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**DOMINANT-FIRM PRICING AND BYPRODUCT SUPPLY  
THE STRUCTURE OF THE U.S. MOLYBDENUM INDUSTRY**

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**WORKING PAPER NO. 50**

**March 1982**

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DOMINANT-FIRM PRICING AND BYPRODUCT SUPPLY  
THE STRUCTURE OF THE U.S. MOLYBDENUM INDUSTRY

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July 1981

\* I am grateful to Daniel Alger and David Scheffman for reviews  
of an earlier draft.

## I: INTRODUCTION

Many markets are characterized by two sorts of production -- primary and byproduct. Byproduct production is distinguished from other sorts of multiproduct production in that byproducts and coproducts are produced in approximately fixed proportions.<sup>1</sup> In contrast, with joint products, a smooth production transformation curve exists and product price ratios determine product production proportions. Fixed-proportion production is very prevalent in the chemicals industries (whenever there is a chemical reaction) and in the metals industries (where ores contain approximately fixed proportions of several metals).<sup>2</sup> One key characteristic of byproducts is that supply may be more responsive to the price of some other commodity than to its own price. For example, byproduct molybdenum production may be more responsive to the price of copper, which occurs in the same ores, than to its own price. This paper examines the market-power implications of byproduct production.

A model of profit-maximizing behavior for a dominant firm (or cartel) facing a competitive fringe is developed where the dominant firm is a primary producer whereas fringe firms may be byproduct producers. The model is intertemporal because the dominant firm knows that the price it chooses today will affect fringe entry in the future. Because byproducts are so prevalent in the metals industries, industry output is assumed to be an exhaustible resource. However, a fully reproducible product is a special case, and the results obtained here can be applied to other industries such as the chemicals industries. The model developed in this paper is in the spirit of dominant-firm competitive-fringe models found in Gaskins (1971), Salant (1976), Pindyck (1977), Gilbert (1978), and Lewis and Schmalensee (1979).

In the next section, the theoretical model is presented and an analysis of

its comparative dynamics is performed. In particular, the price sensitivity of the fringe's response (or entry) function is varied. It is shown that, all else being equal, a decrease in fringe responsiveness leads to a higher price for the industry and a smaller market share for the dominant firm. It therefore looks as if the market is more competitive when entry is insensitive to price, even though the dominant firm has greater power to raise price above marginal cost. In section III, lags in fringe supply are introduced so that entry is not solely a function of current price but is determined by a distributed lag on all past prices set by the dominant firm. An analysis of the extended model's comparative dynamics shows that supply lags also increase the dominant firm's market power and reduce its market share.

The development of the theoretical models is followed by a description of the molybdenum industry, an industry characterized by one large primary producer, AMAX, and several fringe producers that mine copper ores and produce molybdenum as a byproduct.<sup>3</sup> Empirical estimates of the molybdenum industry's byproduct response function are presented in section V under both the static and dynamic (lags in supply) assumptions about entry. The empirical estimates show that fringe entry is very insensitive to own price and that, if there is any significant own-price response, it is very delayed, conditions that facilitate a considerable degree of pricing power for the dominant firm. Finally, in the last section, policy implications are discussed and conclusions are drawn.

## II: THEORETICAL MODEL

We will assume that the market to be modeled consists of one dominant primary producer (or a cartel of joint profit-maximizing primary producers) and a competitive (price-taking) fringe. The fringe may consist of primary producers or of byproduct producers. In either case, the output of the fringe is indistinguishable from the output of the dominant firm. Consumers are assumed to be passive price takers. The dominant firm knows the reaction of the fringe to its own actions and takes this reaction into account in choosing a price path that maximizes its own discounted stream of present and future revenues minus costs. In this equilibrium model, we concentrate on explaining the long-run phenomenon of changes in fringe output over time (i.e., entry or exit)<sup>4</sup> as a function of price rather than explaining changes in fringe output at a point in time (i.e., capacity utilization). The model developed below is an extension of Gaskins (1971) limit-pricing model.

Because the product is an exhaustible resource, we assume that a finite stock of primary reserves is owned by the dominant firm and that cumulative production by the dominant firm does not exceed this stock. These reserves are assumed to be extractible at constant cost. However, the assumption of constant costs for the dominant firm does not significantly affect the results.<sup>5</sup> For the fringe, we do not assume a finite stock of reserves because, in theory, byproduct production could include mining ordinary rock. Instead, we assume that fringe reserves vary in production cost. When reserves vary in cost, less entry will be induced for given price as low-cost reserves are depleted.

In presenting the model, the following notation will be used. Let

$p(t)$  be the price of the product at time  $t$ ,  
 $f(p)$  be the industry demand curve,<sup>6</sup>  
 $c$  be constant average and marginal extraction cost for the dominant firm,  
 $\bar{R}$  be the finite stock of reserves owned by the dominant firm,  
 $x(t)$  be the output of the fringe at time  $t$  (fringe supply is assumed to be completely inelastic in the short run; the effect of price is to induce entry or exit),  
 $X(t)$  be cumulative production by the fringe up to time  $t$ ,  
 $k(p,X,t)$  be the time rate of change of  $x$ , i.e., the reaction of the fringe to the price set by the dominant firm,  
and  $r$  be the dominant firm's rate of time preference.

The problem for the dominant firm is to choose a price path that maximizes the integral

$$\max_p \int_0^{\infty} e^{-rt} (p(t) - c)(f(p) - x(t)) dt \quad (1)$$

subject to

$$\int_0^{\infty} (f(p) - x(t)) dt \leq \bar{R} \quad (2)$$

$$X(t) = \int_0^t x(\tau) d\tau \quad (3)$$

and  $\dot{x}(t) = k(p,X,t), \quad k_p \geq 0, \quad k_X \leq 0, \quad (4)$

where a dot over a variable denotes its time rate of change and a subscripted function denotes the partial derivative of that function with respect to the argument of the subscript. The profit function is assumed to be continuously differentiable.

The precise way in which fringe firms react to the price set by the dominant firm is not specified. It is not assumed that the fringe knows the entire future price path with certainty and optimizes with respect to this path, an assumption

made in most studies using game-theory solution concepts (see Salant (1976), Gilbert (1978) and Lewis and Schmalensee (1979), for example). However, fringe firms may form expectations about future prices from current prices and optimize with respect to their expectations. The area of price expectations, however, will not be dealt with here. What is important is that the dominant firm knows how the fringe will react to its chosen price. The dominant firm and the fringe are thus treated asymmetrically.

The rate of change of fringe output (fringe entry and exit) is both endogenous, a function of price, and an exogenous function of time. Exogenous entry occurs because the fringe is responding to variables determined in some other market. For example, molybdenum byproduct producers may be responding to copper price. The model encompasses the two extremes of entry solely endogenous, as will be the case if fringe firms are primary producers, and entry solely exogenous. The latter will occur if, for example, copper is the dominant-value product and copper and molybdenum are produced in absolutely fixed proportions. In any actual market, it is unlikely that either extreme will prevail and the mix between induced and exogenous entry must be determined empirically.

The optimal-control problem expressed in equations 1-4 can be solved by introducing the costate variables  $\mu(t)$  and  $\eta(t)$  corresponding to the state variables  $x$  and  $X$ , the (constant) Lagrangian multiplier  $\lambda$  corresponding to restriction 2, and forming the Hamiltonian,  $H$

$$H = e^{-rt} (p(t) - c)(f(p) - x(t)) - \lambda(f(p) - x(t)) + \mu(t)k(p, X, t) + \eta(t)x(t). \quad (5)$$

A first-order condition for an interior maximum of (1) is

$$\begin{aligned} \partial H / \partial p &= e^{-rt} [(p(t) - c)f'(p) + f(p) - x(t)] - \lambda f(p) + \\ &\mu(t)k_p = 0 \end{aligned} \quad (6)$$

or 
$$p = c + (x(t) - f(p))/f'(p) - \mu(t)e^{rt}k_p/f'(p) + \lambda e^{rt}. \quad (7)$$

To see how price changes with changes in the fringe's response function, we compute the partial derivative of price with respect to  $k_p$  (where  $k_p$  measures the sensitivity of fringe entry to the price set by the dominant firm). By the implicit function theorem,

$$\partial p / \partial k_p = - (\partial^2 H / \partial p \partial k_p) / (\partial^2 H / \partial p^2) = - \mu(t) / (\partial^2 H / \partial p^2). \quad (8)$$

If we interpret the costate variable in the usual way (i.e.,  $\mu(t)$  is the discounted value to the dominant firm of one additional unit of  $x$  at time  $t$ ) we see that  $\mu(t)$  must be negative. Therefore, the numerator of (8) is greater than zero. The denominator of (8) is less than zero by the second-order conditions for a maximum of (1). Therefore, as  $k_p$  decreases, all being equal, price rises.

We cannot, however, say unambiguously that the price set by the dominant firm will always be higher when fringe entry is unresponsive to price, because the Lagrangian multiplier  $\lambda$  is endogenously determined and, though constant over time, varies with different responses by the fringe. To illustrate this point, we consider the case of linear demand. Let

$$f(p) = a_0 - a_1 p. \quad (9)$$

When (9) is substituted into (7), after algebraic simplification, we obtain

$$p = (a_0 + a_1 c - x + k_p \mu(t) e^{rt} + a_1 \lambda) / (2a_1). \quad (10)$$

Let  $p_2$  be the price at time  $t$  when fringe entry is completely exogenous ( $k_p = 0$ ) and  $p_1$  be the price at  $t$  when  $k_p$  is some positive number. Then, for given  $x$ ,

$$p_2 - p_1 = [a_1(\lambda_2 - \lambda_1) - k_p \mu(t)] / 2a_1 e^{-rt}. \quad (11)$$



If  $\lambda_2$  is greater than  $\lambda_1$ , the price set by the dominant firm will always be higher when fringe entry is exogenous.<sup>7</sup> If  $\lambda_1$  is greater than  $\lambda_2$ , for given  $k_p$ ,  $p_2$  is more likely to be greater than  $p_1$  when  $a_1$  is small (i.e., when demand is not sensitive to price) or when both  $\lambda_1$  and  $\lambda_2$  are small (i.e., if the stock of reserves is very large, for example).

In general we can say that, with linear demand and a fully reproducible product ( $\lambda_i = 0$ ,  $i = 1,2$ ) the price set by the dominant firm for given  $x$  will always be higher when fringe entry is exogenous. When the product is an exhaustible resource, the price set when entry is exogenous is more likely to be the higher price if demand is not very sensitive to price.<sup>8</sup>

Note that, in the case where entry is unresponsive to price and the price set by the dominant firm is higher than it would otherwise be, the dominant firm's market share is smaller for given  $x$  because industry output is restricted. Conversely, the competitive fringe has a larger share of the market. Therefore, the market will appear to be more competitive when the dominant firm has greater power to set price above marginal cost.

The competitive fringe acts as a restraining influence on the dominant firm because the dominant firm knows that a high price will induce entry. It is less obvious that, even in the case where entry is completely exogenous, the fringe provides a restraint. To see this, we calculate  $\partial p/\partial x$ .

$$\partial p/\partial x = - (\partial^2 H/\partial p \partial x)/(\partial^2 H/\partial p^2) = e^{-rt}/(\partial^2 H/\partial p^2). \quad (12)$$

This partial derivative is always negative, even when  $k_p = 0$ . In addition, as fringe output  $x$  approaches industry output  $f(p)$ , price approaches the competitive price. This is obvious from equation 7 with  $k_p = 0$ .

### III: ENTRY LAGS

In section II, fringe entry was assumed to be a function of the current price set by the dominant firm. In the metals industries, however, there are often very long lags between the decision to enter and actual entry. When a company decides to enter a mining industry, if it does not purchase an existing mining firm, it must explore for a deposit, define the deposit (i.e., determine its grade, tonnage, etc.), execute an engineering feasibility study, procure project financing, and finally construct the facility and any needed infrastructure. The total time required can be fifteen years or more. It therefore seems realistic to extend the model of section II to include entry lags.

Suppose that entry is not just a function of current price but is determined by a weighted average of current and all past prices set by the dominant firm (i.e., by a distributed lag on price).<sup>9</sup> Let  $w(j)$  be the probability density function that determines these weights, where

$$\int_0^{\infty} w(j) dj = 1 \quad (13)$$

and  $w(j) \geq 0, \quad j \in [0, \infty).$  (14)

We will assume that  $w$  is continuously differentiable. The weighted average of current and past prices  $\bar{p}(t)$  is then given by

$$\bar{p}(t) = \int_{-\infty}^t w(t - \tau) p(\tau) d\tau \quad (15)$$

and  $\dot{\bar{p}}(t) = \int_{-\infty}^t \dot{w}(t - \tau) p(\tau) d\tau + w(0)p(t).$  (16)

Now the problem is to maximize (1) subject to (2), (3),

$$\dot{x}(t) = k(\bar{p}, X, t), \quad (4')$$

and (15). The Hamiltonian for this problem is

$$H = e^{-rt}(p(t) - c)(f(p) - x(t)) - \lambda(f(p) - x(t)) + \mu(t)k(\bar{p}, X, t) \\ + r(t)x(t) + v(t)\left[\int_{-\infty}^t \dot{w}(t - \tau)p(\tau)d\tau + w(0)p(t)\right], \quad (17)$$

where  $v(t)$  is the costate variable corresponding to  $\bar{p}$ . First-order condition 6 becomes

$$\partial H / \partial p = e^{-rt}[(p(t) - c)f'(p) + f(p) - x(t)] - \lambda f'(p) \\ + v(t)w(0) = 0. \quad (6')$$

$w(0)$  is the intercept of the density function with the vertical axis. To see how price varies with changes in this intercept, we compute  $\partial p / \partial w(0)$ .

$$\partial p / \partial w(0) = - (\partial^2 H / \partial p \partial w(0)) / (\partial^2 H / \partial p^2) = - v(t) / (\partial^2 H / \partial p^2). \quad (18)$$

$v(t)$  is the discounted value to the dominant firm of a unit increase in  $\bar{p}(t)$ . Because  $p$  was chosen optimally,  $v(t)$  will be negative. Therefore,  $\partial p / \partial w(0)$  is negative. Equation 18 states that, all else being equal, the price set by the dominant firm will be higher if the response of the fringe is delayed (because, with the limiting case  $\bar{p}(t) = p(t)$ ,  $w(0)$  is infinite).

To make the analysis more concrete, we consider a particular density function, the exponential. In this case,

$$w(j) = \rho e^{-\rho j} \quad \text{and} \quad w(0) = \rho. \quad (19)$$

With an exponential density function, the intercept  $w(0)$  determines the entire function. In this case, as the response becomes more delayed (as the decay rate  $\rho$  becomes smaller) the intercept becomes smaller and the price set by the dominant

firm increases.<sup>10</sup>

In general, we can say that, all else being equal, entry lags give rise to higher prices. As before, for given  $x$ , a higher price is associated with a smaller market share for the dominant firm so that it looks as if the market is more competitive when the dominant firm's market power increases.<sup>11</sup>

#### IV: THE MOLYBDENUM MARKET

Among the natural-resource markets that appear to correspond to the model developed in sections II and III, the molybdenum market stands out. Molybdenum is a whitish-grey metallic element valued for its hardness, resistance to corrosion, and wear at high temperatures. It is used primarily as an alloying agent in stainless and special alloy steels.

Molybdenum is extracted from both open-pit and underground mines. Approximately two-thirds of U.S. output comes from primary molybdenum mines and the remaining one-third comes from copper mines where molybdenum is a byproduct. The United States produces approximately sixty percent of the world's molybdenum output and Canada and Chile supply most of the remainder. The U.S. is not only self-sufficient in molybdenum, it exports more than half of what it produces.

Molybdenum is a relatively new metal from the point of view of commercial usage. World War I generated the first commercial utilization, and peacetime applications followed in the 1920's. By 1930, world output totaled 4.2 million pounds per year with one mine, the Climax mine in Colorado, accounting for nearly three-quarters of world output. Byproduct recovery began in 1936 at the Bingham open-pit copper mine in Utah. Since the 1930's, world production has increased more than fiftyfold to over 200 million pounds per year.

The molybdenum market has always been dominated by one firm, Climax

Molybdenum Company, a division of AMAX, Inc. AMAX is a diversified natural-resource company engaged in a wide range of mining and related activities. In 1979, AMAX reported assets of more than \$4.0 billion, making it the 115th largest firm in the Fortune 500 in sales and 55th in assets. Molybdenum accounted for 24 percent of AMAX's 1979 sales and contributed 38 percent to its operating profits (AMAX, 1979). In 1978, AMAX's two primary molybdenum mines, Climax and Henderson, produced 65 percent of total U.S. output. The remaining 35 percent was produced by one primary molybdenum company, Molycorp, with five percent of the market, and ten copper companies, including Duval, Kennecott, Newmont, Cyprus, Anaconda, and Asarco.

There are substantial barriers to entering the primary molybdenum industry. Among these are economies of scale and large absolute capital costs. However, by far the most important entry barrier is the possession of a low-cost ore body of sufficient size to sustain the operation of a concentrator for several decades. AMAX's control of molybdenum reserves is even greater than its share of current production.

Byproduct molybdenum is produced from porphyry copper ores in the Southwestern United States. In most of these deposits, copper is the dominant-value commodity.<sup>12</sup> Other byproducts include silver and gold. The percentage of each metal contained in the ore varies geographically within a deposit. However, mining proceeds according to a long-range plan so that, at any given time, production of each metal is in approximately fixed proportions.<sup>13</sup> This is not to say that molybdenum price has no effect on byproduct molybdenum supply ( an issue that will be investigated econometrically in the next section). What it means is that variables such as copper price that are determined outside the molybdenum market may be the principal determinants of byproduct molybdenum production.<sup>14</sup>

## V: EMPIRICAL RESULTS

In this section, the response of molybdenum byproduct production to own and copper price is investigated. The first empirical estimates are based on equation 4 and correspond to the model developed in section II. Thus far, the functional form of the response function  $k$  has remain unspecified. However, for empirical purposes we must adopt a specification. In the absence of any theoretical justification for choosing a particular functional form, the simplest response function, the linear function, was chosen.<sup>15</sup>

Because the copper industry is relatively concentrated,<sup>16</sup> copper producers may have market power in their own market even though they are price takers in the molybdenum market. Therefore, with cyclical changes in demand, producers may vary either price or output. To correct for short-run fluctuations in output caused by demand variations, the index of industrial production was included as an independent variable in the estimated equations.  $X$ , cumulative production by the fringe, was not used as an independent variable because the time period is too short for significant depletion effects to be felt.

The econometric estimates presented below are based on the equation

$$\Delta Q = a_0 + a_1 P_j + a_2 \Delta IP + a_3 \Delta DS_c + \epsilon \quad (20)$$

where  $\Delta$  denotes the annual change in the variable it precedes,  
 $Q$  is byproduct molybdenum production,  
 $P_j$  is real metal price, where  $j = m$  for molybdenum or  $c$  for copper,<sup>17</sup>  
 $IP$  is the index of industrial production,  
 $DS_c$  is a dummy variable that corrects for major strikes in the copper industry,  
and  $\epsilon$  is a random error term that allows for the effects of omitted variables.

In addition to the variables listed in equation 20, a dummy variable was added to the equations for byproduct production that include copper price. D74, a variable equal to one in 1974 and zero elsewhere, corrects for the "commodity boom" of 1974 when the price of copper rose to unprecedented high levels.

Annual time-series data for the period 1954-1978 were used to estimate the equations reported below. These data are described in the data appendix. The equations were estimated by ordinary least squares with the Cochrane-Orcutt correction for first-order serial correlation of the error term.

Empirical estimates of equation 20 are shown in table I. In this table, t-statistics are given in parentheses under the corresponding estimated coefficients. Equations I:1-3 are estimates of the byproduct supply-response function with immediate entry. The first of these was estimated with copper price, the second with molybdenum price, and the last with both prices. In all three equations, changes in supply are positively correlated with industrial production and negatively correlated with industry-wide strikes. Changes in byproduct supply are positively and significantly correlated with copper price, but the coefficient of molybdenum price is not significantly different from zero.

We can conclude that the price of copper is more important in determining molybdenum byproduct production than is the price of molybdenum. However, we cannot conclude that molybdenum price has no effect on byproduct supply. There may be lags between high molybdenum prices and byproduct capacity expansions as discussed in section III. For this reason, estimates of the fringe response function were made based on equation 4' of section III. Equation 20 was modified to allow for dynamic adjustment in the following fashion.

Suppose that the desired rate of change of output,  $\Delta Q^*$ , is a function of price, P, and exogenous variables, Y,

TABLE I: MOLYBDENUM BYPRODUCT SUPPLY-RESPONSE FUNCTIONS

Equation Number	$a_0$	$P_m$	$P_c$	$\Delta IP$	$\Delta DS_c$	D74	$\Delta Q_{-1}$	$R^2$	F	DW
I:1	-8.9		24.3 (3.1)	.28 (3.1)	-5.6 (-2.3)	-12.4 (-4.8)		.72	13	2.2
I:2	-3.0	1.8 (.60)		.35 (2.5)	-8.2 (-2.3)			.40	4.4	2.0
I:3	-11.7	1.8 (.75)	25.0 (3.1)	.25 (2.6)	-5.6 (-2.3)	-12.2 (-4.6)		.73	10	2.2
I:4	-6.3		17.8 (1.8)	.24 (2.2)	-6.5 (-2.4)	-11.9 (-4.4)	.16 (.84)	.74	10	2.2
I:5	-4.8	3.0 (.99)		.29 (1.9)	-9.2 (-2.4)		.25 (1.1)	.38	2.9	2.0
I:6	-9.3	2.2 (.97)	17.4 (1.8)	.20 (1.7)	-6.5 (-2.4)	-11.4 (-4.1)	.18 (.95)	.74	8	2.1



$$\Delta Q^*_t = \alpha_0 + \alpha_1 P_t + \beta Y_t, \quad (21)$$

but that only some fraction  $\delta$  of the desired change in  $\Delta Q$  can take place in any one year,

$$\Delta Q_t - \Delta Q_{t-1} = \delta(\Delta Q^*_t - \Delta Q_{t-1}) + u_t. \quad (22)$$

The error term  $u$  was added to correct for random variations in capacity adjustment. When we substitute (22) into (21), after algebraic simplification we get

$$\Delta Q_t = \alpha_0 \delta + \alpha_1 \delta P_t + \beta \delta Y_t + (1 - \delta) \Delta Q_{t-1} + u_t. \quad (23)$$

Equations 21-23 constitute a "partial adjustment" model for changes in investment rates, a model that is well known in the literature (see Johnston (1972), p. 300, for example).<sup>18</sup> The method used to estimate equation 23, which has a lagged dependent variable and may have a serially correlated error term, is discussed in appendix B.

Equations I:4-6 are the dynamic versions of equations I:1-3. In the three dynamic supply-response equations, the coefficient of copper price is again positive and significant and that of molybdenum price is again not significantly different from zero, implying that, even in the long run, molybdenum price is not the major determinant of byproduct production. With the dynamic equations, as with the static equations, the explanatory power (as measured by either  $R^2$  or the F-statistic) is greatly reduced when own price is used instead of copper price. Other forms of distributed lags were experimented with but no significant fringe response to molybdenum price could be detected empirically.

It seems unlikely, however, that in fact molybdenum price has no effect on byproduct production for two reasons. First, even when metals occur in ores in approximately fixed proportions, the cutoff grade for byproduct recovery (the

grade below which it is uneconomic to recover) will depend on molybdenum price, and second, when copper price is depressed, as it has been in the last few years, copper mines may remain open if byproduct credits (revenues from silver, gold, and molybdenum sales) are high. However, the effect of molybdenum price on byproduct production is too small to be detected by econometric methods, possibly because the response is very delayed and is spread out over a very long period of time.

The empirical evidence indicates that the molybdenum industry fits the model for a nonrenewable natural-resource market where byproduct supply is very insensitive to own price, even in the long run. It was noted in section II that, all else being equal, the lack of price responsiveness of the fringe increases the market power of the dominant firm. However, we could not say unambiguously that the price set by the dominant firm would always be higher under these conditions. It was shown, however, that the price chosen when entry is exogenous is more apt to be higher than when entry is induced if the slope of the demand curve is small (which is equivalent to highly inelastic demand in the observed price and output range).

To test the hypothesis of inelastic demand, several demand-curve specifications were experimented with. In every equation estimated, the slope of the demand curve was not significantly different from zero, both in the short and in the long run. Other econometric studies of the molybdenum industry have reached the same conclusion (see Charles River Assoc. 1974 , for example). Inelastic demand is to be expected because the cost of molybdenum is a small fraction of the price of final products and because there are no good substitutes for molybdenum.<sup>19</sup> Conditions in the molybdenum market therefore facilitate a considerable degree of pricing power on the part of the dominant firm.

## V: SUMMARY AND CONCLUSIONS

In this paper, an intertemporal model of dominant-firm profit maximizing behavior was developed for the case where fringe firms are byproduct producers. The idea that entry is determined at least partially by factors that are exogenous to the market was motivated by the observed frequency of byproduct production in the metals and chemicals industries, where supply appears to respond to the price of the dominant-value commodity. The market-power implications of exogenous entry were examined by varying the fringe's response function. An analysis of the model's comparative dynamics showed that, all else being equal, a decrease in the responsiveness of the fringe to the price set by the dominant firm leads to a higher price for the product and a smaller market share for the dominant firm. The market therefore looks more competitive when the dominant firm's pricing power increases.

The model was then extended to include lags in the fringe's response function. With lags, entry is determined by a weighted average of current and all past prices set by the dominant firm. An analysis of the extended model's comparative dynamics showed that, all else being equal, a delayed response also increases the dominant firm's market power and reduces its market share. Market power, therefore, depends both on the price sensitivity of the response function and on the distribution of the response over time.

The theoretical model was quantified for the molybdenum market, a natural-resource market characterized by one dominant firm, AMAX, and a fringe of firms with much smaller market shares, most of which produce molybdenum as a byproduct of copper mining. The econometric evidence for the molybdenum industry indicates that production by the fringe is very insensitive to own price, even in the long

run. This industry therefore appears to fit the model developed for a nonrenewable natural-resource market with fringe entry and exit determined to a large extent by factors outside the market. Any own-price response of the fringe is likely to be very delayed and has been too small to be detected by econometric methods. In addition, industry demand was seen to be very inelastic, even in the long run. It was argued that the combination of these factors should lead to a considerable degree of pricing power for the dominant firm.

The theoretical results reported here have important policy implications in that they provide one more reason why it is insufficient to look at market shares to determine market power in an industry, a practice that is not justified by economic theory for many reasons. With byproduct production, primary producers have may have greater market power than their market shares indicate, especially if entry barriers are high in the primary industry.<sup>20</sup> It is therefore important to determine what price each firm is responding to in addition to examining concentration and other standard indicators of monopoly power.

FOOTNOTES:

1 The word byproduct is used when there are two or more products, one of which is the dominant-value product and the others are byproducts. The word coproduct is used when products are of fairly equal value.

2 Examples of byproducts include helium, a byproduct of natural gas production (as described in Epple and Lave, 1980), and nitrogen, produced when separating air for its oxygen content. These products are often vented, a good indicator that production does not respond to own price. Examples from the mining industries are extremely prevalent and include silver and molybdenum, byproducts of copper mining, and tungsten, recovered from primary molybdenum ores. Sulfur is produced from mines and as a byproduct of natural gas production.

3 The fringe also includes one primary producer, Molycorp (see the discussion in section IV).

4 The word entry is used here to mean the opening of a new "mine". The "mine" may be owned by an existing firm or by a new firm.

5 If we assume that cost is a function of the remaining stock of reserves,  $R$ , we can replace  $\lambda$  in equations 7, 10 and 11 by  $\lambda - \gamma(t)$ , where  $\gamma(t)$  is the costate variable corresponding to the state variable  $R$ .

6 Demand is assumed to be stationary. However, this assumption is not essential to the theoretical results (i.e., in the first-order conditions we can replace  $f'(p)$  by  $f_p$ ).

7 If demand is linear, there is a finite zero-demand or choke point. If the dominant firm is still producing when this point is reached, we know that  $\lambda_2 < \lambda_1$ . For if  $\lambda_2 > \lambda_1$ ,  $p_2$  is always greater than  $p_1$  and more primary metal would be consumed under  $p_1$  than under  $p_2$ , contradicting the assumption that  $\bar{R}$

is fixed. However, if fringe firms are high-cost producers, it is quite possible that only the fringe will be producing when the choke point is reached. In this case,  $\lambda_2$  can be greater than  $\lambda_1$  and  $p_2$  can be everywhere higher than  $p_1$ .

8 If  $a_1$  is very small, demand is highly inelastic in the observed price and output range. With inelastic demand, the market is less able to accommodate entry and therefore the dominant firm has more incentive to set a low price when price induces entry but will set a higher price otherwise.

9 The model developed here is consistent with a model where fringe firms form price expectations in an adaptive fashion or with a model where capital stocks cannot adjust instantaneously (as well as with a combination of the two).

10 An exponential density function, with largest weight at  $j = 0$ , may not be the best distributed lag to use in modeling investment behavior. However, we can consider convolutions of exponentials, or Erlang density functions. For given type (number of convolutions) the Erlang is also entirely determined by the intercept,  $w(0)$ , and it has the conventional inverted V shape frequently associated with investment lags. In fact, the Erlang is closely related to the discrete Pascal distribution often used in empirical studies of the lag between investment and its underlying determinants.

11 As in section II, when the product is an exhaustible resource, we know the partial effect of a delayed response unambiguously but not the total effect. The analysis of the case of linear demand applies here as well.

12 In one mine, Duval's Sierrita property, copper and molybdenum are considered to be coproducts.

13 Commodities Research Unit (1981) states that

The moly grade at Bingham Canyon rises in the deeper areas that have yet to be exploited. The result is that, although

Kennecott can look forward to higher moly output as a result of improving grades in the years ahead, it does not have flexibility to pick out higher grade ore now in order to offset low copper prices.

14 A more detailed description of the molybdenum industry can be found in U.S. Bureau of Mines (1979).

15 Other functional forms were experimented with but did not change the substance of the results.

16 The 1972 four-firm concentration ratio for copper production in the U.S. was 72 and the eight-firm ratio was 98 (U.S. Bureau of the Census, 1975).

17 Prices were deflated by the Wholesale Price Index (1967 = 1) and are thus in 1967 constant dollars.

18 Equation 23 contains a geometric distributed lag on price. The geometric distribution is the discrete analog of the exponential density function discussed in section III.

19 For a discussion of substitution, see U.S. Bureau of Mines (1979).

20 Barriers are apt to be higher in the primary industry than for the fringe. For example, if there are substantial economies of scale in mining, a primary molybdenum mine must be very large and must supply a sizeable fraction of molybdenum consumption. However, a large copper mine can produce a small amount of molybdenum (which explains why byproduct producers are usually in the fringe).

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## APPENDIX A: DATA SOURCES

The data used in the econometric estimates reported in section V consist of annual time series for the 1954-1978 period. The following data sources were used:

Molybdenum price: U.S. Bureau of Mines, "Mineral Commodity Profiles, Molybdenum," 1979. This price is a weighted average of concentrate prices in dollars per pound.

Copper price: U.S. Bureau of Mines, Minerals Yearbook, various years. The U.S. producer price of copper in dollars per pound.<sup>1</sup>

Molybdenum byproduct production: U.S. Bureau of Mines, Minerals Yearbook, various years, in  $10^6$  pounds.

Industrial production: An index (1967 = 100) computed by the Board of Governors of the Federal Reserve System.

Wholesale price index: An index (1967 = 1) computed by the U.S. Bureau of Labor Statistics.

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1 The U.S. producer price was chosen rather than some exchange price because the producer price is a better indicator of long-run conditions in the copper industry.

APPENDIX B: ESTIMATION METHOD FOR EQUATIONS WITH A LAGGED  
DEPENDENT VARIABLE AND SERIALY CORRELATED ERRORS

Many of the equations reported in section V contain a lagged dependent variable. If, in addition, the error terms for these equations are serially correlated, the estimated coefficients will be biased.<sup>1</sup> However, with a lagged dependent variable, the Durbin-Watson statistic is biased against detecting such serial correlation. For these reasons, the following estimation technique was used.

First, the equations were estimated by ordinary least squares (OLS). Next, the equations were estimated using instruments for the lagged dependent variables and the Cochrane-Orcutt technique to correct for first-order serial correlation of the error term.<sup>2</sup> The estimates,  $\hat{\rho}$ , of the autocorrelation coefficient of the error term obtained in this manner are unbiased. If  $|\hat{\rho}|$  was less than one, the OLS estimates were reported and if  $|\hat{\rho}|$  was greater than or equal to one, the instrumental-variable, Cochrane-Orcutt estimates were reported.

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1 With a "partial adjustment" model, unlike an "adaptive expectations" model, serial correlation is not automatically introduced.

2 The instruments were current and lagged values of a mining cost index plus once and twice lagged values of the exogenous variables in the equation to be estimated.