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ADVERTISING INTENSITY, MARKET SHARE, CONCENTRATION AND DEGREE OF COOPERATION

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The purposes of this study are to assess the role that advertising plays in several explicit models of industrial organization[1/] and to formulate procedures for testing hypotheses which that assessment generates. In addition a technique for the explicit introduction of the degree of cooperation in such models is used and the impact of cooperation is explored.

I. THE GENERAL SETTING

I assume a static "industry" with N firms. By industry I mean a group of firms that produce goods which are perceived as substitutes. The goods may be perfect substitutes, but need not be. Other goods outside the group in question are assumed to be irrelevant.

Using p_i, q_i, and a_i for price, quantity, and advertising for the $i^{\underline{th}}$ firm, respectively, the price of the good is assumed to be a function of all quantities and advertising outlays, i.e.,

(1) $$p_i = p_i(\underline{q}, \underline{a}),$$

where bars under letters indicate vectors. I define γ_{ij} as the elasticity of the $j^{\underline{th}}$ good's price with respect to the $i^{\underline{th}}$ good's advertising, i.e., $\gamma_{ij} = \frac{\partial p_j a_i}{\partial a_i p_j}$. Letting $c_i = c_i(q_i)$ be total production cost for the $i^{\underline{th}}$ firm and π_i its profit,

(2) $$\pi_i = p_i q_i - c_i - a_i.$$

Defining some additional variables, let $s_i = p_i q_i$ be sales and $v_i = a_i/s_i$ be advertising intensity. Define $S = \Sigma s_i$ as industry sales,

$A = \Sigma a_i$ as industry advertising, and $z_i = s_i/S$ as the market share of the i^{th} firm.

II. TWO POLAR CASE OLIGOPOLY MODELS

Cournot[2/] Let each firm maximize its profit independently of all other firms. That is,

(3)
$$\frac{\partial \pi_i}{\partial a_i} = \frac{\partial p_i}{\partial a_i} q_i - 1$$
$$= \left(\frac{\partial p_i}{\partial a_i} \frac{a_i}{p_i}\right)\left(\frac{p_i q_i}{a_i}\right) - 1$$
$$= \gamma_{ii} \frac{s_i}{a_i} - 1 = 0.$$

This implies that

(4)
$$v_i = \frac{a_i}{s_i} = \gamma_{ii}.$$

Let $\Gamma = (\gamma_{ij})$. Further, let A_d be a diagonal matrix with the same diagonal as the matrix A. Then the N equations in (4) may be written compactly as

(5)
$$\underline{v} = \Gamma_d \underline{\iota},$$

where $\underline{\iota}$ is a vector of ones. Since Γ_d is a diagonal matrix, (5) may be rewritten as

(6)
$$\underline{v} = d_{\underline{z}}^{-1} \Gamma_d \underline{z},$$

where, if $\underline{z}$ is a vector, $d_{\underline{z}}$ is a diagonal matrix containing the elements of $\underline{z}$ on the diagonal.[3/]

Chamberlin.[4/] At the other extreme, let each firm maximize total profit for all firms. Letting $\Pi = \Sigma \pi_i$,

(7)
$$\frac{\partial \Pi}{\partial a_i} = \frac{\partial \pi_i}{\partial a_i} + \sum_{j \neq i} \frac{\partial \pi_j}{\partial a_i}$$

$$= \frac{\partial p_i}{\partial a_i} q_i - 1 + \sum_{j \neq i} \frac{\partial p_j}{\partial a_i} q_j$$

$$= \left(\frac{\partial p_i}{\partial a_i} \frac{a_i}{p_i}\right)\left(\frac{p_i q_i}{a_i}\right) - 1 + \sum_{j \neq i} \left(\frac{\partial p_j}{\partial a_i} \frac{a_i}{p_j}\right)\left(\frac{p_j q_j}{a_i}\right)$$

$$= \gamma_{ii}\frac{s_i}{a_i} - 1 + \sum_{j \neq i} \gamma_{ij}\frac{s_j}{a_i} = 0.$$

This implies that

(8)
$$\frac{s_i}{a_i} \gamma_{ii} + \sum_{j \neq i} \frac{s_j}{a_i} \gamma_{ij} = 1$$

$$\gamma_{ii} + \sum_{j \neq i} \frac{s_j}{s_i} \gamma_{ij} = \frac{a_i}{s_i} = v_i$$

$$v_i = \sum_{j=1}^{N} \frac{s_j}{s_i} \gamma_{ij} = \sum_{j=1}^{N} \frac{z_j}{z_i} \gamma_{ij} = z_i^{-1} \sum_{j=1}^{N} z_j \gamma_{ij}$$

$$= z_i^{-1} \begin{bmatrix} \gamma_{i1} & \gamma_{i2} & \cdot & \cdot & \cdot & \gamma_{iN} \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ \cdot \\ \cdot \\ \cdot \\ z_N \end{bmatrix}.$$

Writing (8) compactly gives

(9)

$$\underline{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ \cdot \\ \cdot \\ v_N \end{bmatrix} = \begin{bmatrix} z_1^{-1} \left[\gamma_{11}\ \gamma_{12}\ \cdots\ \gamma_{1N}\right] \\ z_2^{-1} \left[\gamma_{21}\ \gamma_{22}\ \cdots\ \gamma_{2N}\right] \\ \cdot \\ \cdot \\ \cdot \\ z_N^{-1} \left[\gamma_{N1}\ \gamma_{N2}\ \cdots\ \gamma_{NN}\right] \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ \cdot \\ \cdot \\ \cdot \\ z_N \end{bmatrix}$$

$$= \begin{bmatrix} z_1^{-1} & & & & 0 \\ & z_2^{-1} & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & \cdot \\ 0 & & & & z_N^{-1} \end{bmatrix} \Gamma \underline{z}$$

$$= d_{\underline{z}}^{-1} \Gamma \underline{z}.$$

A comparison of (6) and (9) shows that the matrix in the two equations differs only in that all the elasticities are present in (9) but only the own-elasticities are present in (6). Since advertising expense must be non-negative, the first-order requirements are that v_i be the maximum of the amount shown in (6) or (9), and zero.

III. A SIMPLIFIED DEMAND STRUCTURE

So far the demand equation system is completely general. I will now move to a particular demand structure which is characterized by an industry elasticity of price with respect to advertising, a general brand-switching elasticity, and a dependence of the firm-specific elasticity on its relative size. I make the explicit assumption that

(10)
$$\gamma_{ii} = \sigma + (\gamma - \sigma) \quad z_i$$
$$\gamma_{ij} = (\gamma - \sigma) \quad z_i \;,$$

where γ is an industry demand elasticity, σ is a brand-switching elasticity, and z_i is the market share. In matrix notation,

(11)
$$\Gamma = \sigma I + (\gamma - \sigma) \underline{z} \; \underline{\iota}'.$$

Consider the identification of $\boldsymbol{\gamma}$ as an elasticity of industry price with respect to advertising. Noting that $dS/S = \Sigma z_i(dp_i/p_i) = \underline{z}' d_{\underline{p}}^{-1} \overset{o}{\underline{p}}$, and letting $\overset{o}{S}$ denote dS, observe that

(12)
$$\overset{o}{S}/S = S^{-1} \underline{\iota}' \overset{o}{\underline{s}} = S^{-1} \underline{\iota}' d_{\underline{q}} \overset{o}{\underline{p}}$$

$$= S^{-1} \underline{q}' \begin{bmatrix} \frac{\partial p_1}{\partial a_1} & \frac{\partial p_1}{\partial a_2} & \cdots & \frac{\partial p_1}{\partial a_N} \\ \cdot & & & \\ & \cdot & & \\ & & \cdot & \\ \frac{\partial p_N}{\partial a_1} & & & \frac{\partial p_N}{\partial a_N} \end{bmatrix} \overset{o}{\underline{a}}$$

$$= S^{-1} \underline{q}' d_p \begin{bmatrix} \left(\frac{\partial p_1}{\partial a_1} \frac{a_1}{p_1}\right) \left(\frac{\partial p_1}{\partial a_2} \frac{a_2}{p_1}\right) \cdots \left(\frac{\partial p_1}{\partial a_N} \frac{a_N}{p_1}\right) \\ \left(\frac{\partial p_N}{\partial a_1} \frac{a_1}{p_N}\right) \cdots\cdots \left(\frac{\partial p_N}{\partial a_N} \frac{a_N}{p_N}\right) \end{bmatrix} d_{\underline{a}}^{-1} \overset{o}{\underline{a}}$$

$$= S^{-1} \underline{s}' \Gamma' d_{\underline{a}}^{-1} \overset{o}{\underline{a}} = \underline{z}' \Gamma' d_{\underline{a}}^{-1} \overset{o}{\underline{a}} \;,$$

where, if $\underline{x}' = (x_1 \; x_2 \; . \; . \; . \; . \; x_N)$, $\overset{o}{\underline{x}}' = (dx_1 \; dx_2 \; . \; . \; . \; . \; dx_N)$. If $da_i/a_i = dA/A = \overset{o}{A}/A$, then $d_{\underline{a}}^{-1} \overset{o}{\underline{a}} = (\overset{o}{A}/A) \; \underline{\iota}$, and

(13)
$$\frac{\overset{o}{S}}{S} / \frac{\overset{o}{A}}{A} = \underline{z}' \; \Gamma' \; \underline{\iota}.$$

Let the scalar $(\overset{o}{S}/S)/(\overset{o}{A}/A)$ be denoted by γ, so $\gamma = \underline{z}'\Gamma'\underline{\iota}$.

Substituting (11) into (13) gives

$$(14) \qquad \overset{o}{S}/S \Big/ \overset{o}{A}/A = \underline{z}' \left[\sigma I + (\gamma - \sigma)\, \underline{\iota}\, \underline{z}' \right] \underline{\iota}$$

$$= \sigma \underline{z}'\underline{\iota} + (\gamma - \sigma)\, \underline{z}'\underline{\iota}\ \underline{z}'\underline{\iota}$$

$$= \gamma,$$

since $\underline{z}'\underline{\iota} = 1$.

Under these assumptions, then γ is the elasticity of industry sales (and average industry price) with respect to industry advertising. Note that if $\gamma = 0$, so that there is no industry sales effect, there will still be positive advertising if there are brand-switching effects.

Turning now to the brand-switching effect, observe that

$$(15) \qquad d_{\underline{z}}^{-1}\, \overset{o}{\underline{z}} = d_{\underline{z}}^{-1} \begin{bmatrix} \frac{\partial z_1}{\partial a_1} & \frac{\partial z_1}{\partial a_2} & \cdots & \frac{\partial z_1}{\partial a_N} \\ \vdots & & & \\ \frac{\partial z_N}{\partial a_1} & & & \frac{\partial z_N}{\partial a_N} \end{bmatrix} \overset{o}{\underline{a}}.$$

Noting that

$$(16) \qquad \frac{\partial z_i}{\partial a_j} = \frac{\partial}{\partial a_j}(s_i/S) = \frac{\partial}{\partial a_j}(p_i q_i/S) = q_i \frac{\partial}{\partial a_j}(p_i/S)$$

$$= q_i\, S^{-2} \left(S \frac{\partial p_i}{\partial a_j} - p_i \frac{\partial S}{\partial a_j} \right)$$

$$= q_i\, S^{-2} \left[S \left(\frac{\partial p_i}{\partial a_j} \frac{a_j}{p_i} \right) \frac{p_i}{a_j} - p_i \sum_{k=1}^{N} q_k \left(\frac{\partial p_k}{\partial a_j} \frac{a_j}{p_k} \right) \frac{p_k}{a_j} \right]$$

$$= p_i q_i\, a_j^{-1} S^{-2} \left[S\, \gamma_{ji} - \sum_{k=1}^{N} s_k\, \gamma_{jk} \right] = z_i a_j^{-1} \left[\gamma_{ji} - \sum_{k=1}^{N} z_k\, \gamma_{jk} \right],$$

(15) may be written as

$$(17)\quad d_{\underline{z}}^{-1}\underline{\mathring{z}} = d_{z}^{-1}\begin{bmatrix} \left[\gamma_{11} - \sum_k z_k \gamma_{1k}\right]\frac{z_1}{a_1} & \left[\gamma_{21} - \sum_k z_k\gamma_{2k}\right]\frac{z_1}{a_2} & \cdots & \left[\gamma_{N1} - \sum_k z_k\gamma_{Nk}\right]\frac{z_1}{a_N} \\ & & & \\ & & & \left[\gamma_{NN} - \sum_k z_k\gamma_{Nk}\right]\frac{z_N}{a_N} \end{bmatrix}\underline{\mathring{a}}$$

$$= \left[\Gamma' - \begin{bmatrix}1\\1\\ \cdot \\ \cdot \\ 1\end{bmatrix}\begin{bmatrix}\sum_k z_k\gamma_{1k} & \sum_k z_k\gamma_{2k} & \cdots & \sum_k z_k\gamma_{Nk}\end{bmatrix}\right] d_{\underline{a}}^{-1}\underline{\mathring{a}}$$

$$= \left(\Gamma' - \underline{\iota}\,\underline{z}'\,\Gamma'\right) d_{\underline{a}}^{-1}\underline{\mathring{a}}$$

$$= (I - \underline{\iota}\,\underline{z}'\quad \Gamma'\, d_{\underline{a}}^{-1}\underline{\mathring{a}} .$$

Equation (17) is a general statement of the relation between changes in advertising levels and changes in market shares which would accompany them. It may be used to make some general observations about brand-switching effects.

First, assume no industry effect, e.g., $\gamma = \underline{z}' \Gamma' \underline{\iota} = 0$, and assume further that if there are equal percentage changes in all advertising levels, then there will be no changes in market shares. Under these further assumptions,

$\mathring{a}_i/a_i = \mathring{A}/A$, $d_{\underline{a}}^{-1} \mathring{\underline{a}} = (\mathring{A}/A) \underline{\iota}$, and $d_{\underline{z}}^{-1} \mathring{\underline{z}} = (\mathring{A}/A) (\Gamma' \underline{\iota} - \underline{\iota}\ \underline{z}' \Gamma' \underline{\iota})$. By assumption, the second term on the right hand side of this expression is zero. Consequently, for market shares to be unaffected, it is necessary for $\Gamma' \underline{\iota} = 0$ when $\gamma = \underline{z}' \Gamma' \underline{\iota} = 0$. This requirement is satisfied by equation (11).

Secondly, consider effects on market shares if one firm increases its advertising and others do not. As a general assumption, we require such a change to lead to a decrease in the shares of all the other firms. That is, let

$\mathring{a}_i/a_i > 0$ and $\mathring{a}_j/a_j = 0$ for $j \neq i$. For any matrix X with N rows and its transpose X′, let $X' = \{{}_1\underline{x}\ {}_2\underline{x} \cdots {}_N\underline{x}\}$, where ${}_i\underline{x}$ is a column vector which contains the elements of the $i^{\underline{th}}$ row of X. Under the assumptions about the $\mathring{a}_i$'s given just above, it follows that $\Gamma' d_{\underline{a}}^{-1} \mathring{\underline{a}}$ becomes $(\mathring{a}_i/a_i)\ {}_i\underline{\gamma}$.

For $\mathring{z}_j/z_j < 0$ under these conditions, then, it follows that the $j^{\underline{th}}$ element of $(I - \underline{\iota}\ \underline{z}')\ {}_i\underline{\gamma} < 0$, so $\gamma_{ij} - \underline{z}'\ {}_i\underline{\gamma} < 0$, or $\gamma_{ij} - \sum_{k=1}^{N} z_k \gamma_{ik} < 0$. If this condition holds for all γ_{ij} where $j \neq i$, then it follows that $\gamma_{ii} - \sum_{k=1}^{N} z_k \gamma_{ik} > 0$, since the $i^{\underline{th}}$ firm's share must increase to offset the decreases in all other shares.

Turning to the Γ matrix given in (11), we note that $_i\underline{\gamma} = \sigma\, \underline{e}_i + (\gamma - \sigma)\, z_i\, \underline{\iota}$, where $\underline{e}_i$ is the elementary vector with 1 in the $i^{\underline{th}}$ position and 0 elsewhere. This leads to $\underline{z}'\, _i\underline{\gamma} = \sigma + (\gamma - \sigma)\, z_i$. Since $\gamma_{ij} = (\gamma - \sigma)\, z_i$, what is required is that $(\gamma - \sigma) z_i - \{\sigma + (\gamma - \sigma)\, z_i\} < 0$, or $\sigma > 0$.

These observations may be summarized by substituting (11) into (17), giving

$$
\begin{aligned}
(18)\quad d_{\underline{z}}^{-1}\, \overset{o}{\underline{z}} &= (I - \underline{\iota}\, \underline{z}') \left[\sigma I + (\gamma - \sigma)\, \underline{\iota z}'\right] d_{\underline{a}}^{-1}\, \overset{o}{\underline{a}} \\
&= \left[\sigma I + (\gamma - \sigma)\, \underline{\iota z}' - \sigma\, \underline{\iota}\, \underline{z}' - (\gamma - \sigma)\, \underline{\iota z}'\underline{\iota z}'\right] d_{\underline{a}}^{-1}\, \overset{o}{\underline{a}} \\
&= \sigma (I - \underline{\iota}\, \underline{z}')\, d_{\underline{a}}^{-1}\, \overset{o}{\underline{a}}\ .
\end{aligned}
$$

If a_i increases and the other a_j's are const. t, σ must be positive to have a positive effect on z_i. A positive σ also i plies a negative effect of a_j on z_i, other a_k's (including a_i) fixed. If all a_j's, including a_i, change by the same percentage, z_i remains unchanged. Finally, if $\sigma = 0$, there is no brand-switching effect.

For a_j to have no effect on p_i requires $\gamma_{ij} = 0$. From (10), this amounts to $\gamma = \sigma$; that is, the industry demand elasticity and the brand-switching elasticity are equal. If $\gamma > \sigma$, $\gamma_{ij} > 0$; the industry elasticity dominates. If $\gamma < \sigma$, $\gamma_{ij} < 0$; the brand-switching elasticity dominates.

Substituting (11) now in (6), the Cournot case, and in (9), the Chamberlin case, gives

(19) Cournot: $\underline{v}^0 = \sigma \underline{\iota} + (\gamma - \sigma) \underline{z}$

Chamberlin: $\underline{v}^1 = \gamma \underline{\iota}$.

The $i^{\underline{th}}$ element is then

(20) Cournot: $v_i^0 = \sigma + (\gamma - \sigma) z_i$

Chamberlin: $v_i^1 = \gamma$.

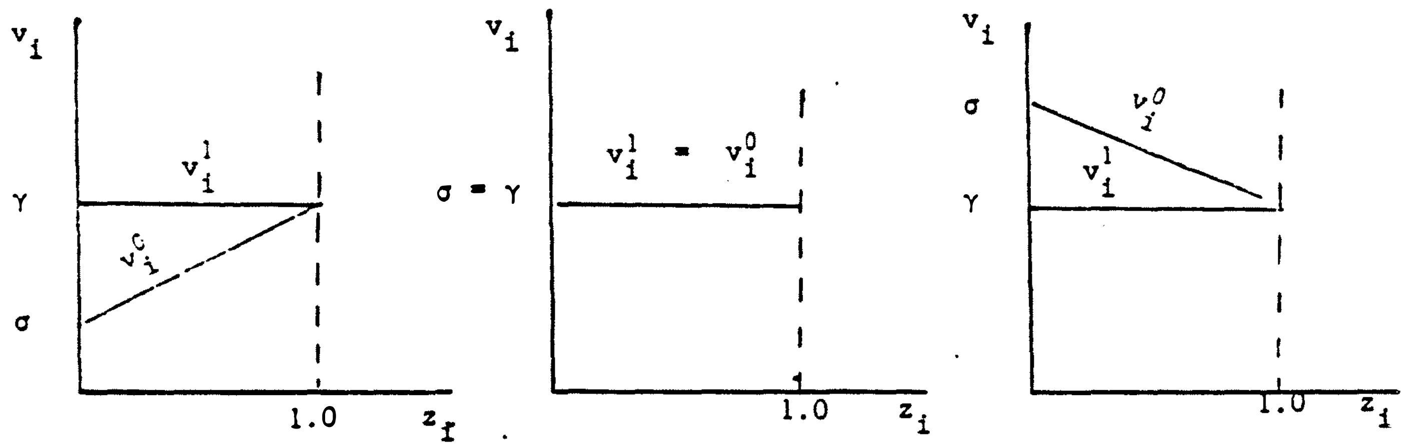
In (19) and (20) the superscript ^{0} is used to denote total independence of decision making and the superscript ^{1} is used to denote total interdependence of decision making

The relation between v_i^0 and v_i^1 depends solely on t e relation between γ and σ. If $\gamma > \sigma$, so that the industry effect domina s, $v_i^1 \geq v_i^0$, with equality in the trivial case of $z_i = 1$. Cooperative irms would have a higher advertising intensity than non-cooperative fir s.

If there is equality between the industry elasticity and the brand-switching elasticity, or $\gamma = \sigma$, we get $v_i^1 = v_i^0 = \gamma$. Whether there is cooperation among the firms has no effect on advertising intensity. And, if brand-switching is more important, with $\gamma < \sigma$, we get $v_i^1 < v_i^0$; cooperative firms will have a lower advertising intensity than non-cooperative firms. The three situations are depicted in Figure 1.

Given the results for the firm level variables, calculation of the corresponding results for the industry is straightforward. Let $H = \Sigma z_i^2 = \underline{z}' \underline{z}$ be the Herfindahl index of concentration, and note that $\underline{z}' \underline{\iota} = \Sigma z_i = 1$ and that $V = A/S = \Sigma a_i/S = \Sigma(s_i/S)(a_i/s_i) = \Sigma z_i v_i = \underline{z}' \underline{v}$. Then for the general case, from (6) and (9),

Figure 1


v_i
v_i^1
γ
v_i^0
σ
1.0
z_i
v_i
$v_i^1 = v_i^0$
$\sigma = \gamma$
1.0
z_i
v_i
σ
v_i^0
v_i^1
γ
1.0
z_i

(21) $$V^0 = \underline{z}' d_{\underline{z}}^{-1} \Gamma_d \underline{z} = \underline{\iota}' \Gamma_d \underline{z} = \Sigma z_i \gamma_{ii},$$

$$V^1 = \underline{z}' d_{\underline{z}}^{-1} \Gamma \underline{z} = \underline{\iota}' \Gamma \underline{z} = \sum_{i=1}^{N} \sum_{j=1}^{N} \gamma_{ij} z_j;$$

and, for the simplified case, from (19),

(22) $$V^0 = \underline{z}' \left[\sigma \underline{\iota} + (\gamma - \sigma) \underline{z} \right] = \sigma + (\gamma - \sigma) H,$$

$$V^1 = \underline{z}' \gamma \underline{\iota} = \gamma.$$

Since the forms of the equations in (22) are the same as in (20) all of the analysis of (20) holds for (22), except that H is substituted for z_i. Advertising intensity will be higher for an industry with cooperative firms than for one with non-cooperative firms if the industry demand elasticity dominates the brand-switching elasticity. If the two elasticities are equal, then so will be advertising intensity for the cooperative firm and non-cooperative firm industries If brand-switching dominates, the cooperative firm industry will have a lower advertising intensity.

In the industry model context the possibility of a critical concentration level may be easily introduced. That is, assume that at values of H below H*, the firms are non-cooperative, but that for values equal to or greater than H*, they are cooperative. That is,

(23) $$V = \sigma + (\gamma - \sigma) H \quad \text{if } H < H^*$$

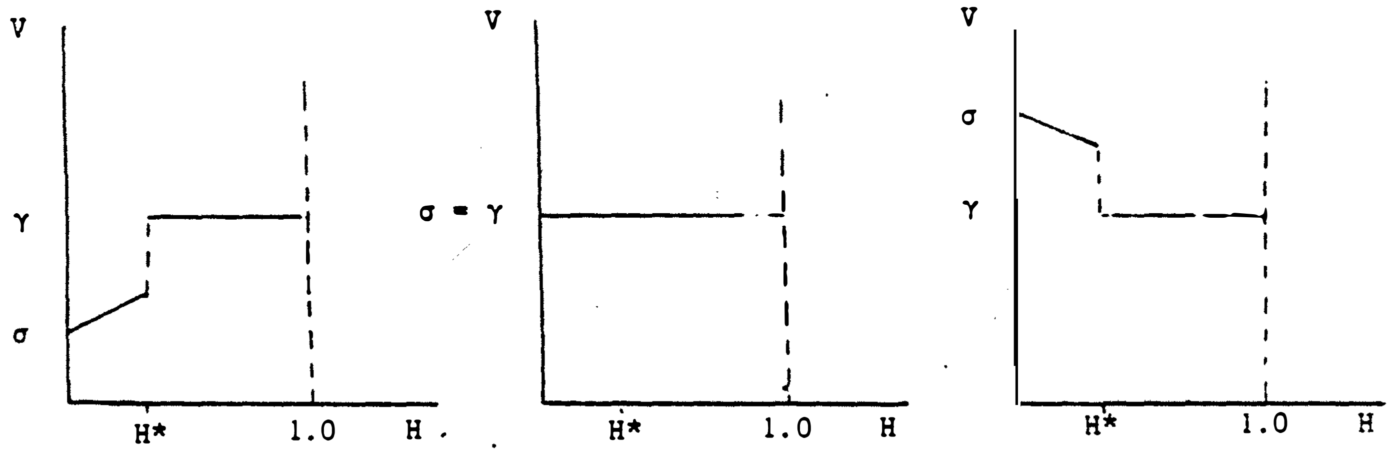
$$= \gamma \quad \text{if } H \geq H^*.$$

This function is shown in Figure 2 for the three relations between γ and σ.

IV. THE DEGREE OF COOPERATION

The models depicted in Figure 2 are based on a very simple notion about the relation between concentration and the cooperation/non-cooperation characteristic. They use what is essentially a step function relating concentra-

<u>Figure 2</u>



tion and the extent or degree of cooperation, as shown in Figure 3. Using δ as a symbol for the degree of cooperation, equation (23) can be rewritten as,

$$(24) \qquad V = (1 - \delta)\, V^0 + \delta V^1$$
$$= (1 - \delta) \left[\sigma + (\gamma - \sigma)\, H \right] + \delta\gamma,$$

where

$$(25) \qquad \delta = \delta(H) = 0 \text{ if } H < H^*$$
$$= 1 \text{ if } H \geq H$$

The specific relation between concentration (H) and the degree of cooperation (δ) shown in Figure 3 is more restrictive than necessary, and has less appeal an one which shows a smooth increase of δ from 0 to 1 as H increases from to 1. Such a smooth function is shown in Figure 4. The degree of cooper.:ion is low until H gets close to H*, increases greatly as H goes through H*, attaining a high level for an H larger than H* but still close to it, and then goes to 1 as H goes to 1. More formally,

$$(26) \qquad \delta = \delta(H)$$
$$= 0 \quad \text{if } H = 0$$
$$= 1 \quad \text{if } H = 1$$
$$\frac{d\delta}{dH} > 0$$
$$\frac{d^2\delta}{dH^2} > 0 \quad \text{if } H < H^*$$
$$= 0 \quad \text{if } H = H^*$$
$$< 0 \quad \text{if } H \geq H^*.$$

If (26) is now substituted for (25) in (24), a corresponding smoothing of the functions shown in Figure 2 is accomplished. The smooth functions

Figure 3

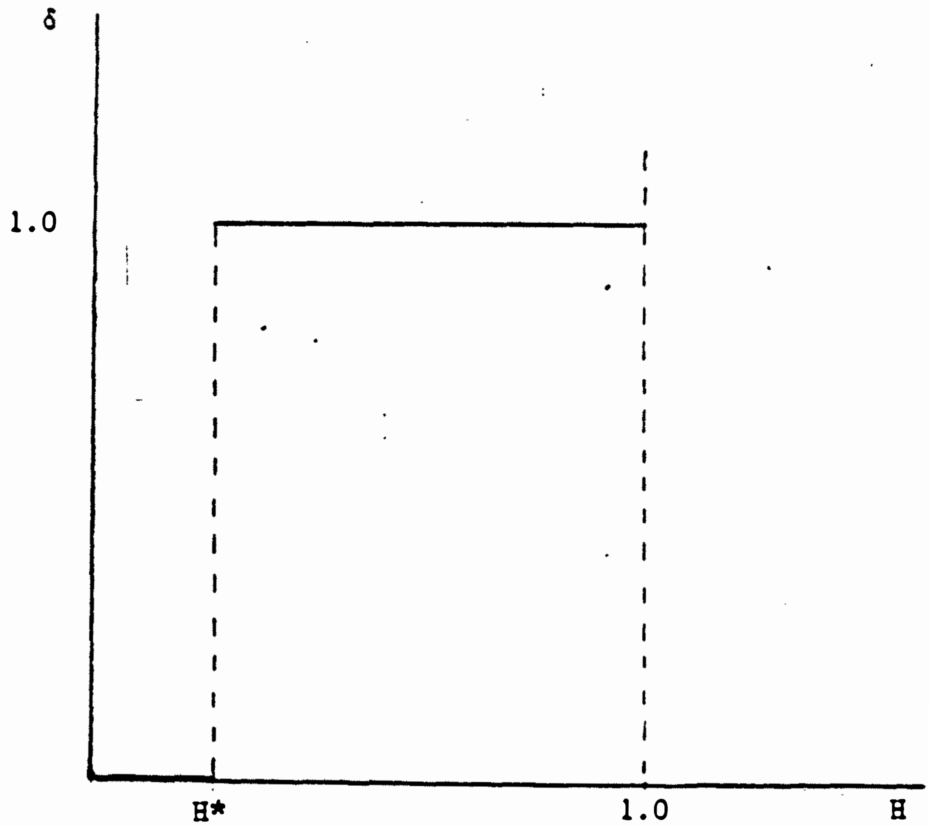
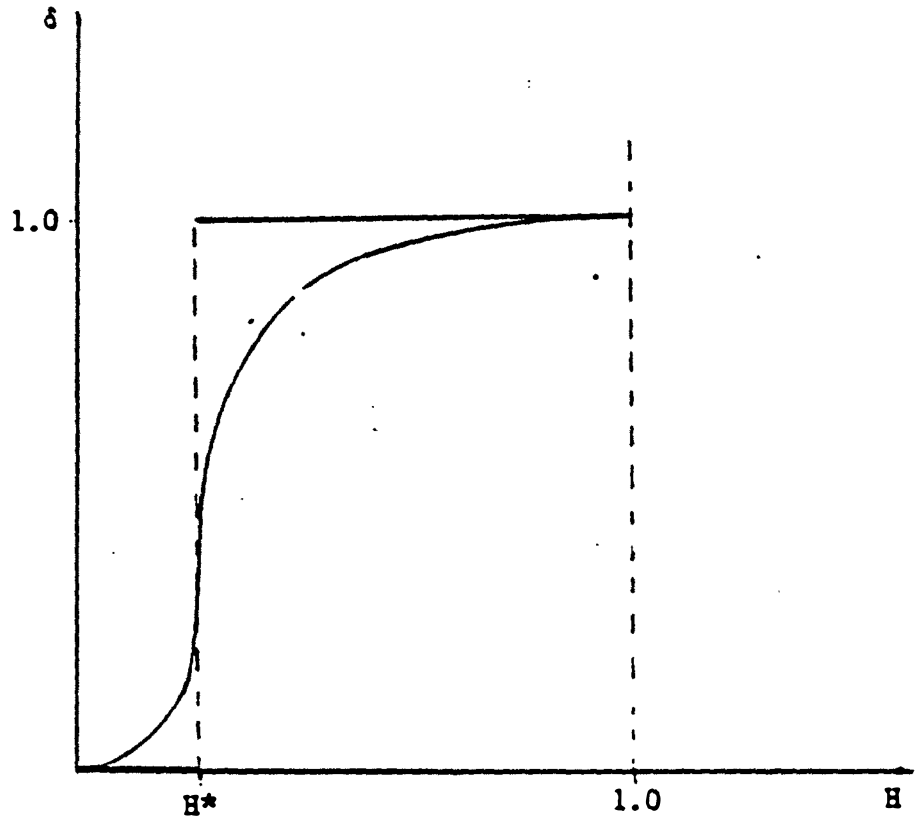


Figure 4



are given in Figure 5.

The equation in (24) was constructed as a hybrid of two results derived from first-order equations for the non-cooperative and cooperative models. It may also be constructed directly by using $l_i = (1 - \delta)\pi_i + \delta\Pi$ as the objective function for the i^{th} firm.[5/] Treating δ as a constant, the first order equation is:

(27) $$\frac{\partial l_i}{\partial a_i} = (1 - \delta)\frac{\partial \pi_i}{\partial a_i} + \delta \frac{\partial \Pi}{\partial a_i}.$$

Using equations (3) and (7) and setting $\frac{\partial l_i}{\partial a_i} = 0$ gives:

(28) $$(1 - \delta)\left(\gamma_{ii}\frac{s_i}{a_i} - 1\right) + \delta\left(\gamma_{ii}\frac{s_i}{a_i} - 1 + \sum_{j \neq i} \gamma_{ij}\frac{s_j}{a_i}\right) = 0.$$

After collecting terms and converting to matrix notation, we get

(29) $$v_i = (1 - \delta)\ \gamma_{ii} + \delta\ z_i^{-1} \sum_{j=1}^{N} \gamma_{ij}\ z_j,$$

(30) $$\underline{v} = (1 - \delta)\ \Gamma_d\ \underline{\iota} + \delta\ d_{\underline{z}}^{-1}\ \Gamma\ \underline{z}$$

$$= (1 - \delta)\ \underline{v}^0 + \delta\ \underline{v}^1$$

$$= d_{\underline{z}}^{-1}\left[(1 - \delta)\ \Gamma_d + \delta\ \Gamma\right]\underline{z},$$

and

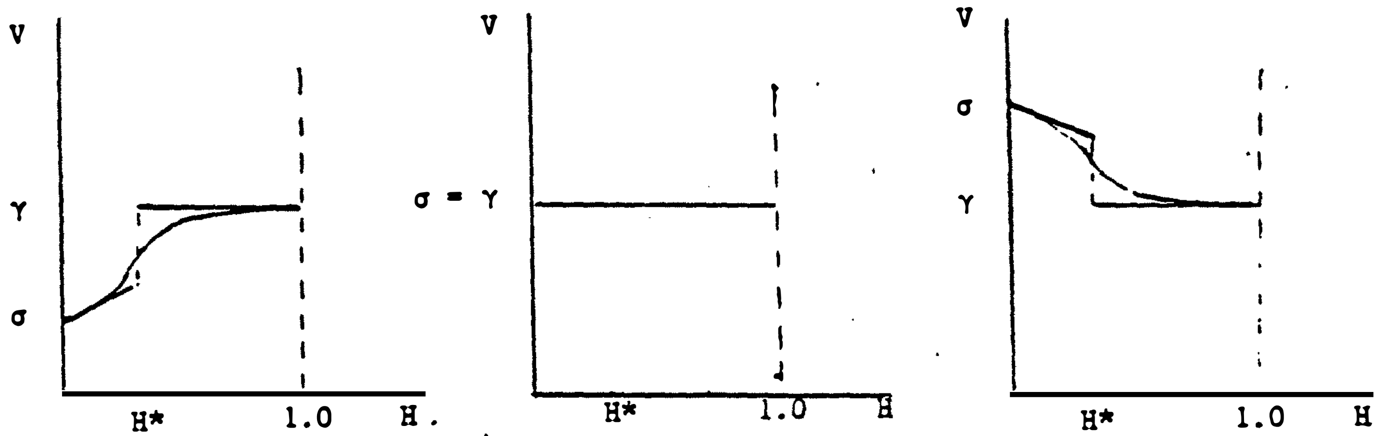
(31) $$V = \underline{z}'\ \underline{v} = \underline{\iota}'\left[(1 - \delta)\ \Gamma_d + \delta\ \Gamma\right]\underline{z}$$

$$= (1 - \delta)\ \underline{\iota}'\ \Gamma_d\ \underline{z} + \delta\ \underline{\iota}'\ \Gamma\ \underline{z}.$$

If we now impose the simplified demand structure by substituting (11), we get

(32) $$v_i = (1 - \delta)\quad \sigma + (\gamma - \sigma)\ z_i \quad + \delta\gamma, \text{ and}$$

Figure 5



(33) $$\underline{v} = (1 - \delta)\left[\sigma\,\underline{\iota} + (\gamma - \sigma)\,\underline{z}\right] + \delta\gamma\underline{\iota}.$$

The corresponding equation for V is given in (24).

If the relation between industry advertising intensity (V) and concentration (H) is non - linear, via the impact of concentration on the degree of cooperation, does the relation take the form of a quadratic, as some other investigators have proposed? (Greer (1971), Martin (1979), Strickland & Weiss (1976)). That question can be approached from two perspectives - conceptual and empirical. In the next section I will try to apply some statistical material to it: here I want to explore the question in the context of the models developed above.

Referring to equation (24), taking the derivative with respect to H, and letting $\delta = \delta(H)$, gives

(33a) $$\frac{\partial V}{\partial H} = (\gamma - \sigma)\,\{(1 - \delta) + (1 - H)\,\frac{d\delta}{dH}\}.$$

Given that δ and H each are bounded by 0 and 1, and that δ is an increasing function of H, the term in the brackets in non-negative. The sign of the deriviative, then, is the same as the sign of $(\gamma - \sigma)$.

If the industry effect is dominant ($\gamma > \sigma$), then advertising intensity will increase with concentration. If the two effects are equal, then advertising intensity will not vary with concentration, and if the brand-switching effect is larger, advertising intensity will decrease with concentration.

For given advertising elasticity parameters, then, it is not possible to show an increase in A/S up to some H and then a decrease. The relation may be non - linear, but it is monotonic over the permissible range of H.

One possibility is to introduce a relation between $(\gamma - \sigma)$ and H. If industries characterized as dominated by the brand-switching effect also were more concentrated, then a cross-industry comparison would show an inverted U shape relation between A/S and H. I know of no support for such an association between $(\gamma - \sigma)$ and H.

V. ERROR SPECIFICATION

So far, I have implicitly assumed that there are no errors in the model. . I now assume that the error variable in the equation for the $i^{\underline{th}}$ firm is additive, that its mean is zero, that its variance is inversely proportionate to its sales, and that all error term covariances are zero. That is,

$$(34) \qquad v_i = (1 - \delta)\left[\sigma + (\gamma - \sigma)\, z_i\right] + \delta\gamma + u_i,$$

$$E(u_i) = 0; \; \mathrm{Var}\,(u_i) = s_i^{-1}\,\xi^2; \; \mathrm{Cov}\,(u_i, u_j) = 0$$

In vector notation,

$$(35) \qquad \underline{v} = (1 - \delta)\left[\sigma\underline{\iota} + (\gamma - \sigma)\underline{z}\right] + \delta\gamma\underline{\iota} + \underline{u},$$

$$E(\underline{u}) = 0; \; \mathrm{Cov}\,(u) = \xi^2\, d_{\underline{s}}^{-1}.$$

If we now calculate $V = \underline{z}'\,\underline{v}$, we get

$$(36) \qquad V = (1 - \delta)\left[\sigma + (\gamma - \sigma)H\right] + \delta\gamma + U,$$

$$U = \underline{z}'\,\underline{u}; \; E(U) = 0, \; \mathrm{Var}\,(U) = \xi^2\,\underline{z}'\,d_{\underline{s}}^{-1}\,\underline{z} = S^{-1}\xi^2.$$

The error term specification, which follows Hall and Weiss, was applied by them to equations with the ratio of profits to equity or assets as dependent variables.[6] Their analysis seems to carry the same weight in the present context as well, particularly given the substantial evidence presented by marketing analysis on the tendency of company decision makers to use the advertising to sales ratio as the decision variable.[7]

The zero covariance assumption, on the other hand, is particularly troublesome. Within a single industry it is probably not true, since many events outside the industry would have similar effects on all the firms in the industry.[8] Between industries it is probably not true either, since many companies produce in more than one industry, and events in the firm would tend to effect all of its activities. The extension of the model to allow for non-zero covariances is certainly called for.

A third characteristic of this specification also deserves some comment; only one equation is shown for the firm or industry. Several investigators have included an advertising equation in a larger system of equations (Comanor and Wilson (1974), Martin (1979), Strickland and Weiss (1976). I did so in my dissertation.[9/] I have two defenses for presenting only single equation results here. One is the suggestion by Comanor and Wilson that simultaneous equation bias may not be very important in this context. The second is that I am willing to see the model extended to include a profitability equation, and I intend to move in that direction in the near future.

On the inclusion of a concentration equation, on the other hand, I am skeptical. The problem is that in some fundamental sense, equations in models of industrial organization should be oriented to the decision-making contexts of firms. If each firm in an industry with N firms has only one decision variable, then there can be only N independent equations which are based on the first-order optimization equations. It is not difficult, of course, to show a large number of functional relations which follow from the first-order conditions. The point here is that only N of them can be independent.

Consider, for illustrative purposes only, a situation in which the N quantities, say $\hat{q}_i$, are set exogeneously and each firm determines its advertising level, say $\bar{a}_i$, where a_i^* is the true optimal value of a_i, and $\bar{a}_i = a_i^* + \Phi_i$. If $p_i = p_i(\underline{q}, \underline{a})$, then the N $\hat{q}_i$'s and the N $\bar{a}_i$'s will determine N p_i's, say $\bar{p}_i$. Now, given $\underline{\bar{p}}$ and $\underline{\bar{q}}$, we can determine both the vector $\underline{s}$ and the vector $\bar{v}$. since $s_i = p_i q_i$ and $v_i = a_i/s_i$. We can also determine $\bar{S} = \Sigma \tilde{s}_i$ and $\underline{z} = S^{-1}\ \underline{\bar{s}}$

Given the observed values of $\underline{\bar{v}}$ and $\underline{\bar{z}}$, we might be tempted to conclude that we have a two equation system in which v_i and z_i are jointly determined. That would be inappropriate, however, since the 2N random variables $(\underline{v}, \underline{z})$ are determined by only N basic random variables $\underline{\Phi}$. The variance-covariance matrix for $(\underline{v}, \underline{z})$ must be singular.

The extension of this observation to industry level variables is straightforward. Since observed industry advertising intensity is $\bar{V} = \underline{\iota}' \underline{\bar{v}}$ and observed industry concentration is $\bar{H} = \underline{\bar{z}}' \underline{\bar{z}}$ (alternatively, $\bar{C4} = \sum_{i=1}^{4} \bar{z}_i$), it follows that $\bar{V}$ and $\bar{H}$ are functions of the same set of random variables $\underline{\Phi}$, and cannot be independent. Furthermore, if there are M industries, then the variables $(\bar{V}_1, \bar{V}_2, \ldots \bar{V}_M)$ and $(\bar{H}_1, \bar{H}_2, \ldots, \bar{H}_M)$ contain, at most, M independent random variables.

VI. STATISTICAL APPLICATIONS

Given the assumptions made above, γ, σ and δ are constant within a given industry. That being the case (34) can be written as

$$(37) \qquad v_i = \beta_0 + \beta_1 z_i + u_i$$

$$\beta_0 = (1 - \delta)\, \sigma + \delta\gamma \quad ; \; \beta_1 = (1 - \delta)\, (\gamma - \sigma).$$

If β_0 and β_1 are estimated for an industry using a suitable statistical procedure, giving $\bar{\beta}_0$ and $\bar{\beta}_1$, an estimate of γ is then available. The sum of β_0 and β_1 is γ, so $\hat{\gamma} = \hat{\beta}_0 + \hat{\beta}_1$ is the estimate of γ.

If δ is assumed to be bounded by zero and one, the sign of β_1 is also the sign of $\gamma - \sigma$. The estimate of β_0 is, of course, an estimate of $(1 - \delta) \sigma + \delta\gamma$. Moreover, if $\gamma > \sigma$, then σ is less than $\beta_0 = (1 - \delta) \sigma + \delta\gamma$. If $\hat{\beta}_1 > 0$, then, we would estimate that $\sigma < \hat{\beta}_0$. Corresponding results would hold if $\gamma = \sigma$ or $\gamma < \sigma$.

Intra-industry analysis.

For purposes of estimating this model, 32 manufacturing industry categories were selected. The data are from the 1974 Line of Business forms filed with the Federal Trade Commission; they are discussed in detail in the Annual Line of Business Report - 1974, which is available from the FTC. The 32 industry categories are identified in Table 1.

Simple regressions for equation (37) were run for each of the 32 industries, with data for each of the firms which filed in the industry constituting an observation. The results are given in Table 2, lines 1A, 2A, ... For all the 'A' equations, generalized least squares was used, with both the intercept and market share being scaled by the square root of sales.

The 32 industries were taken from the larger set of all manufacturing industry categories in the 1974 LB data files. Two selection criteria were used: there had to be at least ten reporting companies; and the industry had to be a consumer goods industry or a producer goods industry which is strongly associated with a related consumer goods industry so that advertising by the firms in the producer goods industry impacts on demand in the related industry (e.g., soft drink syrup and soft drinks). There were 135 industries with at least ten companies - 103 producer goods and 32 consumer goods.

TABLE 1. INDUSTRY CATEGORIES USED

FTC CODE	DESCRIPTION	RELATED 1972 SIC CODE
20.01	MEAT PACKING, SAUSAGES AND OTHER PREPARED MEAT PRODUCTS	2011,3
20.04	DAIRY PRODUCTS EXC. FLUID MILK	202,X2026
20.05	CANNED SPECIALTIES	2032
20.07	FROZEN SPECIALTIES	2038
20.08	CANNED, DRIED, DEHYDRATED, AND PICKLED FRUITS AND VEGETABLES INCLUDING PRESERVES, JAMS, JELLIES, DEHYDRATED SOUP MIXES, VEGETABLE SAUCES AND SEASONINGS, AND SALAD DRESSINGS	2033,4,5
20.10	DOG, CAT, AND OTHER PET FOOD	2047
20.12	FLOUR & OTHER GRAIN MILL PRODUCTS, RICE MILLING, BLENDED AND PREPARED FLOUR	2041,4,5
20.14	BREAD, CAKE, AND RELATED PRODUCTS	2051
20.18	CONFECTIONERY PRODUCTS	2065
20.26	BOTTLED AND CANNED SOFT DRINKS	2086
20.27	FLAVORING EXTRACTS AND SYRUPS, NEC.	2087
20.29	MISC. FOODS AND KINDRED PRODUCTS, EXC. ROASTED COFFEE	209,X2095
23.01	MEN'S AND BOYS' SUITS AND COATS	231
23.02	MEN'S AND BOYS' FURNISHINGS	232
23.03	WOMEN'S AND MISSES' OUTERWEAR	233
25.51	HOUSEHOLD FURNITURE	251
27.02	PERIODICALS	272
27.03	BOOKS	273
27.04	MISC. PUBLISHING	274
28.06	ORGANIC FIBERS	2823,4
28.07	DRUGS, ETHICAL	PT.283
28.08	DRUGS, PROPRIETARY	PT.283
28.09	PERFUMES, COSMETICS, AND OTHER TOILET PREPARATIONS	2844
28.10	SOAP AND OTHER CLEANING PREPARATIONS	284,X2844
28.15	PESTICIDES AND AGRICULTURAL CHEMICALS, NEC.	2879
29.01	PETROLEUM REFINING	291
36.08	HOUSEHOLD COOKING EQUIPMENT	3631
36.12	HOUSEHOLD APPLIANCES, NEC., INCLUDING ELECTRIC HOUSEWARES AND FANS AND SEWING MACHINES	3634,6,9
36.17	RADIO AND TV RECEIVING SETS	3651
38.08	PHOTOGRAPHIC EQUIP. & SUPPLIES, EXC. PHOTOCOPYING EQUIPMENT & SUPPLIES	PT.3861
39.03	SPORTING AND ATHLETIC GOODS, NEC.	3949
39.04	DOLLS, GAMES, TOYS, AND CHILDREN'S VEHICLES	394,X3949

TABLE 2. REGRESSION RESULTS

EQ. NR. (1)	IND. CODE (2)	NR. COS. (3)	INTER- CEPT (4)	MARKET SHARE (5)	ADVER- TISING (6)	SHARE* ADVER. (7)	RSQ (8)	ELASTI- CITY (9)	ELAST./ SHR. COEF. (10)
1A	20.01	16	0.00942(A)	-0.10670(C)			56.6	-0.09727(C)	1.10
B			0.00223(B)		0.00039		48.5		
C			0.00805(B)	-0.17600(B)	0.00072	0.00140	88.2		
2A	20.04	17	0.01494(A)	-0.01050			60.7	0.00449	-2.34
B			0.01032(C)		0.00038		63.5		
C			0.00960(C)	-0.21180	0.00241(A)	-0.00270	90.2		
3A	20.05	12	0.04911(A)	-0.07790			69.8	-0.02883	2.70
B			0.04455(B)		-0.00079		63.3		
C			0.03340(C)	-1.26520	0.00922(B)	0.03790	94.3		
4A	20.07	18	0.05901(A)	-0.38670			41.6	-0.32770	1.18
B			0.00574		0.00670(B)		52.6		
C			0.00700	0.25940	0.02100(A)	-0.27100(A)	92.0		
5A	20.08	26	0.03519(A)	-0.05540			44.1	-0.02019	2.74
B			0.01226(B)		0.00198(A)		60.4		
C			0.02451(C)	-0.81420	0.00390(A)	0.00240	89.1		
6A	20.10	16	0.04224(A)	0.35930(A)			90.9	0.40158(A)	0.89
B			0.05435(A)		0.00118(A)		91.7		
C			0.05351(A)	-0.89850(C)	0.00858(A)	-0.01670(B)	97.1		
7A	20.12	15	0.01862	0.00010			35.1	0.01873	0.01
B			0.00945		0.00106		43.9		
C			0.01109	-0.27050	0.00675(A)	-0.03160(B)	89.7		
8A	20.14	11	0.00757	0.14140(C)			81.4	0.14900(B)	0.95
B			0.00906(C)		0.00069(B)		83.7		
C			0.01323(B)	-0.80910(C)	0.00921(B)	-0.04100(C)	96.0		

LEVELS OF SIGNIFICANCE: A = .995; B = .99; C = .95

TABLE 2. REGRESSION RESULTS (CONT.)

EQ. NR. (1)	IND. CODE (2)	NR. COS. (3)	INTER-CEPT (4)	MARKET SHARE (5)	ADVER-TISING (6)	SHARE* ADVER. (7)	RSQ (8)	ELASTI-CITY (9)	ELAST./ SHR. COEF. (10)
9A	20.18	19	0.02397(B)	-0.07640			36.7	-0.05246	1.46
B			0.00605		0.00550(A)		59.7		
C			-0.00029	0.14180	0.01854(A)	-0.24150(A)	96.7		
10A	20.26	12	0.02890(A)	-0.22290			75.9	-0.19401	1.15
B			0.01897(C)		0.00043		70.5		
C			0.01416(C)	-0.34570	0.00741(A)	-0.07170	96.0		
11A	20.27	10	0.12915(C)	-0.19030			54.2	-0.06114	3.11
B			0.10604		-0.00069		46.6		
C			0.12083	-1.65990	0.00466	0.01760	80.5		
12A	20.29	35	0.03113(A)	0.15920			46.0	0.19037	0.84
B			0.02059(A)		0.00151(A)		61.4		
C			0.01411(B)	-0.46190(A)	0.00847(A)	-0.07430(A)	92.1		
13A	23.01	10	-0.00412	0.35450(A)			83.2	0.35042(A)	1.01
B			0.00293(B)		0.00570(A)		96.1		
C			0.00187	-0.05280	0.01750(A)	-0.16200(A)	99.6		
14A	23.02	20	0.01159(3)	0.02940			42.4	0.04099	0.72
B			0.00684(C)		0.00175(C)		55.0		
C			0.00792(C)	-0.30080(C)	0.00671(A)	-0.04210	91.5		
15A	23.03	19	0.01408(A)	-0.43250(C)			56.1	-0.41844(C)	1.03
B			0.00006		0.01155(A)		65.1		
C			-0.00078	0.33650	0.02207(B)	-1.20770	88.4		
16A	25.51	24	0.01416(A)	-0.19080			49.1	-0.17664	1.08
B			0.00564(C)		0.00467(A)		60.7		
C			0.00741(C)	-0.37880(C)	0.01614(A)	-0.40170(C)	87.3		

TABLE 2. REGRESSION RESULTS (CONT.)

EQ. NR. (1)	IND. CODE (2)	NR. COS. (3)	INTER- CEPT (4)	MARKET SHARE (5)	ADVER- TISING (6)	SHARE* ADVER. (7)	RSQ (8)	ELASTI- CITY (9)	ELAST./ SHR. COEF. (10)
17A	27.02	11	0.07968(B)	-0.79370			56.5	-0.71406	1.11
B			0.04679		0.00007		46.9		
C			0.03266	-0.35760	0.01560	-0.22890	78.0		
18A	27.03	19	0.06574	-0.42360			22.0	-0.35782	1.18
B			0.00746(A)		0.00875(A)		94.6		
C			0.01983(B)	-0.60310(C)	0.01719(A)	-0.33610(A)	98.9		
19A	27.04	10	0.05082	-0.57060			22.6	-0.51977	1.10
B			-0.00080		0.02772(A)		93.0		
C			0.00907	-0.19200	0.04305(A)	-0.46960	99.5		
20A	28.06	13	0.01454(A)	-0.01120			83.7	0.00334	-3.36
B			0.01380(A)		-0.00011		82.1		
C			0.01526(A)	-0.26700(B)	0.00251(A)	0.00530	96.9		
21A	28.07	28	0.04949(A)	-0.08910			53.6	-0.03962	2.25
B			0.02095(C)		0.00173(A)		69.0		
C			0.02965(A)	-0.56650(C)	0.00450(A)	-0.03370(C)	91.1		
22A	28.08	15	0.13306	1.19730			65.5	1.33035	0.90
B			0.07526(C)		0.00396(A)		83.5		
C			0.07523	-0.67230	0.01392(A)	-0.09480(C)	95.3		
23A	28.09	21	0.20873(A)	-1.21810(A)			71.2	-1.00935(C)	1.21
B			0.07874(B)		0.00183(C)		68.6		
C			0.09279(A)	-0.74370(C)	0.00605(A)	-0.04870(A)	95.1		
24A	28.10	32	0.09506(A)	-0.10960			54.5	-0.01450	7.56
B			0.08903(A)		-0.00011		53.3		
C			0.06981(A)	-5.81170(A)	0.00808(A)	0.01520(A)	88.8		

TABLE 2. REGRESSION RESULTS (CONT.)

EQ. NR. (1)	IND. CODE (2)	NR. COS. (3)	INTER-CEPT (4)	MARKET SHARE (5)	ADVER-TISING (6)	SHARE* ADVER. (7)	RSQ (8)	ELASTI-CITY (9)	ELAST./ SHR. COEF. (10)
25A	28.15	16	0.03019	-0.09990			27.5	-0.06969	1.43
B			0.01446		0.00224		28.2		
C			0.02178	-0.54630	0.02221(C)	-0.07500	64.6		
26A	29.01	29	0.00242(A)	0.00520			59.2	0.00761	0.68
B			0.00105(B)		0.00012(A)		80.3		
C			0.00135(A)	-0.01280(B)	0.00027(A)	-0.00080(A)	97.4		
27A	36.08	13	0.03296(A)	-0.08290			59.7	-0.04994	1.66
B			0.01755		0.00203		49.4		
C			0.02499(C)	-0.43820(C)	0.01374(A)	0.06830	94.6		
28A	36.12	24	0.03328(C)	0.13520			39.5	0.16843	0.80
B			0.03042(B)		0.00122		42.4		
C			0.05793(A)	-2.73780(A)	0.02100(A)	-0.03220	81.9		
29A	36.17	13	0.02259	0.09530			64.1	0.11789	0.81
B			0.01448		0.00104(C)		73.2		
C			0.01722(A)	-0.46160(A)	0.00496(A)	-0.01040(B)	98.9		
30A	38.08	14	0.02436(A)	-0.00110			62.8	0.02329(C)	-0.05
B			0.02041(C)		0.00009		63.7		
C			0.01584(C)	-0.16550	0.00219(A)	-0.00060	95.5		
31A	39.03	14	0.02790(C)	0.06590			68.4	0.09384	0.70
B			0.01896(C)		0.00344		74.9		
C			0.02196(C)	-0.43010	0.01323(A)	-0.04800	91.5		
32A	39.04	12	0.04563(B)	0.38300			87.9	0.42865(C)	0.89
B			0.04277(A)		0.00254(B)		91.9		
C			0.04133(B)	-0.71250	0.01188(C)	-0.06270	97.2		

The consumer good/producer good split is sometimes difficult to determine. Some support for the split used in this paper is a comparison of goodness - of - fit measures for the two groups of industries. Table 3 contains distributions of R^2 measures for the 32 consumer goods industries and the 103 producer goods industries. Median R^2 is 57 for the consumer goods set and 40 for the producer goods set.

Estimates of γ, the elasticity of demand with respect to advertising, are given in column 9 of Table 2. The hypothesis that $\gamma = 0$, with a two - tailed alternative, was applied for each industry. As shown in the table, that hypothesis could not be rejected for 24 of the 32 industries. Of the 24, 14 are negative, and 10 are positive. For the remaining eight, three are significantly negative, and five are significantly positive.

Of the three negative elasticities, the largest (in absolute value) is for cosmetics (code 28.09). With an elasticity of -1.01, a one percent increase in industry advertising would generate a decrease in industry sales of slightly more than one percent.

At the other extreme is toys and games (code 39.04). For that industry, a one percent increase in industry advertising would lead to a sales increase of four-tenths of a percent. Though not significantly different from zero, the elasticity estimate for proprietary drugs (code 28.08) is ghest among those which are positive.

Both the cosmetics and proprietary drug industries have very high advertising to sales ratios (22.1 per cent for 28.08 and 13.4 per cent for 28.09). On the other hand flavoring extracts (mainly soft-drink syrups) also has a high industry advertising to sales ratio (8.0 per cent), but its estimated elasticity is very small and not significantly different from zero.

It does not seem unusual to find many elasticities near zero, with a few negative and a few positive. If aggregate consumption is insensitive to aggregate advertising, as seems likely, [10] then what one industry gains must be lost by others.

Finally, a determination concerning the industry elasticity of demand does not fix the individual firm's elasticity. Going back to equation (10), we can write $\gamma_{ii} = (1 - z_i)\,\sigma + z_i\,\gamma$. Since $\sigma > 0$, it follows that if $\gamma \geq 0$, then $\gamma_{ij} > 0$. If $\gamma < 0$, however γ_{ij}'s sign is indeterminate.

Turning next to the coefficient of market share, 20 of them are negative and 12 are positive, with three of each sign being significantly different from zero. The largest positive value which is significant is for pet food (code 20.10), and a close second is men's & boys' suits and coats (code 23.01). The largest significant negative value is for cosmetics (code 28.09).

In the model some relations among γ, σ, δ and β_1 are implied. If σ is negative, $(1 - \frac{\sigma}{\gamma})$ is greater than one. Since $\beta_1/\gamma = (1 - \sigma)\,(1 - \frac{\sigma}{\gamma})$, β_1/γ is positive, but it will be less than one if δ is large enough. Seventeen of the $\hat{\gamma}$'s are negative; in each of the seventeen industries $\hat{\beta}_1$ is also negative and less (algebraicly) than $\hat{\gamma}$, so $\hat{\beta}_1/\hat{\gamma}$ is greater than one. This is not evidence that the degree of cooperation is low or zero in any industry, of course. A high δ together with a negative γ which is small (in absolute terms) relative to σ can give a value of β_1/γ greater than one. On the other hand, if β_1/γ were less than one, δ would have to be greater than zero.

For $\gamma > 0$, two conditions may hold. If $\gamma < \sigma$, it follows that $\beta_1 < 0$, since $(\gamma - \sigma) < 0$. It also follows that $\beta_1/\gamma < 0$, regardless of the value of δ. In addition, the relation is an if-and-only-if one; i.e., if β_1/γ is negative, then $\gamma < \sigma$. Three industries have negative values for β_1/γ: dairy products except milk (code 20.04), synthetic fibers (code 28.06) and photographic equipment (code 38.08). In all three cases γ is quite small, so σ does not have to be very large to give a negative value to β_1.

If $\gamma > \sigma$, the model shows that $\beta_1 > 0$, whatever the value of δ. There are 12 cases where both $\hat{\gamma}$ and $\hat{\beta}_1$ are positive, with $\hat{\beta}_1$ ranging from near zero (grain milling, code 20.12) to 1.20 (proprietary drugs, code 28.08). Several

Table 3 Distributions of R^2-Consumer Goods and Producer Goods Industries

	Consumer Goods	Producer Goods
0 - 9.9	0	3
10 - 19.9	0	11
20 - 29.9	3	20
30 - 39.9	3	17
40 - 49.9	5	16
50 - 59.9	8	15
60 - 69.9	6	10
70 - 79.9	2	6
80 - 89.9	4	4
90 - 100	1	1
Totals	32	103

Notes:

1. R^2's are scaled by 100.
2. Since generalized least squares regression is used, R^2 is calculated as $F/\{F + [(N - K - 1)/K]\}$, where F is the F statistic for the hypothesis that all of the coefficients of non - intercept terms are simultaneously equal to zero.

of these cases have values of β_1/γ greater than 0.8 (pet food (20.10), bread & cakes (20.14), miscellaneous processed foods (20.29), men's and boys' suits and coats (23.01), proprietary drugs (28.08), electric housewares (36.12), radio and TV sets (36.17), and games and toys (39.04)). For β_1/γ to be greater than 0.8, both δ and σ/γ must be less than 0.2. That is, in these industries, the results shown in Table 2 would hold only with very strong dominance of market demand effects over brand switching effects <u>and</u> a very low degree of cooperation. Using the concentration ratios given in Table 4, the average concentration ratio for the eight industries is 38.1. For the remaining 24 industries, it is 43.2. This result would lend some support to the proposition that concentration and cooperation are related if the group of eight industries are identified as having low degrees of cooperation.

One final note on model predictions. Since both σ and δ are assumed to be non-negative, β_0 should also be non-negative. In only one of the 32 cases is β_0 negative, and it is not significant.

TABLE 4. INDUSTRY CATEGORY DATA

FTC CODE (1)	NUM. OBS. (2)	ELASTI-CITY (3)	CONV./ NON-CONV (4)	HERF INDEX (5)
20.01	16	-0.09727	CONV	629
20.04	17	0.00449	CONV	689
20.05	12	-0.02893	CONV	1248
20.07	18	-0.32770	CONV	614
20.08	26	-0.02019	CONV	393
20.10	16	0.40158	CONV	819
20.12	15	0.01973	CONV	719
20.14	11	0.14900	CONV	925
20.18	19	-0.05246	CONV	558
20.26	12	-0.19401	CONV	842
20.27	10	-0.06114	CONV	1191
20.29	35	0.19037	CONV	467
23.01	10	0.35042		1200
23.02	20	0.04097		509
23.03	19	-0.41844		572
25.51	24	-0.17664		443
27.02	11	-0.71406	CONV	912
27.03	19	-0.35782	CONV	559
27.04	10	-0.51977	CONV	1064
28.06	13	0.00334		1336
28.07	28	-0.03962	CONV	632
28.08	15	1.33035	CONV	791
28.09	21	-1.00735	CONV	796
28.10	32	-0.01450	CONV	983
28.15	16	-0.06969	CONV	1008
29.01	29	0.00761	CONV	731
36.08	13	-0.04994		833
36.12	24	0.16843		704
36.17	13	0.11789		783
38.08	14	0.02329		2306
39.03	14	0.09384		751
39.04	12	0.42865		842

Inter-industry analysis.

Equation (36) may be used to formulate a rough test of the cooperation vs. efficiency controversy. If we restricted ourselves to some subset of industries where the relation between γ and σ is the same, we would be looking at a sample of industries for which the dependence of V on H and δ is a quadratic with no squared terms.

If we now substitute (26) into (36), we get

$$(38) \quad V = \sigma + (\gamma - \sigma) H + (\gamma - \sigma) \delta(H) - (\gamma - \sigma) H \delta(H) + U.$$

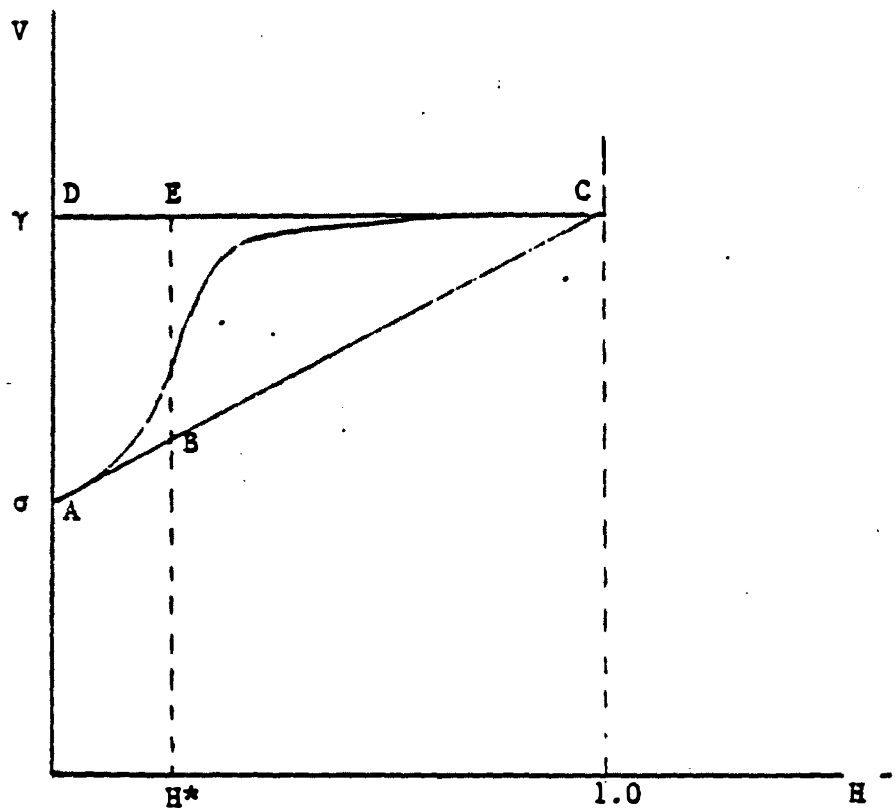
If it is only differences in market shares that affect advertising intensity levels, if concentration does not affect cooperation, and if $\delta = 0$, (38) reduces to

$$(39) \quad V = \sigma + (\gamma - \sigma) H + U.$$

Let this be the null hypothesis, and note that V is a linear function of H. If $\gamma > \sigma$, and the null hypothesis holds, the expected relation between V and H is as shown by the positively sloped straight line in Figure 6, between points A and C.

In this context, the alternative hypothesis is that the degree of cooperation (δ) is a positive function of concentration (H). If the function is as described in (26) and Figure 4, the relation between V and H would be as shown by the curved line connecting points A and C in Figure 6. One way to test the null hypothesis against this alternative is to test for the hypothesized curvature in the function relating V and H.

Figure 6



One characteristic of products which may be useful in defining classes with respect to significance of the brand-switching effect is the convenience/non-convenience distinction developed by Porter (1976). Using his identification of IRS industries as a starting point, I assigned each of the 32 industries used in this study to one of the two classes. The assignments are noted in Table 4.

Using the convenience/non-convenience distinction, the sample was divided into two sub-samples. Four regressions were then run, one for each sub-sample, one for the total sample, and one for the total sample with a dummy variable for convenience goods. Each regression was run with LB's and with industries as observations. The results are given in Table 5.

For the LB level equations (1A - 1D), equation (37) was used. Some rearranging of terms gives $v_i = \gamma - (1-\delta)\ (\gamma - \sigma)\ (1-z_i) + u_i$ $= \gamma - \beta_1(1 - z_i) + u_i$. When observations are pooled across industries, γ is an industry variable. In these regressions the values of γ estimated in the first stage were used. Since industries have been grouped so as to reduce differences in $(\gamma - \sigma)$, that term is treated as a constant within each group. Finally, H, the Herfindahl index, has been substituted for δ, the degree of cooperation. The resultant equation is linear in γ, $(1 - z_i)$, and $H\ x(1 - z_i)$. This is the equation for which results are reported in Table 5. In all regressions all variables were weighted by the square root of sales.

TABLE 5. REGRESSION RESULTS, EXTENSIONS TO TYPE OF PRODUCT AND LEVEL OF CONCENTRATION

EQ. NR. (1)	OBS. TYPE (2)	NR. OBS. (3)	CONV./ NON-CONV (4)	ELASTI- TICITY (5)	ONE - MKT SHARE (6)	HERF. X (1 - M.S.) (7)	KD X (1 - M.S.) (8)	KD X HERF (1 - M.S.) (9)	RSQ (10)
1A	LB	388	CONV	-.256	-.020	.570(A)			19.2
				(-.272)	(-1.49)	(3.07)			
1B	LB	176	NON-CONV	5.82(A)	.0232(A)	.00609			60.9
				(6.70)	(6.66)	(.200)			
1C	LB	564	ALL	.151	.00929	.162(C)			20.4
				(.199)	(1.64)	(2.28)			
1D	LB	564	ALL	.318	.0219(C)	.0356	-.0423(B)	.541(A)	21.6
				(.418)	(2.21)	(.410)	(-2.79)	(3.00)	

EQ. NR. (1)	OBS. TYPE (2)	NR. OBS. (3)	CONV./ NON-CONV (4)	ELASTI- TICITY (5)	ONE - HERF INDEX (6)	HERF. X (1 - HERF) (7)	KD X (1 - HERF) (8)	KD X HERF (1 - HERF) (9)	RSQ (10)
2A	IND	21	CONV	-.341	-1.91	55.0			26.8
				(-.0982)	(-.390)	(.919)			
2B	IND	11	NON-CONV	5.75(C)	2.39(B)	-.0152			85.4
				(2.73)	(2.90)	(-0.002)			
2C	IND	32	ALL	.0907	1.08	13.6			28.5
				(.034)	(.538)	(.602)			
2D	IND	32	ALL	.236	2.31	2.20	-4.25	53.4	30.4
				(.086)	(.665)	(.077)	(-.795)	(.852)	

T-RATIOS ARE GIVEN IN PARENTHESES. LEVELS OF SIGNIFICANCE: A - .995; B - .99; C - .95. SINCE NO INTERCEPT IS USED IN THE EQUATIONS BEFORE ADJUSTMENT FOR HETEROSKEDASTICITY, RSQ IS CALCULATED AS F / F + (N + K)/K , WHERE N IS THE NUMBER OF OBSERVATIONS, F IS THE F-STATISTIC FOR THE HYPOTHESIS THAT ALL COEFFICIENTS ARE EQUAL TO ZERO, AND K IS THE NUMBER OF INDEPENDENT VARIABLES.

Mean values for relevant variables for the two sub-samples are:

	Conv.	Non-Conv.
Adv./Sales	4.7%	2.3%
Mkt. Share	2.7%	3.2%
Elasticity	-.053	.022
Herfindahl	731	860

That the distinction between convenience goods and non - convenience goods matters is supported not only by these means, but also by the regression results. Equations 1A and 1B are different in almost every respect. Only the concentration/market share interaction term seems to matter for convenience goods. Just the opposite holds for non - convenience goods: both elasticity and market share are highly significant, but the concentration/market share interaction term is not.

Given that one minus market share is used in these equations, the coefficient of market share is the negative of what is shown in col. (6) of Table 5. For the regression for non - convenience LB's reported in equation 1B, then, LB's with larger market shares have lower advertising to sales ratios.

On the question of the impact of the degree of cooperation, the story is mixed. For convenience goods, the coefficient is positive, as equation (37) predicts when there is a positive degree of cooperation. For the non-convenience goods LB's concentration has no effect.

Aggregation to the industry level and then rerunning the equations gives the results shown in (2a-2d) of Table 5. As expected, R^2 goes up and levels of significance for individual coefficients go down. In addition, the coefficient of H(1-H) in equation 2A is not significant, though it is positive. Since 2A is the industry level cognate of 1A, the variable H(1-H) plays the same role in 2A as $H(1-z_1)$ plays in 1A.

Economies of scale in advertising.

The literature on advertising contains a number of definitions of economies of scale and of findings concerning its presence and magnitude. Before I look at the data with the hope of commenting about scale effects, then, I want to define the term as I use it.

By economies of scale I mean a decrease in the number of units of advertising which are needed to generate a unit of sales as advertising is increased. Given the definition of the firm's own elasticity of demand with respect to advertising (see the text following equation (1)), the technical definition is that $\gamma_{ii} > 1$. Diseconomies of scale are defined symmetrically; i.e., $\gamma_{ii} < 1$.

With a cross - section of observations on advertising intensity ($v_i = a_i/s_i$) and on advertising (a_i), we may regress the former on the latter. The results of such regressions for the 32 industries are given in Table 2, lines 1B, 2B, ..., 32B. For the 32 cases the regression coefficient is positive in all but one of them, and that one is not significant. For 18 of the 31 positive coefficients, the coefficient is significant.

The question now is whether these results are evidence of pervasive diseconomies of scale in advertising. I think not. Given my assumption that the data I observe are equilibrium results, I may rightly conclude that almost everywhere high levels of advertising are associated, in equilibrium, with high ratios of advertising to sales. I may not conclude anything, however, about the impact that a move by some firm away from its equilibrium level of advertising would have on its advertising to sales ratio.

Given the assumed relation in equation (37), there is a corresponding equilibrium equation relating v_i and a_i. The equation is non - linear, and I have not found a simple way to characterize it. It is possible to determine its deriviative with respect to a_i by taking the deriviative of (37). When that is done, the result is

(40) $$\frac{\partial v_i}{\partial a_i} = \beta_1 \sigma a_i^{-1} z_i (1-z_i).$$

Since all the other terms in (40) are non-negative, the sign depends only on the sign of β_1, which is in turn dependent only on the sign of $(\gamma - \sigma)$. That the signs of the coefficients of a_i in equations 1B, 2B, ..., 32B are not always equal to the signs of the coefficients of z_i in 1A, 2A, ..., 32A is still a mystery to me, and something to be explored.

V. Summary and Conclusions

My purpose in writing this paper was to attempt to integrate explicit microeconomic modelling, reasoned econometric specification of error terms, and high quality data to explore some questions about advertising by large firms. Several quite important improvements could be made in the first two of those areas, and much data massaging is yet to be done in the third.

Concerning the substance of what I have done, I think six things are important:

1. Explicit modeling is worth the effort, if for no other reason than that it provides a basis for choosing from a variety of functional forms.
2. The same is true for error specification. I did in fact look at scatter diagrams for all 32 industry categories, and they virtually all show the heteroscedasticity which I assumed and for which I corrected.
3. The predicted relation between advertising intensity and market share shows up clearly in only 25% of the cases examined. I have yet to explore why that may be the case.
4. Some evidence concerning the presence and impact of cooperation was produced, but it is not clearly pervasive.
5. The distinction between convenience goods and non-convenience goods is unambiguously a good one.
6. Virtually no evidence concerning economies of scale in advertising can be gleaned from this study, given its assumptions.

More work is called for on many of these issues; my study seems to have raised more questions than it has answered. This is probablly due to the richness of the data source, since I had the opportunity to address several issues at once.

Footnotes

* Manager, Line of Business Program, Federal Trade Commission. The author depended heavily on several staff members of the LB Program for statistical and clerical support, in particular, Joe Cholka, George Pascoe, and Harolene Jenkins. Helpful comments were given by Richard Caves, Dennis Mueller, Michael Porter, and F. M. Scherer. When the paper was in its early stages of development, both Steve Garber and Jon Rasmussen, former economists in the LB Program, provided cery useful critiques. The views expressed here are my own, of course.

1. The basic explicit model of the role of advertising is Dorfman and Steiner (1954), and the models in this paper may be seen as another extension of their work.

2. For a review of models of oligopoly which treat prices and quantities as relevant variables, see Shubik (1959). Extensions to take advertising into account are fairly straightforward.

3. Dorfman and Steiner dealt with a price-setting firm. For a monopolist, inverting the demand relation to show $P = P(Q,A)$ instead of $Q = (P,A)$ has no effect on the results of the analysis. For oligopolists who face a system of demand relations, inversion does have some impact on the results. The differences are not trivial; nonetheless, they will not be explored in this study.

4. Shubik (1959), p.

5. From a formal mathematical point of view, the function l_i is similar to a Legrangian function of the form $L_i = \pi_i + \lambda(\sum_{i \neq j} \pi_j - (\sum_{i \neq j} \pi_j)^*)$.

   where the * indicates some fixed level of the profits of the rest of the industry. Following the same logic which results in the identification and income constraint as the marginal utility of the profits of the rest of the industry in the objective of firm i.

6. Hall & Weiss (1967), p. 323.

7. See Schmallensee (1972), Ch.2, pp. 16-47.

8. On this point see Imel & Helmberger (1971).

9. Long (1970), p. 130 - 139.

10. See Schmallensee (1972), Ch. 3, pp. 48-87.

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