithms have been studied to address the question of whether artificial intelligence might arrive at collusive equilibrium strategies (Calvano et al., 2019; Klein, 2019; Salcedo, 2015). The focus of these studies is on how algorithms might affect the sophistication—in terms of prediction or strategies—of players in repeated games of simultaneous play.

By contrast, we consider how algorithms may affect the *nature* of the pricing game. In our context, pricing algorithms can be characterized by a formula to determine prices. The formula performs a calculation based on input variables, which, generically, may include a rich history, including the past play of rivals or the outcomes of experiments. Thus, we study algorithms that may, in principle, also incorporate elements of enhanced prediction or learning. Regardless of its level of sophistication, an algorithm imbues a firm with two significant features, relative to a human agent:

1. *An algorithm lowers the cost of updating prices and facilitates a regular pricing frequency.*

Typically, firms use software to schedule pricing updates at regular intervals, e.g., once per day or every 15 minutes. The frequency with which a firm can update prices depends on investments in pricing technology, which may differ across firms. Algorithms facilitate both regular pricing updates and more frequent updates, as software can better monitor rivals’ prices and can find the solution to a difficult pricing problem in less time and with less error than a human agent.

2. *An algorithm provides a (short-run) commitment device to a pricing strategy.*

When an algorithm calculates price based on the prices of other firms, it can autonomously react to price changes of rivals in the market based on a pre-specified strategy. This serves as a short-run commitment device, as the algorithm itself is typically updated at a lower frequency than it is used to set prices.

These two features map to the innovations of our model. First, we examine the role of pricing frequency in Section 3. This is motivated by the fact that pricing frequency is a key characteristic that differentiates algorithms in practice. Online firms are observed to set prices at regular frequencies and differ widely in the maximum frequency with which they can update prices.¹⁶ By contrast, a human agent cannot analyze rivals’ prices and update prices at high frequency and cannot be expected to maintain a regular pricing frequency.¹⁷ Large online retailers sell several thousands of products; relying on humans to update all prices at regular intervals would be extremely costly.

Second, an algorithm provides a commitment to follow a pricing rule. The software that implements a pricing algorithm is typically updated at a lower frequency than prices. Thus,

¹⁶In addition, third-party “repricers” compete on how frequently they can update prices, e.g., once each day or as fast as every few minutes.

¹⁷The study by Ellison et al. (2018) provides empirical evidence of human inefficiency along these dimensions.

in between updates to the algorithm, the computer changes prices based on a fixed rule. It is widely thought that humans lack this sort of commitment power (e.g. Maskin and Tirole, 1988a). Borrowing a term from contract theory, we tend to expect a human to adhere to the incentive compatibility constraint at every opportunity to set prices. We combine the features of frequency and short-run commitment in a more general model in Section 4.

We assume that firms are fully sophisticated when it comes to monitoring current prices and understanding rivals' algorithms. In practice, firms may use machine learning and experimentation to learn about the pricing algorithms of their rivals. Our environment can be considered the limiting case of an arbitrary (but consistent) learning process. We do limit sophistication in strategies by focusing on Markov perfect equilibria where firms cannot condition on past prices. Our analysis can be contrasted with the literature on algorithmic collusion in which firms employ history-dependent strategies, allowing them to sustain collusion. We find that even if firms are not sophisticated in this dimension, supracompetitive prices can be sustained in equilibrium.

3 Competition with Pricing Frequency

Following the discussion above, we begin by modeling pricing frequency. This is a key characteristic that differentiates algorithms in practice. We show that enabling firms to choose different pricing frequencies has important implications, and it provides some intuition for a richer model where firms can also commit to a pricing strategy in the short run. We present this more general model in Section 4.

3.1 Infinite Horizon Model

Consider two firms with the ability to change prices at different frequencies. Both firms initially set prices at $t = 0$. Firm 1 can update its price at discrete points after each interval of time T_1 , and firm 2 can likewise update its price after T_2 . We assume that $T_1 = \theta T_2$, where $\theta \in \mathbb{N}$. This implies that firm 2 has (weakly) superior technology, allowing it to change its price at least as frequently as firm 1. For example, T_1 may equal one week, while T_2 equals one day ($\theta = 7$). Without loss of generality, we normalize $T_1 = 1$, i.e., we define units of time in terms of the period between firm 1's potential price changes.

In the next section, we formalize the link of this model to a more general model of competition in algorithms. The implicit assumption we make in this section is that firms can revise their algorithms whenever they have the ability to update prices, i.e., they completely re-solve for the optimal price. In other words, firms cannot commit to a fixed pricing rule in intermediate periods. Pricing frequency therefore corresponds to the frequency that firms can update their algorithms. Under the assumption of no commitment, it suffices to analyze the pricing game. We focus on the two-firm case, but our results readily extend to multiple firms.

Demand arrives in continuous time, with a measure $m(t) \geq 0$ of consumers arriving at t . The distribution of consumers is stable over time, so that demand looks identical at any instant t except for the size of the market. Given demand and prices (p_1, p_2) , firm j realizes instantaneous profit flow $\pi_j(p_1, p_2)$. We assume the profit functions are quasiconcave and have a unique maximum with respect to a firm's own price. Firms discount the future exponentially at rate ρ and have an infinite horizon.

Firms choose a sequence of prices to maximize profits, conditional on the flow of consumers $m(t)$, the profit flows π_j , and the behavior of the rival firms. Let $p_1(t)$ and $p_2(t)$ denote the prices of each firm over time, and let P_1 be the discrete sequence of prices chosen by firm 1 at $t = \{0, 1, 2, \dots\}$. For timing purposes, we assume that P_{1s} is relevant for demand over the period $(s, s + 1]$. Firm 1's problem can be written as:

$$\max_{P_1} \sum_{s=0}^{\infty} \int_s^{s+1} e^{-\rho t} \pi_1(P_{1s}, p_2(t)) m(t) dt. \quad (1)$$

Because firm 2 can change its price at every point $s \in \{0, 1, \dots, \infty\}$ in addition to intermediate times, the problem can be expressed as a sequence of single-period stage games. We restrict our attention to subgame perfect equilibrium in each stage game. The resulting equilibrium is the unique (pure-strategy) Markov perfect equilibrium of the infinite horizon problem.

3.2 Stage Game Analysis

As we have shown, the repeated game can be expressed as a sequence of single-period stage games. By solving for the equilibrium of each stage game, we can construct the Markov perfect equilibrium of the repeated game. Firm 1's problem in stage game s is

$$\max_{p_1} \int_s^{s+1} e^{-\rho t} \pi_1(p_1, p_2(t)) m(t) dt. \quad (2)$$

We now analyze the behavior of firm 2 in each period. Firm 2's pricing behavior will satisfy the following two properties in equilibrium: (1) firm 2's price will be constant over the period (despite its ability to update prices), and (2) firm 2's price will lie along its Bertrand best-response function. The first property is a result of $\pi_2(\cdot)$ being time-invariant and p_1 being fixed in the period. The second property arises from the fact that it is optimal for firm 2 to price along the Bertrand best-response function when it is pricing simultaneously with its rival ($t = s$) and also in any later pricing update (e.g., $t = s + 1/\theta$). The Bertrand best-response function for firm 2 treats p_1 as fixed, which is a Nash equilibrium condition at $t = s$ and is literally true at any other point when firm 2 can update its price. Let $R_2(p_1, s)$ denote firm 2's reaction function in period s .

We return to firm 1's problem. Without loss of generality, we focus on the first period

($s = 0$). Let p_2 now denote the price of firm 2, which is time-invariant (in the stage game) in equilibrium, and let $R_2(p_1) = R_2(p_1, 0)$. Firm 1 chooses p_1 recognizing that p_2 can react to its price after a period of $1/\theta$. Firm 1's problem can be expressed as:

$$\max_{p_1} \int_0^{\frac{1}{\theta}} e^{-\rho t} \pi_1(p_1, p_2) m(t) dt + \int_{\frac{1}{\theta}}^1 e^{-\rho t} \pi_1(p_1, R_2(p_1)) m(t) dt. \quad (3)$$

Because the profit flow function is time-invariant, we can write firm 1's stage game problem as:

$$\max_{p_1} (1 - \alpha) \pi_1(p_1, p_2) + \alpha \pi_1(p_1, R_2(p_1)) \quad (4)$$

where $\alpha = \left(\int_0^1 e^{-\rho t} m(t) dt \right)^{-1} \int_{\frac{1}{\theta}}^1 e^{-\rho t} m(t) dt$. The value $1 - \alpha$ describes the relative weight that firm 1 places on the initial period $(0, 1/\theta]$, which is a function of ρ , $m(t)$, and θ .¹⁸ In the initial price-setting phase, the usual Nash-in-price logic holds: firm 1 treats firm 2's price as given over the period $(0, 1/\theta]$. After $t = 1/\theta$, firm 1 recognizes that firm 2 will price optimally against its chosen price when it has the opportunity to update.

There are two special cases of this pricing model that we now highlight. When $\alpha = 0$, firm 1 considers only the current price of firm 2. Roughly speaking, firm 1 places zero weight on the ability of firm 2 to react to a price change by firm 1. This can arise when $\theta = 1$, i.e., when firms have symmetric technology and set prices simultaneously. Thus, our model nests the usual Bertrand-Nash equilibrium assumption that firm set prices while holding fixed the prices of rivals.

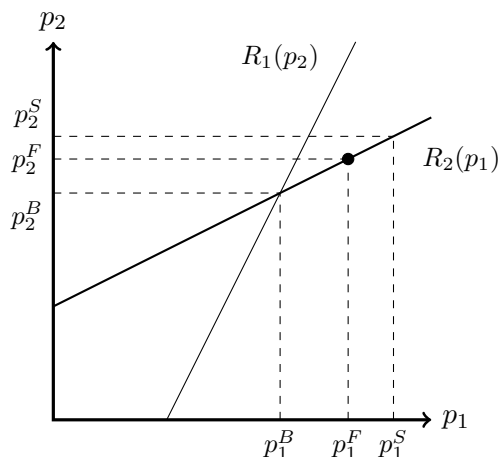
The second special case is when $\alpha = 1$. In this case, firm 1 only considers its profits after firm 2 has a chance to update its price. Roughly speaking, firm 1 fully internalizes the reaction of its rival. This can arise when $\theta \rightarrow \infty$, i.e., when firm 2 has much faster pricing technology than firm 1. The result is equivalent to a sequential pricing model, where first firm 1 chooses a price and then is followed by firm 2. In this way, our model provides a foundation for the sequential pricing game analyzed in the theory literature but rarely in applied work.

Depending on the underlying parameters, the model can capture both simultaneous and sequential price-setting behavior. More generally, the asymmetric technology allowed for in our model provides a foundation for a rich set of equilibrium outcomes that capture a mix of the incentives in these games. We now provide our first proposition, which describes the set of equilibrium outcomes for any value of α :

Proposition 1. *In the pricing frequency game, the equilibrium prices will lie on the faster firm's Bertrand best-response function between the Bertrand equilibrium and the sequential pricing equilibrium.*

¹⁸When the stage game interval is small, it is reasonable to assume that demand arrives uniformly and that $\rho = 0$, in which case we have the simple expression $\alpha = \frac{\theta-1}{\theta}$.

Figure 1: Equilibrium in the Pricing Frequency Game



Notes: Figure plots the best-response functions $R_1(\cdot)$ and $R_2(\cdot)$ for simultaneous price competition with differentiated products. The intersection of these functions produces the Bertrand-Nash equilibrium (p_1^B, p_2^B) . The point (p_1^S, p_2^S) indicates the equilibrium of the sequential pricing game. The point (p_1^F, p_2^F) is the equilibrium of a pricing frequency game, which lies between (p_1^B, p_2^B) and (p_1^S, p_2^S) .

Proof: We have established that firm 2's price will lie along its Bertrand best-response function, as it always treats firm 1's price as given. When $\alpha = 0$, the problem is equivalent to a simultaneous Bertrand pricing game. Note that this is obtained when $\theta = 1$, in which case the game corresponds exactly to simultaneous price setting. Denote the optimal price in this game p_1^B . When $\alpha = 1$, the game is equivalent to a sequential price-setting game, where firm 1 is the leader and firm 2 is the follower, with optimal price p_1^S . Because the profit function is quasiconcave, the price that maximizes the weighted sum of $\pi_1(p_1, p_2)$ and $\pi_1(p_1, R_2(p_1))$ lies in between p_1^B and p_1^S . QED.

Figure 1 illustrates the equilibrium of the game. When firms are very impatient or most consumers arrive before firm 2 can update its price, the equilibrium will resemble Bertrand (p^B). When firms are patient and all consumers arrive after firm 2 can update its price, the equilibrium resembles sequential price setting (p^S). The equilibrium prices p^F can fall anywhere between these points, depending $m(t)$, θ , ρ , and the profit functions. Note that p^F is not necessarily a linear combination of p^B and p^S ; it is in the figure because the best-response function is linear.

We conclude this section by showing that higher prices resulting from asymmetric pricing frequency are a general result for a large class of problems. Consider a typical case where the products are substitutes (i.e., $\frac{\partial q_1}{\partial p_2} > 0$) and prices are strategic complements (with upward-sloping best-response functions in the price-setting game, $\frac{\partial R_2}{\partial p_1} > 0$). Under these conditions, the sequential price-setting equilibrium will have higher prices than the Bertrand equilibrium.

Thus, we obtain our second proposition:

Proposition 2. *Suppose firms produce substitute goods and prices are strategic complements. In the pricing frequency game, both firms realize higher prices compared to the simultaneous price-setting (Bertrand-Nash) equilibrium.*

Proof: Above, we have demonstrated that firm 1's price lies between the Bertrand price p_1^B and the sequential equilibrium price p_1^S . It suffices to show that $p_1^B < p_1^S$, in which case the optimal price lies on $[p_1^B, p_1^S]$.

Consider firm 1's first-order condition to maximize profits (π):

$$\frac{d\pi_1}{dp_1} = \frac{\partial\pi_1}{\partial p_1} + \frac{\partial\pi_1}{\partial p_2} \frac{\partial p_2}{\partial p_1} = 0 \quad (5)$$

In the simultaneous price-setting equilibrium, firm 1 takes firm 2's price as given ($\frac{\partial p_2}{\partial p_1} = 0$), and $\frac{\partial\pi_1}{\partial p_1} = 0$. In the sequential game, firm 1 recognizes that $\frac{\partial p_2}{\partial p_1} = \frac{\partial R_2}{\partial p_1} > 0$ (by strategic complementarity) and $\frac{\partial\pi_1}{\partial p_2} > 0$ (because the products are substitutes). Therefore, relative to the Bertrand-Nash prices, firm 1 has an incentive to raise its price in the sequential game: $\frac{d\pi_1}{dp_1} > 0$. Firm 1's optimal price will be strictly greater than p_1^B when $\alpha > 0$ and the profit function is well-behaved. Higher prices for both firms result from strategic complementarity. QED.

3.3 Pricing Frequency Game: Example

We have described above conditions under which a dynamic game of price competition with asymmetric pricing frequency can be broken down into single-period stage games. We now provide an example to help fix ideas. In this game, firms compete for demand over a single period. Each firm produces a single product and set prices to maximize profits. Firms initially set prices at the beginning of the period, and, depending on the technology, can update prices throughout the period.

We assume that demand is such that products are (imperfect) substitutes and prices are strategic complements. In particular, we use a variant of the Hotelling (1929) model, with fixed locations and an outside option.¹⁹ Where the utility from both goods is positive, the (local) demand for each good has the convenient linear form:

$$q_j(t) = \frac{1}{2}m(t)(1 - p_j + p_{-j}).$$

¹⁹Each consumer i receives utility v from consuming the good and has disutility of τd_{ij} for the distance d_{ij} they travel to purchase from firm j . We set $v = 2$ and $\tau = 1$. Utility is linear in income and is normalized so that the marginal utility of income is 1. Consumer locations are uniformly distributed and the value of not purchasing is normalized to have zero utility.

We assume $\int_0^1 m(t)dt = 2$. Because equilibrium prices are invariant throughout the period, we can integrate over t to obtain $q_j = 1 - p_j + p_{-j}$ for each firm.

As above, firm 1 sets its price at the beginning of each period, whereas firm 2 can update its at a frequency of $\theta \in \mathbb{N}$, corresponding to elapsed intervals of $T_2 = 1/\theta$. Firm 2's price will lie along its best-response function. Firm 1 will internalize the reaction by firm 2, choosing its price to maximize the profit function given by equation (4). In this example, the equilibrium prices are given by

$$\begin{aligned} p_1 &= \frac{3}{3 - \alpha} \\ p_2 &= \frac{6 - \alpha}{6 - 2\alpha}, \end{aligned} \tag{6}$$

where $\alpha = \left(\int_0^1 e^{-\rho t} m(t) dt\right)^{-1} \int_{\frac{1}{\theta}}^1 e^{-\rho t} m(t) dt$. in general, prices depend on the relative level of technology of firm 2 (θ), as well as the discount rate ρ and the arrival rate of consumers $m(t)$.²⁰ Note that, even with linear demand, equilibrium prices may have a nonlinear relationship with α or θ .

To illustrate the impact of pricing technology in this example, we consider three cases. First, consider the standard case where firms have symmetric technology, i.e., $\theta = 1$. This corresponds conceptually to a game in which firms use human agents to set prices. In this case, $\alpha = 0$, and thus equilibrium prices, $p_1 = p_2 = 1$, and profits, $\pi_1 = \pi_2 = 1$, are equivalent to the simultaneous Bertrand-Nash equilibrium.

Now consider the case in which firm 2 adopts new pricing technology and is able to adjust prices at a higher frequency than firm 1. This implies that $\theta > 1$ and $\alpha > 0$. From equation (6), we can see that firm 1 and firm 2 increase their prices, but firm 2 chooses a lower price than firm 1. This result has an intuitive logic: firm 2 commits to “undercut” the price of firm 1, maximizing its own profits conditional on its rival's price. This softens firm 1's incentive to compete on price. For example, when $\alpha = \frac{1}{2}$ (which may correspond to $\theta = 2$), firm 1 chooses a price of 1.2 and firm 2 chooses a price of 1.1. Firm 1 loses market share to firm 2, as equilibrium quantities are (0.9, 1.1), but profits are (1.08, 1.21), which are higher for both firms than in the Bertrand equilibrium.

Finally, consider the case in which firm 2's technology is much more advanced, allowing them to update prices “in real time.” In our model, this corresponds to $\theta \rightarrow \infty$ and $\alpha = 1$. Firm 1 now fully internalizes the reaction of firm 2 and chooses a price of 1.5. This leads firm 2 to price at 1.25. Quantities are (0.75, 1.25), and profits are (1.125, 1.5625), resulting in an equivalent outcome to the sequential pricing game.

The Bertrand-Nash logic uses a dynamic metaphor to rule out the above outcomes: if firm

²⁰When demand arrives uniformly throughout the period and $\rho = 0$, we can represent equilibrium prices as function of the faster firms technology, θ : $p_1 = \frac{3\theta}{1+2\theta}$ and $p_2 = \frac{1+5\theta}{2+4\theta}$.

2's price is fixed at either 1.1 or 1.25, firm 1 has a unilateral incentive to reduce prices, which would then induce a reaction by firm 2, and so on until the Bertrand-Nash equilibrium is obtained. Though both firms may recognize that they would be better off by not undercutting the competitor, they cannot credibly commit not to (especially in a one-shot game). However, since firm 2 is able to undercut firm 1's price through more frequent pricing, firm 1 is able to internalize firm 2's reaction and maintain prices that are above the Bertrand equilibrium. In this way, the model provides a foundation for commitment; such commitment is necessary to generate higher prices than the Bertrand game.

3.4 Endogenous Pricing Technology

We have characterized a pricing game in which firms may differ in their pricing technologies. Here, asymmetry is essential to generating higher prices. If firm 1 adopts technology that enables it to update prices at the same frequency as firm 2, then the equilibrium prices return to the Bertrand-Nash equilibrium. For this reason, firm 1 has a disincentive to upgrade its technology to match that of firm 2.

Thus, when firms can choose the pricing frequency in this model, asymmetric frequencies are the equilibrium outcome. We formalize this result by modeling a first-stage adoption decision in Appendix A, but the result is quite intuitive. Whenever firms choose the same technology, Bertrand prices result. Each firm has a unilateral incentive to move away from symmetric technology, and they would do so if the cost to change technology were not prohibitively high. A firm may adopt costly technology even if its rival gains more from the outcome, as the firm prefers this outcome to the world in which neither firm adopts. Conversely, a firm may even pay to *downgrade* its technology to avoid the Bertrand outcome. In other words, firms may be willing to disadvantage themselves relative to their rivals to gain the benefits of softened price competition. For these reasons, we might not expect simultaneous price-setting behavior to hold in equilibrium.²¹ This result raises some interesting considerations for empirical researchers, for whom simultaneous price-setting behavior is the standard assumption.

In Section 5, we document that asymmetric pricing technology is a key feature of major online retailers in the U.S., which is consistent with the unilateral incentives described above. However, in some cases, symmetric pricing frequencies do arise in the real world. We believe there are other factors that help to maintain symmetric pricing frequency in equilibrium. First, a potential benefit of frequent price changes is the ability to adapt to time-varying demand conditions (so-called "dynamic pricing"). Second, changing one's pricing frequency is not costless; technological or operational costs may maintain symmetric frequencies in equilibrium. These features may dominate the incentive we identify here in certain settings.

²¹Hamilton and Slutsky (1990) show similar incentives in a two-stage game where firms first choose whether to move first or second. They do not address how a firm may commit to only moving once.

4 Algorithms with Commitment

The previous section discussed outcomes in which firms have asymmetries in pricing frequency. We believe the above model captures one of the essential features of pricing algorithms in the real world: namely, the ability to update prices on a more frequent basis. Roughly speaking, the frequency model corresponds to a game where the algorithms employed by firms are “fully rational,” i.e., the algorithms can continually revise their strategies so that they are optimal in every moment. In practice, this maps to an environment where firms are able update their algorithms whenever there is an opportunity to update prices, so that the encoded algorithm is not fixed and does not provide commitment.

Here, we provide a generalization of this game where we allow firms to choose different frequencies for algorithm updates and pricing updates. When prices are updated at higher frequencies than algorithms, an algorithm serves as a short-run commitment device. Roughly speaking, the algorithm enables commitment to a pricing rule that is not “rational” in the short run. From the same general model, we derive a “one-shot” game of competition in algorithms.

4.1 Setup

Two rival firms have the ability to change prices and algorithms at different frequencies. Both firms can update their algorithms and prices at $t = 0$. Firm 1 can update its algorithm at regular intervals, which we normalize to 1 ($\theta_1 = 1$). Firm 2 has weakly superior algorithm technology and can update its algorithm after intervals of $1/\theta_2$, where $\theta_2 \in \mathbb{N}$. Firms can update prices with frequencies $(\gamma_1, \gamma_2) \in \mathbb{N}$. We further assume that $\gamma_j = a_j \theta_j$, where $a_j \in \mathbb{N}$. In other words, whenever a firm can change its algorithm, it can also change its price.²²

In general, a firm’s algorithm may determine its price as a function of rivals’ prices and a rich set of observables. Non-price observables, such as cost shocks or the entire history of play, may be captured by the state vector, x_t . Formally, an algorithm is a function $p_j = \sigma_j(\hat{p}_{-jt}, x_t)$, where \hat{p}_{-jt} is the most recently observed price of the rival firm. Given our focus on Markov perfect equilibrium, we abstract away from x_t and consider algorithms that take the form $\sigma_j(\hat{p}_{-j})$. One can interpret our equilibrium analysis as conditional on a given state in a specific period.

At $t = 0$, both firms have the ability to flexibly change their algorithm, σ_j . Each firm’s strategy at $t = 0$ consists of $(p_{j0}, \sigma_{j0}(\cdot))$, where p_{j0} is the price determined while updating the algorithm and $\sigma_{j0}(\cdot)$ is the pricing rule at future opportunities. Firm 2 submits a new strategy $(p_{jt}, \sigma_{jt}(\cdot))$ when $t \in \{0, 1/\theta_2, 2/\theta_2, \dots\}$. The strategy space captures the fact that whenever a firm can make a revision to its algorithm, its rival does not take the commitment to that algorithm to be credible in that instant.²³

²²This latter assumption also provides expositional clarity. For other values of a_j , similar qualitative results may be obtained. An illustration of timing in this game can be seen in Appendix Figure 12.

²³Think of the programmer as being able to manually override, or “hardcode,” the algorithm.

Firms choose a sequence of prices and algorithms to maximize profits, conditional on the flow of consumers $m(t)$, the profit flows π_j , and the behavior of the rival firms. Let $p_1(t)$ and $p_2(t)$ denote the prices of each firm over time, and let $S_1 = \{(p_{1t}, \sigma_{1t})\}$ be the sequence of strategies chosen by firm 1 at $t = \{0, 1, 2, \dots\}$. Demand adheres to the same conditions as the previous section.

When pricing updates correspond to algorithm updates ($\gamma_1 = 1$ and $\gamma_2 = \theta_2$), we obtain the pricing frequency game of Section 3. In this game, there is no opportunity to rely on the pricing rule $\sigma_j(\cdot)$ to set prices.

In this section, we focus on two additional special cases of pricing technology:

- **Asymmetric Commitment:** We can consider a game with asymmetric commitment, where only one firm has an algorithm that commits to automatic updates as a function of its rival's price ($\gamma_1 = \theta_1 = 1$ and $\gamma_2 > \theta_2$). This game closely corresponds to the pricing frequency model. We discuss this game and the connections to the frequency game in Section 4.2.
- **Symmetric Commitment:** We consider a case with symmetric short-run commitment, which allows us to highlight the role of commitment in algorithmic pricing. We turn our attention to this case in Section 4.3.

In each case, we restrict attention to Markov perfect equilibria. Because of the synchronous nature of the updates, it suffices to analyze subgame perfect equilibrium of a single-period stage game. Using these cases, we illustrate how the changes to frequency and commitment brought about by algorithms can lead to higher prices in competitive equilibrium.

4.2 Asymmetric Competition in Pricing Algorithms

We first focus on the case in which firm 2 can commit to an algorithm that conditions on the price of firm 1, but firm 1 does not have this capability. We call this game the *asymmetric commitment game* to refer to the asymmetry in the nature of the algorithms. Though firm 1 does not automate its response to firm 2's prices, it may, in general, have an algorithm that responds to demand shocks and cost shocks, or other observables. In the absence of such features, i.e., when demand is stable, its algorithm reduces to standard price-setting behavior.

The asymmetric game is of particular interest because the real world features asymmetry in the ability of firms to monitor rivals and adjust prices. Thus, characterizing the equilibrium may help us understand real-world phenomena. Formally, the model differs from the frequency game of Section 3 by allowing the firm with superior technology to commit to a pricing function. It is a case of the general model with $\gamma_1 = 1$, $\theta_2 = 1$, and $\gamma_2 > 1$.

Conditional on firm 2's strategy $S_2 = (p_2, \sigma_2)$, firm 1's problem in the first period can be

expressed as:

$$\max_{p_1} \int_0^{\frac{1}{\gamma_2}} e^{-\rho t} \pi_1(p_1, p_2) m(t) dt + \int_{\frac{1}{\gamma_2}}^1 e^{-\rho t} \pi_1(p_1, \sigma_2(p_1)) m(t) dt. \quad (7)$$

As before, we can write firm 1's stage game problem as a weighted average of the pre-update period $(0, 1/\gamma_2]$ and the post-update period $(1/\gamma_2, 1]$:

$$\max_{p_1} (1 - \alpha) \pi_1(p_1, p_2) + \alpha \pi_1(p_1, \sigma_2(p_1)) \quad (8)$$

where $\alpha = \left(\int_0^1 e^{-\rho t} m(t) dt \right)^{-1} \int_{\frac{1}{\gamma_2}}^1 e^{-\rho t} m(t) dt$. In the asymmetric commitment game, σ_2 depends on p_1 . The duration $\frac{1}{\gamma_2}$ represents the time lag between firm 1's pricing decision and the response of the algorithm by firm 2. In this game, it is a (weakly) dominant strategy for σ_2 to mirror firm 2's best-response function. We use this result to highlight a special equilibrium where firm 2 submits its best-response function.

Proposition 3. *There exists an equilibrium to the asymmetric commitment game in which the second firm submits its best-response function as its algorithm. This strategy is weakly dominant. The first firm submits a price that maximizes its own profit along the second firm's best-response function.*

It is readily apparent that no profitable deviation exists. The firm that submits a price-dependent algorithm cannot do better than submitting its Bertrand best-response function as its algorithm, regardless of the price chosen by firm 1. Thus, this is the unique equilibrium after eliminating weakly dominated strategies.²⁴ At this equilibrium, equation (8) is equivalent to (4). Thus, the asymmetric commitment game mirrors the asymmetry pricing frequency game from Section 3. Indeed, we present our second result for this section as a corollary to Proposition 2:

Corollary. *When firms produce substitute goods and prices are strategic complements, then, in the asymmetric equilibrium where one firm submits its best-response function as its algorithm, both firms realize higher prices compared to the price-setting (Bertrand-Nash) equilibrium.*

We have shown that asymmetries in pricing technologies are sufficient to generate higher prices than the in the simultaneous price-setting equilibrium. The results from this section highlight a somewhat surprising result: asymmetries arising from either frequency or commitment generate the same outcomes in equilibrium. Thus, understanding the exact nature of the pricing strategies may matter less than accounting for asymmetries. One can model a firm with a

²⁴There are many Nash equilibria where firm 2 has an algorithm that, local to the equilibrium, the algorithm maps to the best-response function. There are fewer limitations on how the algorithm looks away from the equilibrium.

superior algorithm that conditions its rival's price as simply having the ability to update prices more frequently.

As we show next, these similarities end when considering symmetric technology. Symmetric pricing frequency leads uniquely to Bertrand prices. By contrast, when both firms have algorithms with short-run commitment, firms are able to realize higher prices and profits than the Bertrand equilibrium, despite possessing symmetric technology.

4.3 Symmetric Competition in Pricing Algorithms

We now consider the case in which both firms have algorithms that can depend on the prices of rivals. Further, these algorithms update prices at a higher frequency than the frequency which firms can update their strategies, generating short-run commitment to the strategies. Without loss of generality, we consider the first period, $t \in (0, 1]$. Our objective is to characterize the equilibrium strategies that would be chosen by both firms.

Suppose that firm 1 and firm 2 can both update their algorithms with equal frequency ($\theta_2 = 1$). Firms are also able to commit to an algorithmic pricing rule for future price updates, which occur simultaneously, with $\gamma_1 = \gamma_2 = \gamma$. Thus, initial price-setting behavior determines prices until $t = 1/\gamma$, after which the algorithms determine prices. For expositional clarity, we assume that there is no mass point in $m(t)$ at $t = 1/\gamma$ and that algorithms instantaneously converge to the “steady-state” prices, so the transition has no impact on profits. In other words, we allow the dynamic process of tâtonnement to play out in every instant.²⁵

As before, we can write firm 1's stage game problem as a weighted average of the pre-update period $(0, 1/\gamma]$ and the post-update period $(1/\gamma, 1]$:

$$\max_{p_1, \sigma_1} (1 - \alpha)\pi_1(p_1, p_2) + \alpha\pi_1(\sigma_1, \sigma_2) \quad (9)$$

where $\alpha = \left(\int_0^1 e^{-\rho t} m(t) dt\right)^{-1} \int_{\frac{1}{\gamma}}^1 e^{-\rho t} m(t) dt$.²⁶ The value $1 - \alpha$ describes the relative weight that firm 1 places on the initial period $(0, 1/\gamma]$, which is a function of ρ , $m(t)$, and θ_2 . In the initial price-setting phase, the usual Nash-in-price logic holds: firm 1 treats firm 2's price as given over the period $(0, 1/\gamma]$. After $t = 1/\gamma$, firm 1 recognizes that firm 2's algorithm will control the pricing updates, and it will choose σ_1 optimally with that in mind.

As in the asymmetric game, each firm chooses a strategy that maximizes a weighted average of two profit components. As before, the first component is equivalent to the profit function

²⁵Alternatively, one could explicitly model this process over discrete pricing updates determined by γ . Our focus is on the case where γ is large; for this case, the process has no impact on firm profits or strategies.

²⁶The simplification is possible because the profit flow function is time-invariant. The full problem is

$$\max_{p_1, \sigma_1} \int_0^{\frac{1}{\gamma}} e^{-\rho t} \pi_1(p_1, p_2) m(t) dt + \int_{\frac{1}{\gamma}}^1 e^{-\rho t} \pi_1(\sigma_1, \sigma_2) m(t) dt.$$

for the Bertrand model. The second component is different, as firm 1 chooses σ_1 while taking into account the choice of σ_2 . To make progress on understanding the equilibria of the general setup, we analyze the equilibria of the subgame in which firms choose algorithms (σ_1, σ_2) . We can treat this component as a subgame because our setup is equivalent to a model in which firms first choose prices at $t = 0$ and then choose (σ_1, σ_2) at $t = 1/\gamma$.

We now develop the equilibrium of this subgame. When $\alpha = 1$, the equilibrium will correspond to the stage game equilibrium of the full model. We consider the case of $\alpha = 1$ to be a fair approximation to price competition in which both firms have very high-frequency algorithms, e.g., algorithms that can update hundreds of prices within a 15-minute window.

4.4 Stage Game Analysis

We now define a competitive game—*competition in pricing algorithms*—and its equilibrium concept. Firms compete in pricing algorithms by submitting a pricing strategy $\sigma(\cdot)$, or “algorithm”, to a market coordinator. The algorithms may condition directly on the prices of rivals. The algorithm may also be a function of variables that are observable to the firm, but they cannot be functions of other player’s algorithms.

After receiving the pricing algorithms, the market coordinator solves the system of equations set by the algorithms to determine prices. Based on the general model developed above, the market coordinator may be thought of as the process of tâtonnement arising from an initial price vector. Without further restrictions, the game thus far described may suffer from an *indeterminacy problem*: there may be multiple solutions to the system of equations set by the algorithms. For example, consider the case where both firms submit an algorithm of the form

$$\sigma(p_{-j}) = \begin{cases} p^C, & \text{for } p_{-j} = p^C \\ p^B, & \text{otherwise} \end{cases} \quad (10)$$

where p^C is the collusive price and p^B is the punishment (Bertrand) price. Both (p^B, p^B) and (p^C, p^C) are equilibria of the system, depending on the initial price vector.

A second issue is that cooperate-or-punishment strategies like this one would raise immediate antitrust concerns if made public. We wish to analyze, fundamentally, the impact of algorithmic competition on prices. Do they lead to higher prices in the absence of behavior that “looks collusive?”

To resolve these issues, we provide a modification to the general game that results in a unique solution conditional on algorithms. When multiple solutions are possible, the market coordinator picks the solution that minimizes the profits of the firms. If multiple such solutions exist, the coordinator randomizes among them. Effectively, we allow an adversarial market coordinator to choose the initial price vector.

Restriction (Profit-Minimizing Coordinator). *In the pricing algorithm game, the market coordinator selects the solution to the system of equations determined by the algorithms that minimizes joint profits. Formally, the market coordinator chooses $p = (p_1, p_2)$ to solve*

$$\begin{aligned} \min_p \quad & \sum_{j \in \{1,2\}} \pi_j(\sigma_j(p_{-j}), \sigma_{-j}(p_j)) \\ \text{s.t.} \quad & p_j = \sigma_j(p_{-j}) \quad \forall j. \end{aligned} \tag{11}$$

This selection procedure is a natural choice for us because it provides conservative results regarding prices. We tie our hands, eliminating equilibria that mirror typical collusive strategies. In the real world, this selection procedure reflects pro-consumer market mechanisms to discipline firms.

We now define the equilibrium concept for the algorithm-setting game. In equilibrium, each firm's algorithm maximizes its own profit, conditional on the algorithms submitted by the other firms and subject to a market coordinator that minimize the joint profits when multiple solutions to the algorithms exist. We formalize this below.

Equilibrium definition: *When firms compete in pricing algorithms, equilibrium algorithms $\{\sigma_j^*\}$ satisfy*

$$\begin{aligned} \sigma_j^* &= \arg \max_{\sigma_j | \sigma_{-j}^*} \pi_j(\sigma_j(p_{-j}^*), \sigma_{-j}^*(p_j^*)) \quad \forall j \\ \text{s.t.} \quad p^* &= \arg \min_{p \in \tilde{P}} \sum_{j \in \{1,2\}} \pi_j(\sigma_j^*(p_{-j}), \sigma_{-j}^*(p_j)) \\ \tilde{P} &\equiv \{p : p_j = \sigma_j^*(p_{-j}) \quad \forall j\}, \end{aligned} \tag{12}$$

resulting in equilibrium prices $p^ = (p_1^*, p_2^*)$.*

Even subject to the profit-minimizing coordinator, many equilibrium strategies can be supported. Note that any equilibrium of the pricing algorithm game has the following property: in equilibrium, no firm can do better by submitting a single price, conditional on the algorithms of its rivals. Formally,

$$\pi_j(\sigma_j^*(p_{-j}^*), \sigma_{-j}^*(p_j^*)) \geq \pi_j(p_j, \sigma_{-j}^*(p_j^*)) \quad \forall p_j, j. \tag{13}$$

Therefore, any equilibrium lies at the intersection of modified best-response functions for price, where the best-response functions take into account the algorithms of the rivals.

Given the equilibrium concept, we now illustrate some of the similarities and differences to the asymmetric commitment game from Section 4.2. Consider a scenario in the pricing algorithm game in which firm 1 submits algorithm $\sigma_1(\cdot) = p_1^S$ and firm 2 submits algorithm $\sigma_2(p_1) = R_2(p_1)$, where $p_1^S = \arg \max_{p_1} \pi_1(p_1, R_2(p_1))$ and $R_2(\cdot)$ is firm 2's best-response func-

tion. Recall that p_1^S is equivalent to the equilibrium price of the first-mover in a sequential pricing game. As in Section 4.2, neither firm can do better with a unilateral deviation. Thus, this asymmetric case—where one firm submits the price, and the other a function of that price—is an equilibrium of a game even when both firms have the technology to condition on the prices of rivals.

If both firms were to instead submit their best-response functions from the price-setting game, $\sigma_j(p_{-j}) = R_j(p_{-j})$, the unique price vector that satisfies both algorithms is the Bertrand equilibrium. Thus, as in Section 4.2, firm 1 can do strictly better by submitting $\sigma_1(\cdot) = p_1^S$ instead of $\sigma_1(\cdot) = R_1(p_2)$. Therefore, $(\sigma_1, \sigma_2) = (R_1, R_2)$ is not an equilibrium of the algorithm-setting game. This is a central negative result of our model.

Proposition 4. *When firms compete in a one-shot game by submitting pricing algorithms, it is (in general) not an equilibrium for each firm to submit their price-setting best-response function.*

Proof: By the above reasoning, individual firms can realize a profitable deviation by submitting a price that lies along their rival’s best-response function and results in greater profits to the firm. QED.

When firms compete in algorithms, the algorithms will not reflect the price-setting best-response functions in equilibrium. That is, if each firm’s algorithm conditions on its rival’s price, algorithms cannot be “competitive” in equilibrium.²⁷

Though it is not an equilibrium for each firm to submit their Bertrand best-response functions, the symmetric commitment game admits a multitude of possible equilibria. We provide a formal proof and a discussion of equilibrium selection in Appendix B. Despite this result, we expect algorithms to result in higher prices than the Bertrand-Nash equilibrium for three reasons. First, when algorithms have positive slope coefficients on rivals’ prices, higher prices result. Imposing this restriction on firms’ choices seems reasonable a priori when prices are strategic complements. In other words, prices that are lower than Bertrand-Nash are supported only when an algorithm treats the rival prices as strategic substitutes, despite the complementarity.

Second, many of these equilibria are “knife-edge” cases. To examine which equilibria are, in some sense, more robust, we simulate a simple learning process in Appendix B. Firms experiment with algorithms that are linear functions of rivals’ prices, updating the parameters if profits increase. From a starting point of randomly-chosen algorithms, firms disproportionately arrive at equilibria that are bounded from below by their best-response functions and bounded from above by the profit Pareto frontier. Our simulations show that higher prices result.

²⁷Interestingly, the Bertrand-Nash solution is still an equilibrium of the game. However, it is only an equilibrium if the algorithms do not depend (locally) on rivals’ prices. For example, $p^B = (p_1^B, p_2^B)$ is obtained in equilibrium if both firms resort to simple price-setting technology, with algorithms $\sigma_j(p_{-j}) = p_j^B$. More generally, when $\sigma_j(\cdot)$ is differentiable at p_{-j}^B , a necessary condition to obtain p^B in equilibrium is that $\partial\sigma_j(p_{-j})/\partial p_{-j} = 0 \ \forall j$. Otherwise, the reaction by rivals creates an incentive to deviate from the Bertrand price.

4.5 Algorithms, Supracompetitive Prices, and Collusive Prices

We have, thus far, address two questions related to the use of algorithms and supracompetitive prices. First, we have demonstrated that asymmetries in frequency and commitment—key features of pricing algorithms—lead to higher prices than the Bertrand equilibrium. Thus, by unilaterally changing one’s pricing technology, a firm can increase its prices and profits above the usual competitive benchmark. In other words, technology provides firms with a means to increase profits without resorting to collusion.

Second, we have shown that algorithms that depend on rivals’ prices *cannot* be competitive in equilibrium. Thus, if all firms use algorithms that condition on rivals’ prices, we might expect supracompetitive prices to result. As discussed above, sensible restrictions on equilibrium strategies do result in higher prices. Simulations that provide firms with a simple reinforcement learning rule to select strategies provide additional support for this conclusion.

We now address a third question: Can algorithms be used to obtain collusive outcomes in competitive equilibrium? In other words, are collusive profits possible in Markov perfect equilibrium? We again focus on one-shot mechanics of the symmetric commitment game, allowing each firm to commit to an algorithm that is optimal conditional on the algorithm of its rival.

As discussed above, our restrictions rule out the typical strategies to sustain collusive behavior. However, the collusive outcome can be supported by algorithms that satisfy the restrictions. For example, in the model of demand in Section 3.3, the collusive outcome is $(p_1, p_2) = (\frac{3}{2}, \frac{3}{2})$. This is an equilibrium with the following strategies:

$$\sigma_j(p_{-j}) = 1 + \frac{1}{3}p_{-j} \quad (14)$$

It is straightforward to verify that, conditional on these algorithms, no firm wishes to deviate in its algorithm and the collusive price results. In fact, the collusive outcome $p^C = (p_1^C, p_2^C)$ can be achieved in equilibrium in general with simple linear algorithms. These algorithms take the form

$$\sigma_j(p_{-j}) = p_j^C + b_j(p_{-j} - p_{-j}^C), \quad (15)$$

where b_j is chosen to eliminate any incentive for the rival firm ($-j$) to deviate in prices.²⁸

Thus, simple linear strategies, which may not raise competitive concerns *prima facie*, can support fully collusive prices when all firms employ algorithms. These results suggest that the widespread concerns about the level of sophistication in algorithms may be somewhat misplaced. With commitment, firms can sustain collusive profits with algorithms that appear to be quite unsophisticated.

The intuition behind higher prices in our model is related to the logic of how price-matching guarantees may lead to higher prices: if a firm (credibly) commits to match the price of its rival,

²⁸Specifically, $b_j = - \frac{\partial \pi_{-j} / \partial p_{-j}}{\partial \pi_{-j} / \partial p_j} \Big|_{p^C}$. See derivation in Appendix B.

then the rival has a reduced incentive to lower its price. Our model allows price matching as a possible strategy, and it is straightforward to show that pure price-matching algorithms do not arise in equilibrium. If one firm chooses the price-matching algorithm $\sigma(p_{-j}) = p_{-j}$, the other will pick the collusive price. But, conditional on the second firm's price, the first firm will want to deviate along its best-response function. If both firms choose price-matching algorithms, then the adversarial market coordinator is free to pick any price that delivers the lowest profits.

Our model of symmetric commitment is also related to the analysis of conjectural variations. One important distinction is that the conjectural variations literature has attempted to restrict the set of equilibria to those in which the conjectural variations are consistent with the beliefs and actions of the other players (e.g., Bresnahan, 1981; Kamien and Schwartz, 1983; Daughety, 1985; Lindh, 1992). In the equilibria of our model of pricing algorithms, firm's beliefs are consistent with the pricing strategies (algorithms) played by other firms, yet *any* conjectural variation equilibrium may be supported, regardless of whether it is an equilibrium in consistent conjectures with the price-setting game.

Thus, our general model unifies several different pricing games (e.g., Bertrand, sequential pricing, conjectural variations) under the same set of primitives. We view algorithms as providing a real-world foundation for many classic models of price competition. By nesting these models under a common structure, we also provide a framework for firms to choose among different models of competition by changing their pricing technology. Our model also provides a basis for more flexible assumptions of price competition that can be adapted to empirical settings. We demonstrate the importance of accounting for pricing technology when examining competition empirically in Section 6.

4.6 Pricing Algorithm Game: Oligopoly Example

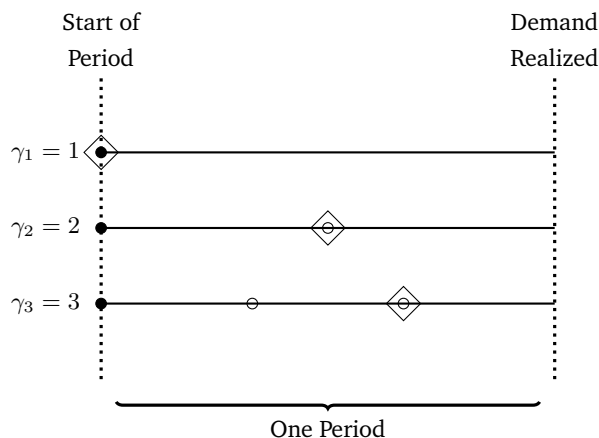
To extend the intuition of asymmetry in pricing algorithms beyond duopolistic competition, we consider an oligopoly setting with three firms. We simulate equilibrium prices in the model with the aim of comparing model predictions to our empirical findings in Section 5. Similar qualitative results can be obtained for any number of firms.

Demand remains similar to that of the model in section 3.3, but the three firms are now located at equidistant 1-unit intervals along a circle with circumference of 3. Thus, we use the Salop (1979) model to characterize demand. Each unit of the circle's circumference contains a mass of 1 consumers. Consumers maintain travel costs as before. Where the utility from both goods is positive, the (local) demand for each good is:

$$q_j = 1 - p_j + \frac{1}{2} \sum_{k \neq j} p_k \quad (16)$$

As before, the Bertrand-Nash equilibrium is $p_j = 1$ and the collusive price is $p_j = \frac{3}{2}$ (for all j).

Figure 2: Timing for Oligopoly Example



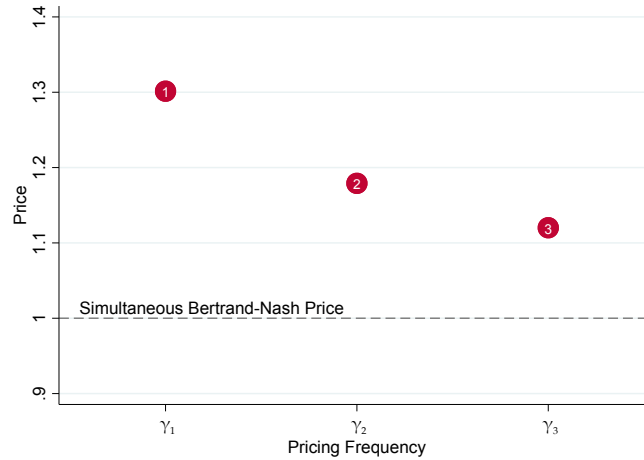
Notes: Solid black markers represent opportunities to adjust algorithms and update prices. Open circles indicate opportunities to update prices based on the previously-determined algorithm. Algorithm updates are governed by $\theta = 1$ and pricing updates are governed by γ . Diamonds indicate consequential opportunities to adjust prices when all pricing updates can occur before demand is realized.

Now assume that there are three levels of pricing technology. Firm 1 has inferior pricing technology and can update prices only at the beginning of the period. Firm 2 has more frequent pricing, allowing it to react to firm 1 before the end of the period. Firm 3 has superior technology and can update prices in response to both firm 1 and firm 2. In particular, assume $\gamma_3 > \gamma_2 > \gamma_1 = \theta$ and $\theta_j = \theta \forall j$. Furthermore, assume the differences in pricing frequency are large so that it is as if firms with faster algorithms react instantly to slower rivals. In other words, the faster algorithms can react before demand is realized.

Figure 2 illustrates how timing works in this oligopoly example. When the algorithms can react faster than demand is realized, any set of technology satisfying $\gamma_3 > \gamma_2 > \gamma_1$ will have equivalent strategic effects. In the figure, we show the edge case when $(\gamma_1, \gamma_2, \gamma_3) = (1, 2, 3)$ and all demand is realized at the end of the period. Open circles indicate pricing updates determined by the algorithms for firms 2 and 3. Diamonds indicate pricing decisions that are consequential for the realized demand. Effectively, firms with superior technology have a last-mover advantage for price. Variation in pricing technology can sort firms into a sequential pricing game, with the pricing order given by γ_j . Thus, pricing frequency provides a simple economic mechanism for firms to commit to a specific sequence, even in oligopoly settings.

Figure 3 demonstrates the equilibrium prices of the model compared to the simultaneous price-setting benchmark. Firm 1, which has the slowest pricing technology, has the highest price. Firm 3, which has the fastest pricing technology, has the lowest price. The model implies that prices are monotonically decreasing in pricing algorithm frequency. Furthermore, all prices in the pricing algorithm equilibrium are higher than those from the Bertrand-Nash equilibrium. Firms with inferior technology choose to compete less aggressively, as firms with superior tech-

Figure 3: Simulated Pricing Algorithm Equilibrium



Notes: Figure displays the prices for three firms from a simulation of competition in pricing algorithms with heterogeneous pricing technology. Pricing frequency of 3 is the superior (i.e., fastest) pricing technology. Marker labels indicate the firm.

nology can credibly commit to offering lower prices. Within a pricing algorithm equilibrium, more frequent pricing is correlated with lower prices, but all prices are elevated relative to the case where all firms have the slowest technology.

5 Empirical Evidence on Pricing Technology and Prices

In this section, we introduce a novel dataset of high-frequency prices for competing retailers. We then explore the differences in pricing technology across the retailers in our dataset. We document systematic differences in when price changes occur. We then examine the linkages between pricing technology and the predictions of our theoretical model.

5.1 Data

For our empirical analysis, we collected a novel dataset of hourly prices for over-the-counter allergy drugs from five online retailers. The retailers are the five largest in the allergy category based on Google search data and are among the largest retailers overall by e-commerce revenues.²⁹ We have kept the identities of the retailers anonymous, calling them *A*, *B*, *C*, *D*, and *E*. For each of these retailers, allergy drugs represent an important product category. All five retailers sell products in many other categories, and four of the five have a large in-store presence in addition to their online channel. Prices are set uniformly for online shoppers across the United States.

²⁹According to ecommerceDB (<https://ecommercedb.com/>), these five retailers combined for \$6 billion in e-commerce revenues for personal care, which includes medicine, cosmetics, and personal care products.

Table 1: Price Observations by Website and Brand

Retailer	Allegra	Benadryl	Claritin	Flonase	Nasacort	Xyzal	Zyrtec	Total
A	309,554	208,422	509,404	104,634	68,858	108,854	234,903	1,544,629
B	125,095	58,270	144,098	46,584	12,517	34,177	75,096	495,837
C	89,477	99,608	171,782	80,772	34,633	32,508	90,858	599,638
D	112,281	68,459	128,394	50,130	2,411	47,321	128,123	537,119
E	71,061	47,799	125,171	51,732	38,051	23,185	62,600	419,599
Total	707,468	482,558	1,078,849	333,852	156,470	246,045	591,580	3,596,822

Notes: Count of price observations for the sample period from April 10, 2018 through October 1, 2019.

We focus on the seven brands of allergy drugs that are sold by all five retailers: Allegra, Benadryl, Claritin, Flonase, Nasacort, Xyzal, and Zyrtec.³⁰ This set of products provides a relatively straight-forward set of competing products in which to examining pricing technology in detail, however we believe our analysis of firms' pricing technology applies more broadly to other products sold by the retailers.

We define a product to be a drug-brand-form-(variant-)size combination, e.g. Loratadine-Claritin-Tablet-20. Each of the retained brands specializes in one drug, but they often offer the products in multiple forms (e.g., Liquid Gels, Liquid, or Tablets). Each brand offers many different size options, so there are several products per brands. In addition, most brands offer variants with different amounts of the active drug, targeted for children, 12-hour or 24-hour use. There are also versions of the drug that are combined a decongestant. These varieties are captured by the *variant* of the drug. Finally, we distinguish products that are sold in a twinpack, so that twinpack of 12 tablets is a different product than a single pack of 24 tablets.³¹ When a retailer sells multiple versions of the same product, we select the most popular version by retaining the version that has the greatest number of reviews, on average, in our sample. Our dataset spans April 10, 2018 through October 1, 2019, resulting in 3,596,822 price observations across the five websites. See Table 1 for a tabulation. Retailers *A* and *B* offer significantly more product varieties than the other retailers. This is primarily due to the number of size options offered for each brand.

Obtaining online prices can be challenging, as updates to price information may take a while to propagate through the network, retailers can have complicated websites that take time to load, and the websites tend to change over time. These features are reflected in our raw data, and we have taken steps to eliminate measurement error. First, we have focused on high-volume brands, helping to ensure the availability of price information. Second, we use supplemental information obtained at the time of our price sample to rule out price changes brought about by a lag in the website. For example, we can see if the description of the product

³⁰Our sample consists of products sold directly by retailers and not products in which a third-party seller sets the price. Third-party sellers are less popular for allergy products.

³¹We drop multipacks that are of greater size than a twinpack, as they are not common across retailers.

Table 2: Summary Statistics for Hourly Prices by Retailer

	Retailer					All
	A	B	C	D	E	
<i>Daily Mean</i>						
Count of Products	124.2	41.1	49.9	42.5	35.1	58.6
<i>Daily Mean per Product</i>						
Observations	20.9	20.4	19.0	21.1	19.1	20.1
Count of Reviews	101.1	231.9	258.5	241.3	302.1	219.4
<i>Price Statistics</i>						
Mean Price	27.354	16.185	17.628	20.925	21.742	20.750
Mean Abs. Price Change	1.358	2.236	1.124	3.281	3.063	1.891
Count of Price Changes	1.858	0.285	0.008	0.021	0.025	0.441
Any Price Change	0.372	0.088	0.008	0.020	0.024	0.103

Notes: Statistics are calculated by website by day.

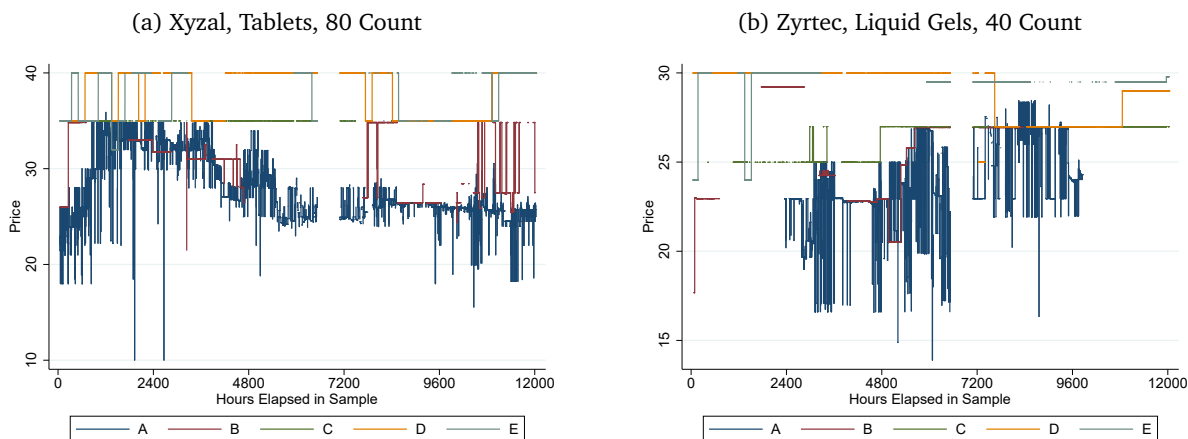
is consistent over time. Third, we impute missing prices by filling in missing prices with the most recently observed price if the gap of missing prices is fewer than six hours. Finally, for the three retailers that do not change prices hourly, we smooth over single-period blips in price that revert back to the earlier price.³² Figure 13 in the Appendix displays the count of products in our sample over time.

Summary statistics for our data are presented in Table 2. On average, we observe 124 products each day for retailer *A*, compared to 41 products for retailer *B*. Across all retailers, we observe the price for each product in 20 out of 24 hours on average. The mean price for these products is \$20.75, with a mean (absolute) price change of \$1.89. The table indicates stark differences in the frequency of price changes. Retailer *A* changes the prices of 37 percent of its products in a given day, with an average count of 1.9 price changes per product. At the other extreme, retailer *C* only changes the price of 0.8 percent of its products each day, making a single change when it does so.

Figure 4 displays example time series for two products in our sample: Xyzal-Tablet-80 and Zyrtec-Liquid Gel-40. These two examples illustrate fundamentally different pricing patterns across the five retailers. Retailer *A* has frequent price changes of a large magnitude, but prices that are on average lower than its competitors. Retailer *B* has price movements that are closer to *A*, though less frequent, whereas *C*, *D*, and *E* tend to have more similar prices.

³²Overall, 7.8 percent of the prices are imputed in our analysis sample.

Figure 4: Example Time Series of Prices



Notes: Figure displays the time series of hourly prices in our dataset for two example products across five retailers. Panel (a) displays the prices for an 80-count package of Xyzal tablets. Panel (b) displays the prices for a 40-count package of Zyrtec liquid gels.

5.2 Heterogeneity in Pricing Technology

The previous section showed that there is variation across the five retailers in terms of how frequently they change prices. This fact alone does not demonstrate that the retailers possess different technologies, in terms of the capability to change prices quickly. Whether or not a price actually changes may not reflect the underlying capability to change price; for example, some products at retailer A have long periods of stable prices in the data.

However, examining the data further reveals that the five retailers possess very different pricing technologies, which we define as the maximum frequency with which firms can adjust prices. Figure 5 displays the heterogeneity in price changes by day of the week and hour of the day. First, panel (a) of Figure 5 presents the occurrence of price changes by day of the week. Though A, B, and C have roughly equal amounts of price changes throughout the week, retailers D and E realize nearly all of their price changes on Sunday.

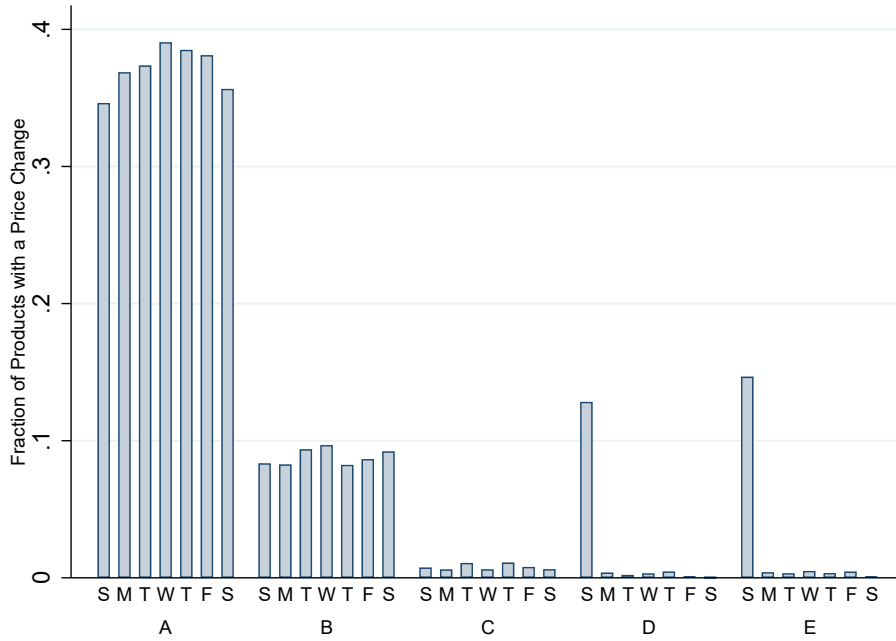
Second, panel (b) presents the distribution of price changes across hours of the day. Retailers A and B have well-dispersed price changes across the 24 hours of the day. By contrast, C, D, and E have nearly all observed price changes occurring within a few hours in the morning.³³ Firms D and E begin their price update script around midnight EDT. Thus, we observe that A and B have pricing technology that allows for updates at any hour of the day, C has technology that allows for a daily update each morning, and D and E have technology that allow them to update their prices once per week (on Sundays). Table 3 summarizes these findings.

Pricing technology, in the sense of this paper, is directly linked to the frequency with which firms are able to update prices. We highlight the stark differences in the distribution of observed

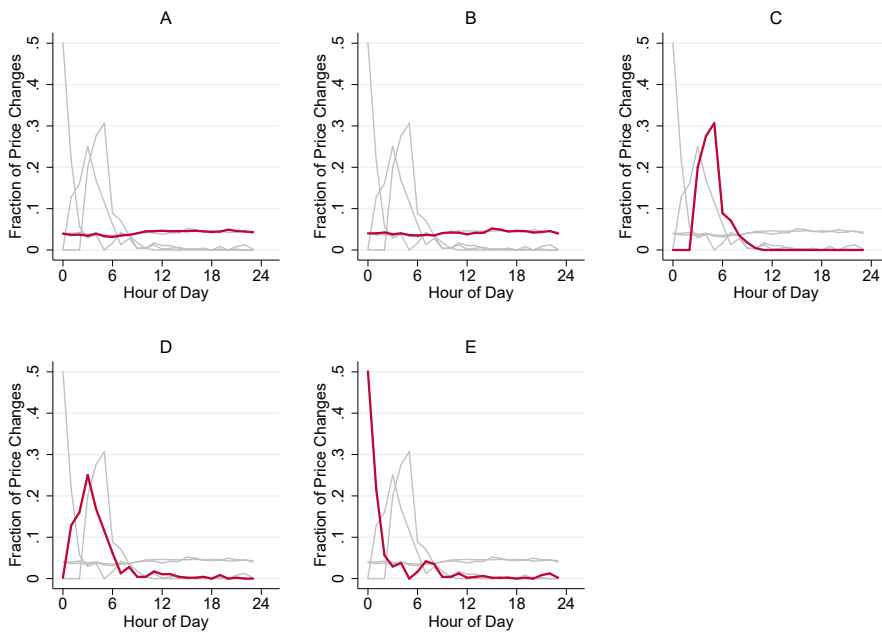
³³Several of the changes that occur away from these peaks are likely due to measurement error.

Figure 5: Heterogeneity in Pricing Technology

(a) Daily Price Changes, by Retailer and Day of Week



(b) Hourly Price Changes, by Retailer



Notes: Panel (a) displays the fraction of products with a price change in each day of the week, by day of week and retailer. Panel (b) displays the fraction of all price changes that occur at a given hour of the day, by retailer. Hours are reported in Eastern Time.

Table 3: Pricing Frequency by Online Retailers

Retailer	Frequency	Period
A	Hourly	Any time
B	Hourly	Any time
C	Daily	3:00 AM to 6:00 AM EDT
D	Weekly on Sunday	1:00 AM to 6:00 AM EDT
E	Weekly on Sunday	12:00 AM to 2:00 AM EDT

Notes: Table summarizes the pricing technology of the five retailers in our data.

price changes as pointing to heterogeneity in pricing technology. Though firms do not use every opportunity to change prices—recall that firm *C* changes the prices of less than one percent of its products each day—we find the consistency in the times that price changes occur as compelling evidence of technological constraints.

Of course, our definition of technology is not merely the set of hardware and software that functionally updates a price on website. Technology also includes managerial and operational constraints that restrict a firm from updating prices on a more frequent basis. Put differently, even if firm *C* had access to the same hardware and software as *A*, it would take significant operational changes to enable the firm to update its prices as frequently.

5.3 Evidence of Competitive Effects

Having established that the five retailers in our data have different technologies affecting the frequency at which they can update prices, we now examine the pricing patterns in more detail to determine whether the data are consistent with the model of Section 3. The theory generates a stark prediction: firms that have higher-frequency pricing technology will have lower prices. Again, the intuition is higher-frequency pricing allows a firm to commit to meet its rival’s best price; as a consequence, the rival prices less aggressively.

To examine this prediction, we wish to compare the price of identical products across retailers. We use a regression in order to account for differences in product assortment in the cross-section and over time. More specifically, we regress log prices on indicators for each retailer, while including product and period (hourly) fixed effects. The resulting coefficients reflect the average difference in (log) price for identical products (brand-drug-form-variant-size) sold across different retailers at the same point in time.

Table 4 presents the results. Retailer *A* serves as a baseline, so the coefficients reflect the average difference in log price relative to *A*. Relative to retailer *A*, products are typically sold at a 6.8 percent (0.066 log point) premium at *B* and a 9.5 percent (0.091 log point) premium at *C*. These same products are sold at a substantial premium at retailers *D* and *E*, who have average price differences of 28 percent and 33 percent, respectively. We observe the same qualitative

Table 4: Log Price Differences Across Retailers, Relative to A

	(1)	(2)	(3)	(4)
B	0.066*** (0.000)	0.047*** (0.001)	0.146*** (0.000)	0.117*** (0.001)
C	0.091*** (0.000)	0.107*** (0.001)	0.171*** (0.000)	0.187*** (0.001)
D	0.249*** (0.000)	0.289*** (0.001)	0.307*** (0.000)	0.337*** (0.001)
E	0.284*** (0.000)	0.366*** (0.001)	0.340*** (0.000)	0.419*** (0.001)
Product FEs	X	X	X	X
Period FEs	X	X	X	X
Sold at All Retailers			X	X
On or After Jul 1 2019		X		X
Observations	3,596,822	673,771	1,186,534	234,696

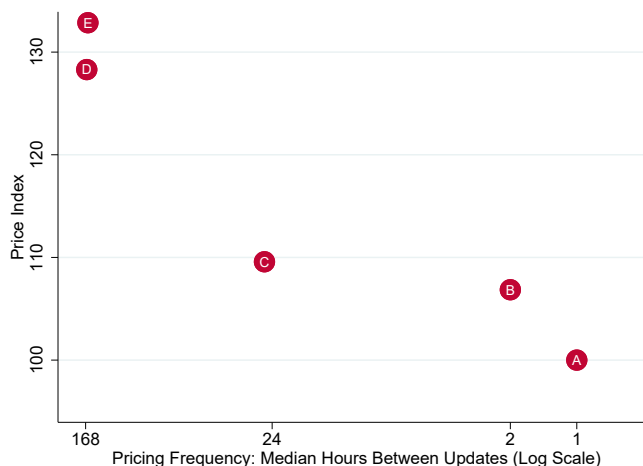
Standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

patterns if we vary our estimation sample. Models (2) and (4) use observations from the most recent three months of the data, and models (3) and (4) includes only products sold by all five retailers. The results remain qualitatively similar, though the price differences between A and the rest increase when we restrict the sample.

We plot the (scaled) coefficients from specification (1) against a measure of pricing technology in 6. The x -axis captures the pricing frequency, which increases along the x -axis. We report the frequency as the median number of hours between any pricing update on each website; the axis values are reversed so that superior (more frequent) technology is to the right. Firm *E* has a median approximately equal to the number of hours in a week (168), whereas firm *A* has a median of 1. The resulting price patterns are consistent with the model described in Section 3. Firm *A* has implemented a pricing technology that enables them to perform frequent updates, and *A* has the lowest prices. This is in line with the prediction that a faster pricing algorithm enables a firm to best respond to its competitors, resulting in a relatively lower price. The pattern also holds up if we look at firms with weekly pricing technology (*D* and *E*). These firms sell at a price substantially higher than *B* and *C*, who have more frequent pricing technology. Lastly, we note that *E* sells at a slightly lower price than *D*, and it updates its prices a few hours later.

Consistent with the model, we find that higher-frequency technology is correlated with lower prices. This is initial evidence of competition in pricing algorithms, implying higher prices than what would result from a simultaneous price-setting game. However, it is important to note that there are other reasons why prices could be higher for firms with low-frequency pricing. The primary concern in our setting is that demand may not symmetric, i.e., consumers prefer to purchase from certain firms or engage in directed search. We address this concern in the following section by accounting for flexible substitution patterns in our model of demand.

Figure 6: Price Differences Across Retailers



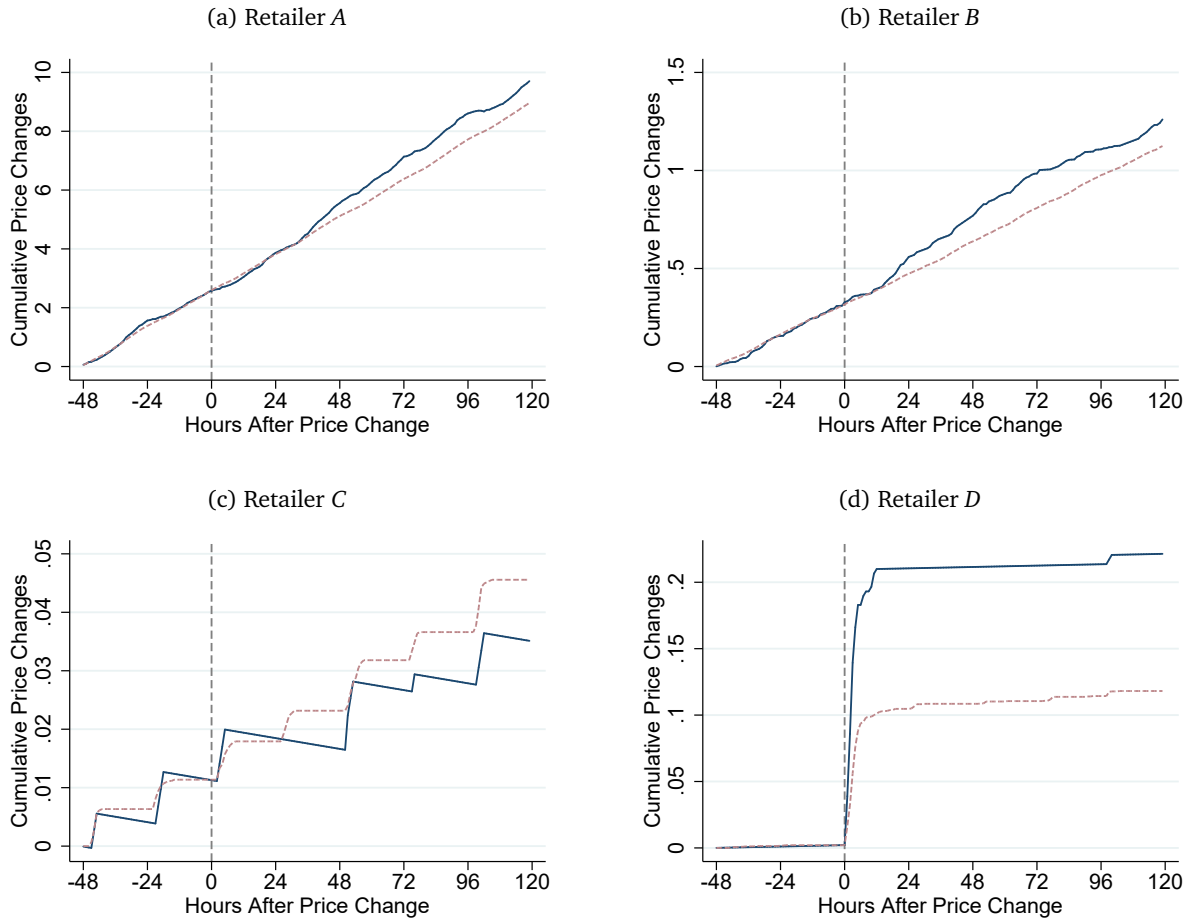
Notes: Figure displays the relative prices (Firm A = 100) plotted against the pricing frequency of each retailer. We report the frequency as the median number of hours between pricing updates. 168 hours corresponds to one week. The relative prices are obtained from the estimated coefficients in specification (3) of Table 4.

The model of asymmetric pricing technology generates a second set of predictions: If firms’ algorithms depend on rivals’ prices, then we should expect a unilateral price change by a low-frequency firm to increase the probability of a price change by the high-frequency firms. In order to examine this prediction, we analyze the timing of price changes by faster firms in response to price changes by a slower rival. In particular, a slow firm may experience an idiosyncratic cost shock due to shipping delays or have low stock in warehouses. In response to the cost shock, the slow firm may adjust prices, causing faster rivals to react. The primary concern is that price changes are endogenous and firms may be responding to common shocks. However, to the extent that firms are responding to a demand or cost shock that affects all firms, price changes at higher-frequency pricing technology firms would anticipate those of a slower rival.

To examine the reaction of prices to other firms, we take price changes occurring at retailer *E*, one of the two firms with weekly pricing technology, as the impulse. We observe 374 price changes in our data occurring between midnight and 5 AM on Sunday. We partition the weeks into Friday through Thursday blocks, giving us a two-day pre period and a five-day post period around each price change. We then measure cumulative price changes of the same product occurring at rival retailers during each week. We capture “treated” product-weeks in which the product changed its price at retailer *E* and “control” weeks in which the product did not change its price.

Figure 7 plots the cumulative price changes before and after midnight on Sunday across each product-week. The blue line corresponds to treated product-weeks, i.e., weeks in which the price of a particular product changed at retailer *E*. The dashed line corresponds to control

Figure 7: Price Changes in Response to a Price Change by Retailer *E*



Notes: Figure displays the cumulative price changes of four retailers in response to a price change occurring at retailer *E*. The blue line displays the cumulative price change when retailer *E* changes a price of the same product in that week. The dashed line plots the cumulative price changes when the product at retailer *E* does not have a price change. The pre-period differences are netted out so that the difference is zero at period 0.

product-weeks that had no price change. The gap between the blue line and the dashed line is the marginal increase in price changes when a price change occurs at retailer *E*, and is analogous to a difference-in-differences estimate. We adjust for pre-period differences so the gap is zero in period 0.

Retailers *A* and *B* have an increased probability of a price change within 48 hours after a price change at retailer *E*. Retailer *A* realizes approximately 1 additional price change on average, whereas *B* realizes roughly 0.1 additional price changes. The baseline rate of price changes at *A* is approximately 10 times that of *B*, so the proportional increase is roughly the same across the two retailers.

Retailer *D* is twice as likely to change the price of its product when the price changes at *E*. Since they update their prices only a few hours after *E*, it is likely that these changes are

determined by a common unobserved factor, though it is plausible that D has technology in place that allows it to respond to a change at E . The point estimates for retailer C indicate that it is slightly less likely to change its price after a change at E . However, the estimate is not precise because price changes at C occur at a much lower rate. We only observe 16 price changes for C during a product-week where E realizes a price change.

The evidence suggests that the two retailers with the most frequent pricing technology, A and B , are responding to other firms' prices. There is not enough data to conclude about retailer C . Retailer D , the second slowest firm, may be responding to price changes that occur a few hours earlier at E , or they may be determined simultaneously by an unobserved factor, such as a wholesale cost shock or a demand shock.

For the high-frequency firms, it is quite possible that this analysis underreports the degree to which they respond to E 's prices. The above figure captures an increased rate of price changes. The high-frequency retailers may plausibly react through the magnitudes of the price changes, while maintaining the same rate of price changes.

6 Quantifying The Impact of Algorithmic Competition

While previous empirical work has assumed that firms have symmetric price-setting technology, we find that differences in pricing technology is an important feature of the market we examine. As a first step towards quantifying the impact of algorithmic technology on prices, we perform a counterfactual exercise where study how equilibrium prices would change if firms competed via simultaneous Bertrand competition. The simulated differences in prices may interpreted as the impact of adopting the asymmetric pricing technologies we observe. The exercise also suggests that fitting a (misspecified) Bertrand model could generate biased estimates of markups in online markets.

To calculate counterfactual prices, we simulate Bertrand competition using a model of spatial demand that is calibrated to aggregate prices and shares in our data. We generalize the Hotelling (1929) spatial demand model to allow for an arbitrary number of firms and flexible demand substitution patterns. We then apply the model to the five firms in our sample, taking into account the pricing technology of each firm.

One potential challenge for the empirical analysis of algorithmic competition is that the game can become computationally intractable, as the solution for one firm is an input into another firm's problem. A feature of our spatial demand model is that it generates analytical solutions for both the algorithm game and the simultaneous Bertrand game. This allows us to feasibly match the model predictions to the data and simulate alternative forms of competition.

6.1 Demand with Spatial Differentiation

We introduce a model of demand for products that are spatially differentiated. Consumers vary in their proximity to each firm, therefore the “travel” costs associated with each firm varies across consumers. In our setting, travel costs represent psychological costs and hassle costs of visiting each website. This may roughly be interpreted as search costs, though we provide no formal connection. These costs allow for positive markups when products are homogeneous.

The model is a generalization of the Hotelling (1929) line. Unlike the circle model of Salop (1979), firms compete with all other firms, not just their closest neighbors. In this way, the model is related to the pyramid model of von Ungern-Sternberg (1991) and the spokes model of Chen and Riordan (2007). Unlike previous models, our approach allows for the mass of consumers on each segment to be different, including the mass of consumers on the outside option segments. This feature is important since it allows for flexible substitution patterns that could explain differences in prices across retailers. This is also an advantage over models of vertical differentiation, such as the logit model, which restrict the horizontal substitution patterns to be symmetric across firms.

Each firm j lies in a $(J - 1)$ -dimensional space. A mass of consumers μ_{jk} lie along the line segment connecting j to k .³⁴ The distance between each firm is 1 unit. Each firm sells a single product, which consumers value at $v_j > 0$, and each firm chooses a price p_j . Each firm also has a mass of consumers on a line segment of distance D_0 connecting to an outside option ($j = 0$), with $p_0 = 0$ and $v_0 = 0$. Consumers lie on these segments with mass $\mu_{j0}D_0$. D_0 may be arbitrarily large, so that the firm never captures the full segment.

Each consumer i is indexed by its location and bears a travel cost τd_{ij} for traveling a distance d_{ij} to firm j to purchase its product. A consumer along segment jk will choose j if $u_{ij} > u_{ik}$, or

$$(v_j - p_j) - (v_k - p_k) > \tau(d_{ij} - d_{ik}). \quad (17)$$

That is, the consumer will prefer j to k if the added value of product j is greater than the additional travel cost of visiting firm j . The consumer also has the option to stay home and get $u_{i0} = 0$, which he will do if $u_{ij} < 0$ and $u_{ik} < 0$.

For our calibration exercise, we assume that consumer locations are distributed uniformly within each segment. We also assume that the products are homogeneous (but for the travel costs), so that $v_j = v$ for all j except for the outside option, for which $v_0 = 0$. Finally, we assume that consumer valuations are sufficiently high that all consumers on the inside segments purchase a product.³⁵ Demand for retailer j is equal to

$$q_j = \sum_{k \neq j, 0} \mu_{jk} \left(\frac{1}{2} - \frac{1}{2\tau} (p_j - p_k) \right) + \mu_{j0} \frac{1}{\tau} (v - p_j). \quad (18)$$

³⁴Demand can be represented by a graph. The graph is complete if $\mu_{jk} > 0$ for all $\{j, k\}$.

³⁵In a slight abuse of notation, we omit the arrival rate of consumers $m(t)$.

The model flexibly captures horizontal differentiation through the distribution of consumers across segments: for all consumers that could choose product j , there are a fraction of consumers $\frac{\mu_{jk}}{\sum_{k'} \mu_{jk'}}$ that have product k as the next-best option. A mass of consumers μ_{j0} will substitute only between j and the outside option, though all consumers would choose not to buy if prices were high enough ($p_j > v$). For additional details of this model, see Appendix C.

6.2 Calibration

To estimate the parameters of the demand system, we leverage the supply-side restrictions arising from firms' profit-maximizing behavior. In contrast to the standard assumptions in applied work, we allow firms to be asymmetric in their pricing technology, corresponding to the observed patterns found in Section 5.1. Retailers D and E set prices simultaneously, followed by retailer C , then B , and, finally, A . The sequence can be interpreted as arising from asymmetries in frequency (as in Section 3) or from asymmetric commitment (as in Section 4). What matters strategically is that the faster firms can change their prices in response to slower rivals. Given the large differences in technology across firms, we assume that faster firms can react before rivals realize any (meaningful) demand.

Under these assumptions, the firms' best-response functions are

$$\begin{aligned} R_A(p_B, p_C, p_D, p_E) &= \arg \max_{p_A} (p_A - c) q_A(p_A, p_B, p_C, p_D, p_E) \\ R_B(p_C, p_D, p_E) &= \arg \max_{p_B} (p_B - c) q_B(R_A(\cdot), p_B, p_C, p_D, p_E) \\ R_C(p_D, p_E) &= \arg \max_{p_C} (p_C - c) q_C(R_A(\cdot), R_B(\cdot), p_C, p_D, p_E) \\ R_D(p_E) &= \arg \max_{p_D} (p_D - c) q_D(R_A(\cdot), R_B(\cdot), R_C(\cdot), p_D, p_E) \\ R_E(p_D) &= \arg \max_{p_E} (p_E - c) q_E(R_A(\cdot), R_B(\cdot), R_C(\cdot), p_D, p_E). \end{aligned}$$

Equilibrium prices are determined by the solution to the system of equations above. A key advantage of the demand system in equation (18) is that it admits an analytical solution for prices.³⁶

The goal of the calibration exercise is to find demand parameters in order to match each retailer's price index, p_j , and aggregate shares, q_j . Each firm's price index is calculated by averaging over the price of all products and then constructing an index relative to retailer A as in Figure 6. A key challenge in online markets is that market shares for individual products are rarely observed by researchers. We construct a proxy for aggregate market shares using the share of Google searches for the retailer name and the word "allergy."³⁷ In order to help validate this measure of market share, we also obtain market shares of online personal care

³⁶The expressions for prices are several pages long and are available upon request.

³⁷We use the average of Google searches for the retailer name alone as well as the retailer name in addition to "allergy." See Appendix Table 8. The data were obtained from Google Trends (trends.google.com).

Table 5: Calibrated Segment Weights

		Retailer k					Outside
		A	B	C	D	E	
Retailer j	A	0.00	11.22	2.05	0.52	0.52	0.00
	B	11.22	0.00	2.05	0.52	0.52	1.79
	C	2.05	2.05	0.00	0.52	0.52	1.42
	D	0.52	0.52	0.52	0.00	0.52	3.02
	E	0.52	0.52	0.52	0.52	0.00	3.93

Notes: Row j column k shows the mass of customers on the segment between retailer j and k (μ_{jk}). The weights are symmetric; for convenience, they are displayed twice ($\mu_{jk} = \mu_{kj}$), representing the perspective of each firm. The outside segment weights represent the share of customers captured from the outside segments at the equilibrium prices.

products for the retailers from ecommerceDB. Appendix Table 8 shows that the implied market shares are quite similar. We also assume firms have identical marginal cost, which we normalize to 1.³⁸ Price-cost margins are determined by the calibrated prices in the model.

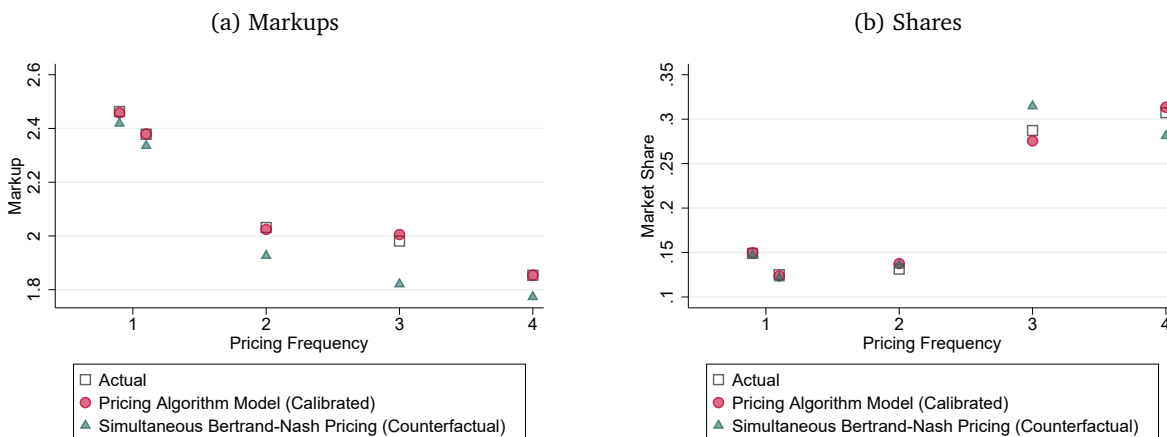
The unknown parameters to be recovered are the value of the product v , the travel cost parameter τ , and the relative weights on the segments $\{\mu_{jk}\}$. We parameterize the J by $(J+1)$ μ matrix with six parameters: $\{m_1, m_2, m_3, m_4, m_5, m_6\}$. While the fact that prices are negatively correlated with higher-pricing frequency is consistent with the model, this may also be due in part due to the fact that demand is not symmetric. In other words, consumers may have a preference for firms with lower pricing frequency. In the calibration, we allow substitution patterns that could explain differential pricing across firms. Thus, we can use our model to capture the impacts of both preferences and pricing technology on price differences across firms.

Specifically, we choose a parameterization for the segment weights so that differences in preferences can account for differences in prices and quantities we observe in the data. For the slower firms, D and E , we constrain the segment weights so that substitution is symmetric to all other retailers: $m_1 = \{\mu_{AD}, \mu_{BD}, \mu_{CD}, \mu_{AE}, \mu_{BE}, \mu_{CE}\}$. The firm with daily pricing, C , has symmetric weights with the faster firms, $m_2 = \{\mu_{AC}, \mu_{BC}\}$. The two fastest firms have a unique weight $m_3 = \mu_{AB}$. Finally, we give each firm a unique mass for the outside option, normalizing the mass for E to 1. We also set the mass along the outside option for A to zero.³⁹ This assumption is made because this retailer does not have any in-store sales for this market;

³⁸In the context of allergy drugs, we argue that differences in marginal costs across retailers for identical products are relatively small. As in Ellison et al. (2018), we take wholesale costs to be common across retailers. All five retailers sell large quantities of these brands across online and brick-and-mortar channels. Shipping costs may differ among retailers, but shipping costs are a relatively small portion of the total price. The average price ranges from \$16 to \$27 across retailers, and the products are small and light. Overall, differences in marginal cost are unlikely to generate the price differences seen in Figure 6.

³⁹Thus, $(\mu_{A0}, \mu_{B0}, \mu_{C0}, \mu_{D0}, \mu_{E0}) = (0, m_6, m_5, m_4, 1)$.

Figure 8: Calibration Fit for Markups and Shares



Notes: Figure displays the markups (panel (a)) and the relative shares (panel (b)) plotted against the pricing frequency of each retailer. Frequency is normalized to the relative sequence. The black squares indicate the data, and the red dots are the fitted prices from a calibration exercise. The relative prices are obtained from the estimated coefficients in specification (1) of Table 4. The markup level is pinned down by the calibrated model. The green triangles display the counterfactual simultaneous Bertrand markups at the calibrated parameters and the corresponding shares.

we are imposing that the all of A 's marginal customers would substitute to one of the other four online retailers at the equilibrium prices.

We use the method of moments to choose the parameters $(v, \tau, \{\mu_{jk}\})$ that best fit the relative prices and shares we observe in the data. We minimize the sum of squared deviations from relative average prices, taken from specification (1) of Table 4, and relative average shares using our proxy for quantities.⁴⁰

The calibrated parameters for the value of the product and travel costs are $v = 5.09$ and $\tau = 0.67$. The calibrated segment weights are displayed in Table 5. These parameters generate an equilibrium mean price of 2.07. As marginal costs are normalized to 1, prices may be interpreted as markups (price over cost). Mean realized travel costs are 0.61. Thus, we estimate that, net of travel costs, willingness to pay is roughly twice the equilibrium price.

The fit of the calibration exercise is displayed in Figure 8. In panel (a), squares indicate the relative prices in the data; these prices are translated to markups based on the calibrated model. The x -axis displays the pricing frequency in terms of the relative sequence. The red dots indicate the markups from the calibrated model. Likewise, the black squares in panel (b) represent observed shares, and the red dots indicated the predicted shares from the model. Our eight-

⁴⁰In calibration, we impose a penalty if the parameters result in a firm capturing more than 95 percent of the consumers on a given segment. This ensures that the counterfactual simultaneous Bertrand prices have an interior solution. The resulting penalty is small and the constraint does not meaningfully affect our estimates. Our counterfactual effects are robust to alternative share definitions that are based on category revenues or a combination of revenues and search data.

Table 6: Own-Price and Cross-Price Demand Elasticities

		Retailer Price				
		A	B	C	D	E
Retailer Share	A	-2.17	1.84	0.34	0.10	0.10
	B	1.93	-2.81	0.39	0.12	0.12
	C	0.71	0.77	-2.18	0.23	0.24
	D	0.20	0.22	0.22	-1.76	0.27
	E	0.17	0.18	0.18	0.21	-1.72

Notes: Row j column k shows $(\partial q_j / \partial p_k)(p_k / q_j)$.

parameter model fits prices and shares quite well. Allowing for flexible substitution patterns is important; if we had instead assumed symmetric demand, we would not be able to rationalize the data. Though we fit relative prices among the firms, underlying marginal costs play an important role in determining equilibrium in the model. Marginal costs are pinned down by the first-order conditions, allowing us to recover an estimate of markups. The calibrated parameters imply reasonable price-cost margins between 0.461 (retailer *A*) and 0.593 (retailer *E*).

Table 6 shows a matrix of elasticity of demand estimates from the model. Own-price elasticities range from -1.7 to -2.8 , consistent with other estimates from online goods.⁴¹ Our estimated cross-price elasticities indicate that, when the price of a product increases, consumers are more likely to substitute towards similar firms, e.g., consumers from retailer *A* are more likely to substitute to *B* and consumers from retailer *E* are more likely to substitute to *D*.

6.3 Counterfactual

To illustrate the potential impact of pricing algorithms on prices, we use our calibrated model to predict equilibrium prices if all firms instead had simultaneous price-setting technology. The Bertrand equilibrium prices and shares are displayed with green triangles in Figure 8. Our model indicates that algorithmic competition increases the average price by 5.2 percent above the counterfactual Bertrand equilibrium. These price changes differ across firms. Firms *D* and *E* realize more modest price changes of 1.9 and 1.6 percent. Based on our calibrated demand parameters, these firms receive a greater relative share of consumers from outside segments, rendering their behavior closer to that of a (local) monopolist. Competition for customers is more intense between the other three firms, who realize price increases between 4.5 and 10.1 percent as a result of algorithmic competition.

The results from the counterfactual exercise are presented in Table 7. Algorithmic competition has the biggest impact on shares for firm *B*, which sees a 3.9 percentage point (12 percent) decline in market share relative to the counterfactual Bertrand environment. The majority of this shift in share accrues to Firm *A*, which increases market share by 3.2 percentage points.

⁴¹See, for instance, De los Santos et al. (2012).

Table 7: Counterfactual Effects on Markups and Profits

Firm	Simultaneous Bertrand			Algorithmic Competition			Percent Change		
	Markup	Share	Profit	Markup	Share	Profit	Markup	Share	Profit
A	1.77	0.281	6.4	1.85	0.313	7.8	4.6	11.5	22.0
B	1.82	0.315	7.6	2.01	0.276	8.1	10.1	-12.4	6.3
C	1.93	0.136	3.7	2.02	0.138	4.1	5.1	1.3	11.1
D	2.34	0.121	4.8	2.38	0.124	5.0	1.9	2.0	4.4
E	2.42	0.147	6.1	2.46	0.150	6.4	1.6	1.8	3.7
Aggregate	1.97	1	28.6	2.07	1	31.3	5.2	0	9.6

Notes: Table displays the implied markups, shares, and profits from the calibrated model. The first three columns report the counterfactual estimates with simultaneous Bertrand price-setting behavior. The middle three columns report the predicted values from the model of algorithmic competition that is fitted to the data. The final three columns report the percent changes of moving from simultaneous Bertrand to algorithmic competition. Profits are arbitrarily scaled so that 1 unit corresponds to \$100 million of e-commerce in the Personal Care category.

The remaining 0.7 percent lost by Firm *B* result in modest increases for the other three firms. The differential effects on prices and quantities generate heterogeneous effects on firm profits. Because retailer *A* realizes meaningful increases in both price and quantity as a result of algorithmic competition, it sees the largest gain in profits (22 percent). Despite lower quantities, retailer *B*'s price increase is great enough to generate a 6 percent increase in profits from asymmetric technology. By contrast, retailers *D* and *E* realize profit gains of about 4 percent from more modest increases in both price and quantity. Consistent with the stylized results in Section 3, all firms profit as a result of algorithmic competition.

Our model predicts that algorithmic competition results in a modest decline in market-level quantities of 0.9 percent. This limited substitution to the outside option means that effects on total welfare are small (a decline of 0.3 percent). Algorithmic competition in our calibrated model serves primarily as a transfer between firms and consumers: consumer surplus falls by 4.1 percent, and firm profits increase by 9.6 percent. To assign a dollar value to these effects, we can do a rough back-of-the-envelope calculation. These five firms have annual e-commerce revenues of approximately \$6 billion in the category of Personal Care. If we assume that our estimated price effects apply to the entire category, then consumer surplus for the category would improve by \$300 million annually by moving from algorithmic competition to simultaneous Bertrand price setting.

7 Implications

As online sales grow and pricing algorithms become more prevalent, it is important to understand the implications for competition. While differentiated Bertrand has become the canonical model of competition in applied work, the model does not account for the fact that firms have

asymmetries in pricing frequency and employ algorithms that condition on rivals' prices. With these features, firms have unilateral incentives to move away from Bertrand competition. We show that firms would choose technology that results in asymmetric frequency, even if it provided a greater advantage to their rivals. In addition, the Bertrand best-response functions are *not* equilibrium strategies when firms compete in algorithms. These findings suggest that the Bertrand equilibrium may be the exception in online markets, rather than the rule.

Our theoretical results demonstrate how changes to pricing technology can lead to higher equilibrium prices relative to the Bertrand equilibrium. We then document that five large online retailers have asymmetric pricing technology and show that the pricing patterns in the data are consistent with the predictions of our theoretical results. In particular, we find that firms with higher-frequency pricing technology have, on average, substantially lower prices.

We take a first step towards quantifying the effects of competition in pricing algorithms by calibrating a model that incorporates both flexible substitution patterns and heterogeneous pricing technology. The model implies that algorithmic competition increases average prices by 5.2 percent relative to the simulated counterfactual in which firms engage in simultaneous Bertrand competition. Our simulation indicates that algorithmic pricing may meaningfully increase prices—even in markets with several firms in competitive equilibrium.

Thus, if policymakers are concerned that algorithms will raise prices, then the concern is much more broad than that of collusion. Policymakers could potentially shift firms to the Bertrand counterfactual by limiting the ability of firms to respond to rivals' prices. One solution would be to prohibit algorithms from directly conditioning on rivals' prices, while still allowing firms to have frequent price updates as a function of other factors, such as demand shocks. Besides prohibiting the behavior, policymakers could limit the scraping of rival firms' prices or restrict the storage of recent prices by other firms; either of these policies may be more feasible to implement and yield similar results. Alternatively, policymakers could regulate the frequency with which firms update their algorithms and their prices. Simultaneous price-setting conduct could be restored if price updates occurred with an industry-standard (symmetric) frequency, such as once per day.⁴²

Such enforcement measures raise conceptual and legal challenges, as they do not fit neatly into the existing regulatory and antitrust environment of the United States. Our results potentially raise new considerations for future policies about digital markets. However, the above measures may prove to be less burdensome than trying to determine whether algorithms have reached an “agreement” to collude. Moreover, we have shown that a regulator searching for the existence of an agreement may miss many instances of algorithms actually raising prices; indeed, very simple algorithms can support the fully collusive outcome in competitive equilibrium.

⁴²The standard could be jointly determined by market participants and a regulatory authority, per a suggestion from Fiona Scott Morton.

In this paper, we focus on the fact that pricing technology allows firms to condition their strategies on rivals' prices. What is special about algorithms that are a function of prices? Because retail prices are public and immediately available to rival firms, they allow for short-run commitment that shapes the nature of competition. If firms were prohibited from using rivals' prices, one could imagine firms using algorithms based on rivals' quantities, inventories, or other factors. However, these data are rarely made public at the frequency necessary to support a short-run commitment. When firms can condition their strategies on the actions of rivals, they have several instruments to discipline price competition.

Though we focus on competitive equilibria, our study also has important implications for collusion. First, the competitive equilibrium is typically used as "punishment" in a collusive equilibrium. In our model, pricing algorithms can support a competitive equilibrium with higher profits than the Bertrand equilibrium. Thus, pricing algorithms can make punishment less severe, reducing the likelihood of collusion. On the other hand, our model explicitly considers the ability of firms to increase their pricing frequency. As both firms increase in frequency, the ability to capture profits by deviating from a collusive price falls, thus increasing the likelihood of collusion or coordination. Finally, the model offers a new set of dimension of the strategy space that firms can use to increase prices. As an alternative to collusion, firms could instead choose to adopt different pricing technologies. As we show in the paper, firms need not cooperate on this outcome; asymmetric technology is in fact the equilibrium outcome when the choice is endogenous.

Online sales represent an increasing share of many diverse markets, including insurance, accommodations, and automobiles, in addition to retail goods. In all of these sectors, the shift online coincides with an increased availability of publicly posted prices and pricing technology that uses these prices as inputs. Though we view the issues raised in this paper as quite general, there is a large scope for future research that incorporates other features of these markets and examines additional implications of competition in pricing algorithms.

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Online Appendix

A Endogenous Pricing Frequency

A.1 Adoption Game

In this appendix, we provide a two-stage game in which firms can initially choose their pricing technology, before choosing prices. Firms are characterized by pricing technology $\theta_j \in \{1, 2, 3, \dots, \bar{\theta}\}$, where a higher value represents superior technology and $\bar{\theta}$ represents the best available technology. Firms can adopt $\theta_j = 1$ at zero cost or pay an adoption cost A to choose any other feasible technology. Firms compete in the pricing game after determining their technology.

In the model, the profits do not depend directly on the technology each firm has, but rather on their relative order. Denote the profits for the superior technology firm as π^H , the profits for the inferior technology firm as π^D , and the profits for when they have the same technology as π^S . Following the results from the main text, $\pi^H > \pi^D > \pi^S$. We assume that $\pi^H - \pi^S > A$, so that it can be profitable for one firm to adopt costly technology.

We now characterize equilibria of the game. Without loss of generality, let firm 2 represent the firm with (weakly) superior technology in equilibrium. To characterize the equilibria, there are two relevant cases to consider.

Case 1: $\pi^H - \pi^D \geq A$. Under these conditions, a pure-strategy equilibrium is for firm 2 to choose the best available technology ($\theta_2 = \bar{\theta}$) while firm 1 chooses $\theta_1 = 1$. It must be both profitable for firm 2 to adopt a superior technology, relative to symmetric technologies (this is true by assumption), and firm 2 must choose a technology so that firm 1 would not want to “leapfrog” firm 2’s choice. As the adoption cost is the same for any technological improvement, firm 2 must choose the best possible technology. The firm with superior technology has higher profits.

Case 2: $\pi^H - \pi^D < A$. The pure-strategy equilibria are characterized by firm 2 adopting any technology $\theta_2 > 1$ and by firm 1 choosing $\theta_1 = 1$. Firm 2 is indifferent to the exact level of technology because firm 1 has no incentive to invest in superior technology in equilibrium. In fact, the firm with inferior technology has higher profits (net of adoption costs) in this scenario. Thus, the firm that adopts superior technology is only motivated to do so to break the symmetric outcome, in which both realize lower profits. Though it competes more aggressively and realizes higher profits in the pricing game, it would prefer to be in firm 1’s position.

The pure strategy equilibria result in higher prices and higher profits for both firms, compared to the simultaneous price-setting equilibrium. As a corollary, any mixed strategy equi-

Figure 9: Example Pricing Frequency Adoption Game

		Firm 2		
		Low	Moderate	High
Firm 1	Low	(1.00, 1.00)	(1.08, 1.21)	(1.08, 1.21)
	Moderate	(1.21, 1.08)	(1.00, 1.00)	(1.08, 1.21)
	High	(1.21, 1.08)	(1.21, 1.08)	(1.00, 1.00)

librium also has higher expected prices and profits than the simultaneous price-setting equilibrium. Firms have a positive profit incentive to endogenously sort into asymmetric pricing technologies.

To illustrate this point, consider the three-by-three first-stage game where firms can choose pricing frequency and adoption is costless ($A = 0$). Firms know the profits for each subgame when they choose a low frequency, a moderate frequency, or a high frequency ($\theta \in \{1, 2, 3\}$). Figure 9 presents the payoffs based on the illustrative model in Section 3.3 when $\alpha = 0.5$. Any scenario where both firms choose the same frequency—low, moderate, or high—is not an equilibrium, because each firm has an incentive to deviate by choosing either a faster or a slower pricing technology. The only equilibria of the game are asymmetric where only one player chooses the highest frequency.

A.2 Adoption with an Initial Endowment of Technology

To further highlight the motivation for firms to make asymmetric choices in technology, we now consider a variant of the game above where both firms are initially endowed with technology $\theta^e > 1$. To change to a different technology, firms pay an adoption cost A as before, but they may costlessly retain their endowment or costlessly switch to $\theta = 1$. The costs for the initial endowment are sunk, so there is no salvage value for the endowed technology.

Without loss of generality, suppose that firms are initially endowed with $\theta^e = 2$. If $\pi^H - \pi^D \geq A$, then, similarly to case 1 above, the equilibrium has firm 2 choosing $\bar{\theta}$, while firm 1 keeps its initial endowment $\theta_1 = \theta^e$.⁴³

Now suppose that $\pi^H - \pi^D < A$, so that surpassing your rival with costly investments is not profitable. In this scenario, the unique pure-strategy equilibrium is for firm 1 to *downgrade* its technology to $\theta_1 = 1$ and for firm 2 to maintain its endowment. Here, firms willingly choose inferior technology to generate asymmetry. This is profitable for both firms, but it is less profitable for the firm that gives up its initial endowment. Perhaps surprisingly, this result holds even when there is some cost to downgrade (a), provided that the asymmetric outcome is still

⁴³If firm 1 were to costlessly reduce its technology to $\theta_1 = 1$, firm 2 would prefer to keep its initial endowment. But this is not an equilibrium because firm 1 would then optimally leapfrog firm 2.

more profitable for firm 1 than the symmetric outcome ($\pi^D - a > \pi^S$, and also $\pi^D - a > \pi^H - A$).

A.3 Discussion

The simple adoption game highlights a few properties of the price competition when firms vary in pricing frequency. First, the incentive to have asymmetric technologies is quite robust. A firm may adopt costly technology even if its rival gains more from the outcome, as the firm prefers this outcome to the world in which neither firm adopts. A firm may even pay a cost to downgrade its technology, if the firm and its rival are endowed with similar technology to begin with. Thus, though the most salient case for asymmetry is one in which the investing firm gains vis-a-vis its rivals, firms may even be willing to disadvantage themselves relative to their rivals to gain the benefits of softened price competition.

The above equilibrium results also apply if technology adoption is costless. Thus, if firms can choose their pricing technology at costs that are not prohibitively high, then we should not expect simultaneous price-setting behavior to hold in equilibrium. This raises some interesting considerations for empirical researchers, where a simultaneous price-setting behavior is the standard assumption.

When extending the analysis to dynamic settings, the model provides potentially interesting interpretations of observed phenomena. In the first case discussed above, we have one firm adopting the best available technology, and the other firm choosing to not invest at all in costly technology. Thus, this model has flavor of a one-sided “arms race,” where the superior technology firm over-invests in technology to prevent being bested by its rival. This over-investment can be quantified in a more general model where the cost of adoption depends on the technology level, i.e., as a (weakly increasing) function, $A(\theta)$. We omit an exposition of the model here, as it can complicate the analysis by eliminating all pure-strategy equilibria.

Over multiple periods, it would be possible to observe an arms race if the best-available technology were increasing over time, and firms maintained their technology from the previous period. With an increase in $\bar{\theta}$ from one period to the next, firm 1 would find it profitable to leapfrog firm 2, and, if the positions switch, a future increase in $\bar{\theta}$ would allow firm 2 to again overtake firm 1.

B Equilibrium Selection

B.1 A Multitude of Equilibria

It is possible to show that a multitude of equilibria can exist when firms compete in algorithms. To demonstrate this, we further restrict the class of algorithms to a special case: algorithms that are linear in other firms' prices. Even with these straightforward algorithms, we can show that many equilibria exist:

Proposition 5. *When firms compete in a one-shot game by submitting pricing algorithms, any price vector can be supported by algorithms that are linear functions of rivals' prices, provided the derivatives of profits with respect to prices exist at those prices.*

Proof: For the two-firm case, consider the price vector $\hat{p} = (\hat{p}_1, \hat{p}_2)$. Recall that, in equilibrium, it must be the case that a firm cannot do better by reverting to price-setting behavior. Firm 1's equilibrium price-setting first-order condition can be rewritten as:

$$\left. \frac{d\pi_1}{dp_1} \right|_{\hat{p}} = \frac{\partial \pi_1}{\partial p_1} + \frac{\partial \pi_1}{\partial p_2} \frac{\partial \sigma_2}{\partial p_1} \Big|_{\hat{p}} = 0 \quad (19)$$

$$\implies \left. \frac{\partial \sigma_2}{\partial p_1} \right|_{\hat{p}} = - \left. \frac{\partial \pi_1 / \partial p_1}{\partial \pi_1 / \partial p_2} \right|_{\hat{p}} \quad (20)$$

Likewise, $\left. \frac{\partial \sigma_1}{\partial p_2} \right|_{\hat{p}} = - \left. \frac{\partial \pi_2 / \partial p_2}{\partial \pi_2 / \partial p_1} \right|_{\hat{p}}$ when evaluated at \hat{p} . To support the prices (\hat{p}_1, \hat{p}_2) with algorithms that are linear in rivals' prices, one can solve the system of equations so that beliefs and strategies are consistent:

$$\hat{p}_1 = \sigma_1(\hat{p}_2) = a_1 + b_1 \hat{p}_2 \quad (21)$$

$$\hat{p}_2 = \sigma_2(\hat{p}_1) = a_2 + b_2 \hat{p}_1 \quad (22)$$

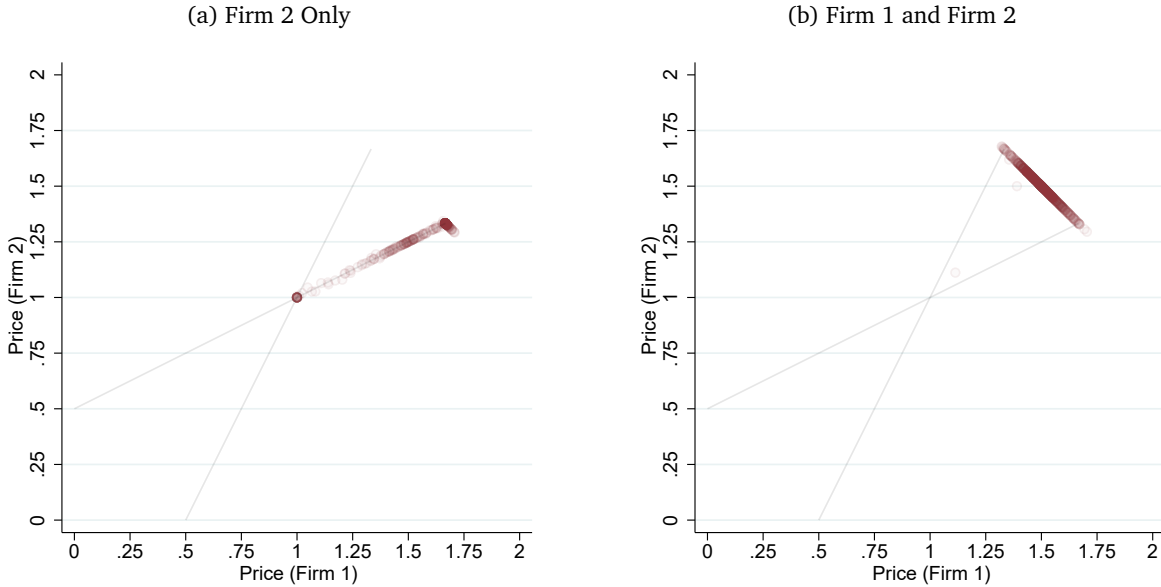
It is apparent that the solution has $b_1 = - \left. \frac{\partial \pi_2 / \partial p_2}{\partial \pi_2 / \partial p_1} \right|_{\hat{p}}$ and $b_2 = - \left. \frac{\partial \pi_1 / \partial p_1}{\partial \pi_1 / \partial p_2} \right|_{\hat{p}}$. Thus, each equation has one unknown, and the system has a unique solution for the parameters a_1 and a_2 . It is straightforward to extend the argument to many firms.⁴⁴

B.2 Simulations

Despite this multiplicity result, we expect algorithms to result in higher prices than the Bertrand-Nash equilibrium. We discuss these reasons in the main text. Here, we highlight one of the

⁴⁴For example, one solution to the J -firm problem would be to allow each firm's algorithm to depend only on one other firm's price: $R_j(p) = a_j + b_{jk} p_k$, where $k = j + 1 \forall j < J$ and $k = 1$ if $j = J$. The solution is $b_{jk} = - \left. \frac{\partial \pi_j / \partial p_j}{\partial \pi_j / \partial p_k} \right|_{\hat{p}}$ and $a_j = \hat{p}_j - b_{jk} p_k$.

Figure 10: Equilibrium Selection with Pricing Algorithms



Notes: Figure displays the resulting prices from 500 simulated duopoly markets when firms use a simple learning rule to update their prices or pricing algorithms. Each firm will update its algorithm if a random deviation in the algorithm parameters improve profits. Any stable point in simulation is an equilibrium (no profitable deviation exists). Each point displays the prices after 10,000 experiments. Panel (a) displays the results from the asymmetric algorithm game (firm 1 chooses price). Panel (b) displays the results from the game where both have algorithms. The plotted lines indicate the two price-setting best-response functions; their intersection is the unique Bertrand-Nash equilibrium.

reasons: many of these equilibria are “knife-edge” cases. To examine which equilibria are, in some sense, more robust, we simulate a simple learning process. We allow firms to experiment with linear algorithms, updating the parameters if profits increase. From a starting point of randomly-chosen algorithms, firms disproportionately arrive at equilibria that are bounded from below by their best-response functions and bounded from above by the profit Pareto frontier. Our simulation shows that higher prices result than those of the Bertrand equilibrium.

To test this intuition, we simulate a simple learning process to select equilibria. We follow the duopoly setup of Section 3.3 and allow firms to choose linear algorithms: $p_{jt} = a_{jt} + b_{jt}p_{kt}$. We initialize each firm with random parameters a_{j0} and b_{j0} . Each period, one (randomly-chosen) firm runs an experiment, modifying their parameters: $\tilde{a}_{jt+1} = a_{jt} + \varepsilon_t^1$ and $\tilde{b}_{jt+1} = b_{jt} + \varepsilon_t^2$. If this experiment improves profits, the firm updates their benchmark to the new parameters $((a_{jt+1}, b_{jt+1}) = (\tilde{a}_{jt+1}, \tilde{b}_{jt+1}))$, otherwise, they revert to the previous parameters $((a_{jt+1}, b_{jt+1}) = (a_{jt}, b_{jt}))$.

A “rest point” of this game is an equilibrium, i.e., where no unilateral deviation exists. To find the rest points, we simulate 10,000 experiments in each of 500 duopoly markets. The resulting prices are displayed in Figure 10. Panel (a) displays the results from the asymmetric

game in which firm 1 is a price-setter and firm 2 chooses an algorithm. The resulting prices, as would be expected, lie along firm 2's best-response function and are (weakly) higher than the simultaneous Bertrand-Nash equilibrium, $(1, 1)$. There is a mass at the Bertrand-Nash equilibrium, at firm 1's optimal choice conditional on the best-response of firm 2, and at the joint profit-maximizing point along firm 2's best-response function. Some simulations arrive at the Bertrand-Nash equilibrium because the firms never realize more profitable algorithms strategies. The second mass point corresponds to the equilibrium of the sequential pricing game.

Panel (b) shows the resulting prices from the game in which both firms have pricing algorithms. The prices are centered around the collusive equilibrium, $(1.5, 1.5)$, and lie along the profit Pareto frontier. The equilibria are bounded by the two firms' best-response functions.

Our simulation of a simple learning process selects equilibria with higher prices. The resulting prices are bounded from below by each firm's best-response function and bounded from above by the profit Pareto frontier. This is supported by the simple intuition that firms only have the incentive to adopt these algorithms if it would improve profits above the price-setting equilibrium.

C Details of Spatial Differentiation Model

We introduce a model of demand for products that are spatially differentiated. Consumers vary in their proximity to each firm, therefore the “travel” costs associated with each firm varies across consumers. In this section, we present additional formal details about the model. For further motivation, see Section 6.1.

Each firm j lies in a $(J - 1)$ -dimensional space. A mass of consumers μ_{jk} lie along the line segment connecting j to k .⁴⁵ The distance between each firm is 1 unit. Each firm sells a single product, which consumers value at $v_j > 0$, and each firm chooses a price p_j . Each firm also has a mass of consumers on a line segment of distance D_0 connecting to an outside option ($j = 0$), with $p_0 = 0$ and $v_0 = 0$. Consumers lie on these segment with density μ_{j0} and mass $\mu_{j0}D_0$. D_0 may be arbitrarily large, so that the firm never captures the full segment. Figure 11 provides a visual representation of the demand system for the case of three firms.

Each consumer i is indexed by its location and bears a travel cost τd_{ij} for traveling a distance d_{ij} to firm j to purchase its product. A consumer along segment jk will choose j if $u_{ij} > u_{ik}$, or

$$(v_j - p_j) - (v_k - p_k) > \tau(d_{ij} - d_{ik}). \quad (23)$$

That is, the consumer will prefer j to k if the added value of product j is greater than the additional travel cost of visiting firm j . The consumer also has the option to stay home and get $u_{i0} = 0$, which he will do if $u_{ij} < 0$ and $u_{ik} < 0$.

Consumers are distributed along each line segment connecting j to k according to a distribution F_{jk} with support $[0, 1]$. We assume that the distribution is symmetric about the midpoint of the segment. Symmetry implies $F_{jk} = F_{kj}$, so the direction of the connection is arbitrary. We also assume that the same distribution is applied to all segments: $F_{jk} = F$, though this could easily be relaxed. Demand along each segment can then be characterized by the distribution function F .

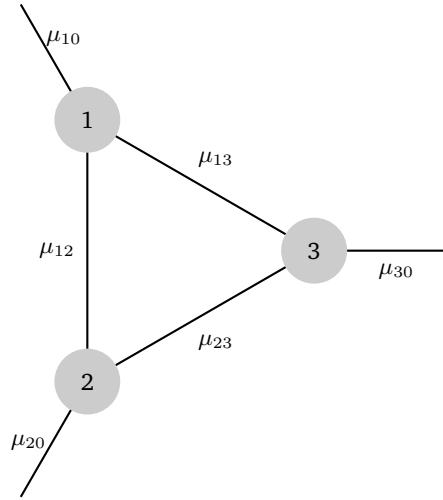
Noting that $d_{ik} = 1 - d_{ij}$ for a consumer on segment jk , a consumer on this segment will choose j if $u_{ij} > u_{ik}$ and if $u_{ij} \geq 0$, i.e., $\frac{1}{2} + \frac{1}{2\tau} ((v_j - p_j) - (v_k - p_k)) > d_{ij}$ and $\frac{1}{\tau} (v_j - p_j) \geq d_{ij}$. Firm j receives customers for which d_{ij} satisfies both conditions. Therefore, firm j receives a quantity of $\mu_{jk}F(y_{jk})$ from line segment jk , where

$$y_{jk} = \min \left\{ \frac{1}{2} + \frac{1}{2\tau} ((v_j - p_j) - (v_k - p_k)), \frac{1}{\tau} (v_j - p_j) \right\}. \quad (24)$$

For the outside segments, $y_{j0} = \frac{1}{D_0} \frac{1}{\tau} (v_j - p_j)$, as these segments have length D_0 instead of 1. The parameter D_0 can also be interpreted as the relative travel cost of choosing the outside option relative to an inside good, as the model has an isomorphic parameterization with outside travel costs $\tilde{\tau}_0 = D_0\tau$.

⁴⁵Demand can be represented by a graph. The graph is complete if $\mu_{jk} > 0$ for all $\{j, k\}$.

Figure 11: Spatial Differentiation Model with Three Firms



Notes: Example of demand for three firms with an outside option. The mass of consumers along each segment is given by μ_{jk} . The segments with mass μ_{10} , μ_{20} , and μ_{30} represent consumers whose next-best alternative to the linked firm is the outside option.

Overall, quantities are given by

$$q_j = \sum_{k \neq j} \mu_{jk} F(y_{jk}). \quad (25)$$

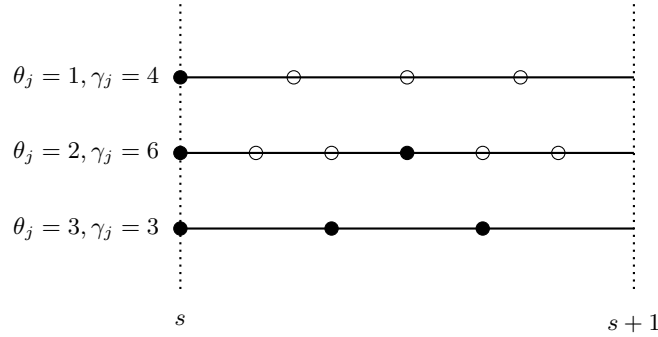
The flexibility in substitution patterns from this relatively parsimonious model comes primarily through the mass of consumers on each segment $\{\mu_{jk}\}$ and the choice of distribution F . In equilibrium, the consumers $\{\mu_{j0}\}$ that have no next-best alternative other than the outside option are also important in determining substitution patterns.

We introduce some terminology to facilitate discussion of the model. When $\max(u_{ij}, u_{ik}) \geq 0$ for all i on segment jk and $y_{jk} < 1$, the segment is *contested*.⁴⁶ When some consumers prefer to stay home, rather than purchase, the segment is *uncontested*. If segment jk is uncontested, there is no consumer indifferent between j and k , so those firms have local monopoly power over a portion of consumers on that segment. That is, a change in the price of firm k does not affect demand for firm j at the margin. When all segments between firms (the “inside” segments) are contested, we say the market is *covered*. For a covered market, all consumers on inside segments purchase.

⁴⁶When $y_{jk} \geq 1$, the segment is *dominated* by j .

D Additional Tables and Figures

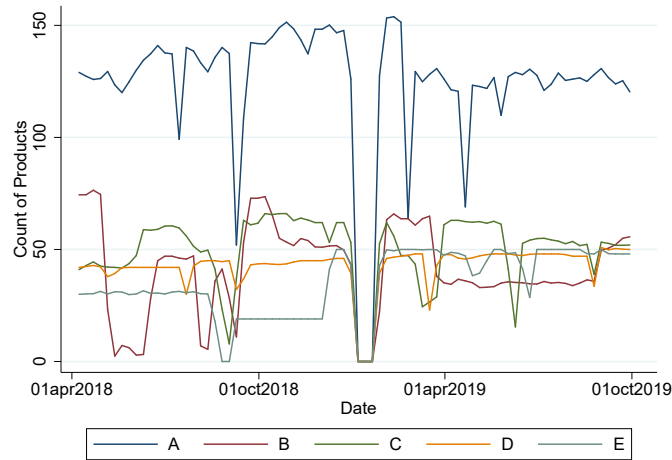
Figure 12: Timing with Pricing Technology (θ, γ)



Notes: Solid black markers represent opportunities to adjust algorithms and update prices. Open circles indicate opportunities to update prices based on the previously-determined algorithm. Algorithm updates are governed by θ and pricing updates are governed by γ .

Figure 12 illustrates the timing of pricing decisions in period s of the repeated pricing algorithm game. Pricing technology for firm j is governed by the frequency with which the firm can update its algorithm (θ_j) and the frequency that it can update prices (γ_j). When $\gamma_j > \theta_j$, the firm has a short-run commitment to update prices according to the previously-determined algorithm, $\sigma_j(\cdot)$.

Figure 13: Observed Products Over Time



Notes: Figure displays the average daily count of observed products in our sample by week and by retailer. Dips in the data correspond to changes to the retailer website and issues with the researchers' servers. Retailers A and B offer significantly more product varieties than the other retailers. This is primarily due to the number of size options offered for each brand.

Figure 13 illustrates the challenge of capturing high-frequency price data over an extended period. Dips in the data correspond to changes to the retailer website and issues with the researchers' servers. We note that we have several periods of many thousands of observations for which we have a consistent sample, and the periods of missing data do not meaningfully affect our results once we account for period fixed effects. We also include specifications using only data from July 1, 2019 through October 1, 2019, which are the most recent three months and for which we have a fairly consistent panel.

Table 8: Measures of Retailer Market Share

Retailer	Share of Online Personal Care	Google Search		
		“Retailer name”	“Retailer name” + Allergy	Mean
A	0.338	0.427	0.188	0.307
B	0.252	0.311	0.263	0.287
C	0.084	0.139	0.123	0.131
D	0.119	0.062	0.188	0.125
E	0.207	0.061	0.237	0.149

Notes: Share of personal care category reflect 2019 revenue figures from ecommerceDB.com. This includes online sales of medical, pharmaceutical, and cosmetic products for each of the retailers, including sales through mobile channels. Google search figures refer to the searches over the sample period as a share of total searches for all of the five retailers. Google search data are obtained from Google Trends (trends.google.com).

Table 8 provides measures of aggregate shares for the retailers in our data. We calibrate our model to Google search shares, using the mean of search shares for the retailer name and search shares for the retailer name along with the word “allergy.” We cross-check these shares against revenue shares provided by ecommerceDB.com. The measures of online revenue shares are obtained for the category of personal care, which includes all medical, pharmaceutical, and cosmetic products. Four of our retailers are in the top five for the personal care category by revenue, and all are in the top ten. The other retailers in the top ten have a focus on cosmetics.