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EXPECTATIONS AND STABILITY WITH GROSS COMPLEMENTS

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Arrow and Nerlove [1] have found that when all commodities, both present and future, in a continuous time general equilibrium model of a competitive economy, are gross substitutes, then stability with "static" expectations is equivalent to stability with "adaptive" expectations. It is found here that we may extend the Arrow-Nerlove result that the stability behavior of the system is unaffected by the selection of the more sophisticated adaptive expectations over static expectations, provided the substitute-complement relationship of commodities, both presented and future, fits into the Morishima matrix pattern. The Morishima pattern [10], allows gross complements provided that substitutes, of substitutes are substitutes, complements of complements are substitutes, and substitutes of complements are complements.

In general the excess demand for a commodity depends not only on current prices but also on expected prices of commodities in the future. Expected future prices effect the excess demands for the commodities currently traded. Hicks, [7] and [8, chapter 6], was one of the first authors to investigate such models in detail and this paper's analysis corresponds to the "temporary equilibrium" analysis developed by Hicks.

I follow Arrow-Nerlove [2] and consider a two period model: the present and the future. The specific model I employed is due to Arrow and Hahn [1, pp. 33-40], who have established conditions under which an equilibrium exists.

In order to allow for the possibility of obtaining more of a good now, by promising to deliver some of a good in the future, a single futures market is assumed to exist. Thus there are (n+1) goods: n current goods and one future good. Let the (n+1)-st good be contracts for sale and delivery of the n-th good in the future and let the (n+1)-st good be the numeraire. Letting all prices be relative prices expressed in terms of the numeraire, write the excess demand for the i-th good in the competitive economy as:

$$(1) \quad x_i = x_i[P_1,\ldots,P_n,P_1^*,\ldots,P_n^*] \qquad i = 1,\ldots,n$$

where P_i = the current price of the i-th good

P_i^* = the expected future price of the i-th good. 1/

Due to Walras' Law we need only examine the excess demands of the nonnumeraire commodities.

Expectations are said to be adaptively formed if the expected price increases (decreases) when the actual price is greater (less) than the expected price i.e.

$$(2) \quad \dot{P}_i^* = r_i[P_i - P_i^*] \qquad i = 1,\ldots,n$$

where P_i^* is the time derivative of P_i^* and the r_i are positive constants.

Suppose the dynamic behavior of prices can be described by

$$(3) \quad \dot{P}_i = K_i x_i \qquad i = 1,\ldots,n$$

where the K_i are positive constants representing "speeds of adjustment." The dynamic behavior of the markets is described by the 2n equations (2) and (3) together. An equilibrium vector of current and expected prices is one which yields no excess demand and expectations are fulfilled:

$$(4) \quad x_i = 0$$

$$(5) \quad P_i^* = P_i \qquad i = 1,..,n.$$

Let $P = [P_i]$, $P^* = [P_i^*]$ and $\bar{P}$ = the first n components of an equilibrium vector, which from (5) must equal the second n components.

Let R be an nxn diagonal matrix with the r_i along the diagonal and

$$A = \left[a_{ij}\right] = \left[K_i \quad \frac{\partial x_i}{\partial P_j}\right]$$

$$i, j = 1,\ldots,n$$

and

$$B = \left[b_{ij}\right] = \left[K_i \quad \frac{\partial x_i}{\partial P^*_j}\right] \qquad i,j = 1,\ldots,n$$

where both are evaluated at equilibrium.

Then the Jacobian matrix of (2)-(3) evaluated at the equilibrium is

$$(6) \quad C = \begin{bmatrix} A & B \\ R & -R \end{bmatrix} = [c_{ij}] \qquad i,j = 1,\ldots,2n$$

and the dynamic behavior of (2)-(3) may be approximated in the neighborhood of equilibrium by

$$(7) \quad \begin{bmatrix} \dot{P} \\ \dot{P}^* \end{bmatrix} = C \begin{bmatrix} P - \bar{P} \\ P^* - \bar{P} \end{bmatrix}$$

When expectations are "static," $P = P^*$ and (7) reduces to

$$(8) \quad \dot{P} = [A+B] \quad [P-\bar{P}]$$

Consequently the question of whether adaptive expectations are stabilizing or destabilizing relative to static expectations reduces to the question of when the stability of (7) implies that (8) is stable and when the stability of (8) implies the stability of (7).

It has been shown by Arrow and Nerlove that when all commodities, both present and future, are gross substitutes for one another, i.e. $a_{ij} \geqq 0$ $i \neq j$, $b_{ij} \geqq 0$ for all i, j and $a_{ii} < 0$ for all i, that (7) is stable if and only if (8) is stable.

Clearly it is desirable to relax the strict gross substitutes assumption and permit some commodities to be gross complements. For that purpose the following definitions are introduced:

Definition 1. Morishima Matrix. An nxn matrix M is said to be a Morishima matrix (denoted MR matrix) if there exists a permutation matrix P such that:

$$M^* = P'MP = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$$

where $M_{11} \geqq 0$ is kxk, $M_{22} \geqq 0$ is (n-k)x(n-k), $M_{12} \leqq 0$ is kx(n-k) and $M_{21} \leqq 0$ is (n-k)xk with $1 \leqq k \leqq n$.

Define $I^* = \begin{bmatrix} I_k & 0 \\ 0 & -I_{n-k} \end{bmatrix}$ where I_k and $I_{(n-k)}$ are k and (n-k) order identity matrices respectively. Then $I^*M^*I^* \geqq 0$. Note that we employ the notation for vectors that $x > 0$ if $x_i > 0$ for all i, $x \geqq 0$ if $x_i \geqq 0$ for all i, and $x \geq 0$ if $x \geqq 0$ but $x \neq 0$ and similarly for matrices.

Definition 2. Morishima Matrix. An nxn matrix $M = [m_{ij}]$ is a MR matrix if there exists two subsets I and J of N = (1,2,...,n) such that:

(i) $m_{ij} \geqq 0$ if either $i \varepsilon I$ $j \varepsilon I$ or $i \varepsilon J$ $j \varepsilon J$

and (ii) $m_{ij} \leqq 0$ if either $i \varepsilon I$ $j \varepsilon J$ or $i \varepsilon J$ $j \varepsilon I$ where $I \cup J = N$, $I \cap J = \emptyset$

The equivalence of definitions 1 and 2 is easily established by letting the first k columns of the permutation matrix in definition 1 be the unit vectors with indices in I.

Since a MR matrix is similar to a nonnegative matrix, it has identical characteristic roots. If we denote by $\lambda(M)$ the largest real root of M, we have from the Frobenius Theorem that if M is a MR matrix:

(i) $\lambda(M) \geqq |\lambda| \geqq 0$ for all λ a characteristic root of M and

(ii) $\lambda(M) \geq m_{ii}$ for all i.

We are now prepared to assume that C of (6) satisfies:

(9) $C = M - sI$

where $M = [\ m_{ij}\ ]$ is a MR matrix and $s > m_{ii}$ for all i. In addition we require that the effect of a change in the current price of the i-th commodity on the excess demand for the i-th commodity exceed the effect of a change in the expected future price of the i-th commodity on the excess demand for the i-th commodity, when evaluated at equilibrium, i.e.

(10) $|a_{ii}| > |b_{ii}| \qquad i = 1,\ldots,n.$

Equation (9) relaxes the gross substitute assumption. Developed by Morishima, [10] it permits gross complements to enter the system provided that substitutes of substitutes are substitutes, complements of complements are substitutes and substitutes of complements are complements. 2/ The gross substitute case is a special case of (9) when $M \geq 0$.

Theorem: Suppose that the weak gross substitute, weak gross complement relationship of goods, both present and future, satisfies the Morishima matrix pattern, i.e. equations (9) and (10) are satisfied. The linearized version of the dynamic adjustment process of current and expected future prices is stable with adaptive expectations if and only if it is stable with static expectations, i.e. the system (7) is stable if and only if the system (8) is stable.

Proof: First suppose (7) is stable. λ is a characteristic root of M if and only if $\lambda = s+\mu$ where μ is a characteristic root of C. Denote by $R(\lambda)$ the real part of the root. Then $R(\lambda) = s+R(\mu)$ and $\max_i R(\lambda_i) = s + \max_i R(\mu_i)$. Since $R(\mu_i) < 0$ for all i by virtue of the stability of (7), we have $s - \lambda(M) = -\max_i R(\mu_i) > 0$. We now invoke Lemma 1 of the Appendix; $s > \lambda(M)$ implies $-C$ has a positive dominant diagonal (p.d.d.). 3/

There exists a permutation matrix P and a corresponding matrix I* defined in definition 1 such that:

$$(11)\quad M^{**} = I^*P'MPI^* \geqq 0 ;$$

then by Lemma 2 of the Appendix

$$(12)\quad I^*P'[-C]PI^* = sI-M^{**} \quad \text{has a p.d.d.}$$

From (6) and (9) we have that:

$$(13)\quad m_{ij} = c_{ij} = r_i > 0 \qquad i = n+j \qquad j=1,\ldots,n.$$

It follows that:

$$(14)\quad c_{ij} \geqq 0 \qquad j = n+i \qquad i = 1,\ldots,n$$

and any I and J satisfying definition 2, must satisfy

$$(15)\quad i \in I \Rightarrow n+i \in I \qquad 1 \leqq i \leqq n.$$

From (15) we have:

$$(16)\quad \text{sgn } a_{ij} = \text{sgn } b_{ij} \qquad i \neq j.$$ 4/

Suppose P of (11) is

$$(17)\quad P = [e_1,\ldots,e_k,e_{n+1},\ldots,e_{n+k},e_{k+1},\ldots,e_n,e_{n+k+1},\ldots,e_{2n}]$$

where e_i is a 2n component vector with zeroes everywhere except for a one in the i-th component. The general case follows in a straightforward manner from this case.

Partition A, B, and R into four submatrices:

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \quad B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \quad R = \begin{bmatrix} R_{11} & 0 \\ 0 & R_{22} \end{bmatrix},$$ where the upper left hand corner matrix is kxk in each case. Then we have:

$$(18) \qquad -I^*P'CPI^* = \begin{bmatrix} -A_{11} & -B_{11} & A_{12} & B_{12} \\ -R_{11} & R_{11} & 0 & 0 \\ A_{21} & B_{21} & -A_{22} & -B_{22} \\ 0 & 0 & -R_{22} & R_{22} \end{bmatrix}$$

Note that the matrix in (18) has positive diagonal and nonpositive off diagonal elements. Since it possesses a p.d.d., from (12), there exists a positive 2n component vector $x = (x_i)$, (which we partition into $x = \begin{bmatrix} x^1 \\ x^2 \\ x^3 \\ x^4 \end{bmatrix}$ where x^1 and x^2 are kxl and x^3 and x^4 are (n-k) component vectors), such that $-I^*P'CPI^*x > 0$.

Then we have:

$$(19) \quad \text{(a)} \quad -A_{11}\, x^1 - B_{11}\, x^2 + A_{12}\, x^3 + B_{12}\, x^4 > 0$$
$$\text{(b)} \quad A_{21}\, x^1 + B_{21}\, x^2 - A_{22}\, x^3 - B_{22}\, x^4 > 0$$

and

$$(20) \quad \text{(a)} \quad -R_{11}\, x^1 + R_{11}\, x^2 > 0$$
$$\text{(b)} \quad -R_{22}\, x^3 + R_{22}\, x^4 > 0$$

20 (a) and (b) imply $x^2 = x^1 + \varepsilon^1$ and $x^4 = x^3 + \varepsilon^3$ with $\varepsilon^1 > 0$ and $\varepsilon^3 > 0$.

Substituting into (19) yields:

$$(21) \quad \text{(a)} \quad -A_{11}\, x^1 - B_{11}\, x^1 - B_{11}\, \varepsilon^1 + A_{12}\, x^3 + B_{12}\, x^3 + B_{12}\, \varepsilon^3 > 0$$
$$\text{(b)} \quad A_{21}\, x^1 + B_{21}\, x^1 + B_{21}\, \varepsilon^1 - A_{22}\, x^3 - B_{22}\, x^3 - B_{22}\, \varepsilon^3 > 0.$$

Since the terms involving ε^1 and ε^3 are nonpositive we have:

$$(22) \quad \text{(a)} \quad -(A_{11} + B_{11})x^1 + (A_{12} + B_{12})x^3 > 0$$
$$\text{(b)} \quad (A_{21} + B_{21})x^1 - (A_{22} + B_{22})x^3 > 0.$$

Define $y^1 = x^1$ and $y^2 = x^3$.

It follows from (22) after, multiplication, rearrangement and recognition of the sign pattern of the partitioned matrices that:

$$(23)\ \text{(a)}\ -y_i(a_{ii}+b_{ii}) > \sum_{\substack{j=1\\ j\neq 1}}^{k} [a_{ij}+b_{ij}]y_j - \sum_{j=k+1}^{n} [a_{ij}+b_{ij}]y_j = \sum_{\substack{j=1\\ j\neq i}}^{n} |a_{ij}+b_{ij}|\ y_j \qquad i=1,..,k$$

$$\text{(b)}\ -y_i(a_{ii}+b_{ii}) > -\sum_{j=1}^{k} [a_{ij}+b_{ij}]y_j + \sum_{\substack{j=k+1\\ j\neq i}}^{n} [a_{ij}+b_{ij}]y_j = \sum_{\substack{j=1\\ j\neq i}}^{n} |a_{ij}+b_{ij}|\ y_j \qquad i=k+1,\dots,n$$

Since from (9) and (10) we have that $a_{ii} + b_{ii} < 0$, (23)(a) and (b) imply that the matrix [A+B] has a negative dominant diagonal. Consequently (8) is stable.

Now suppose (8) is stable. Define $\bar{C} = [\bar{c}_{ij}] = A+B$. From (9) it follows that there must exist a MR matrix $\bar{M} = [\bar{m}_{ij}]$ and a $\bar{s} > \bar{m}_{ii}$ for all i such that

$$(24)\quad \bar{C} = \bar{M} - \bar{s}I.$$

(24) must hold since from (9) and definition 2 there exists sets I and J such that:

$$(25)\quad \begin{array}{lllll} c_{ij} \geqq 0 & i\varepsilon I\ \ j\varepsilon I & \text{or} & i\varepsilon J\ \ j\varepsilon J & i\neq j \\ c_{ij} \leqq 0 & i\varepsilon I\ \ j\varepsilon J & \text{or} & i\varepsilon J\ \ j\varepsilon I. & \end{array}$$

Define $\bar{N} = (1, 2,\dots, n)$, $R = I \cap \bar{N}$ and $S = J \cap \bar{N}$.

Since $\bar{c}_{ij} = a_{ij} + b_{ij}$ and sgn a_{ij} = sgn b_{ij} $i \neq j$ by (16), then sgn $\bar{c}_{ij}$ = sgn a_{ij} $i \neq j$. Then (26) holds:

$$(26)\quad \begin{array}{lllll} \bar{c}_{ij} \geqq 0 & i\varepsilon R\ \ j\varepsilon R & \text{or} & i\varepsilon S\ \ j\varepsilon S & i\neq j \\ \bar{c}_{ij} \leqq 0 & i\varepsilon R\ \ j\varepsilon S & \text{or} & i\varepsilon S\ \ j\varepsilon R. & \end{array}$$

Define $\bar{s} = \max_i [-\bar{c}_{ii}]$ and $\bar{m}_{ii} = \bar{s} + \bar{c}_{ii}$. Since we know that $\bar{c}_{ii} < 0$ for all i, we have that $\bar{m}_{ii} \geqq 0$ and $\bar{s} > \bar{m}_{ii}$ for all i. This fact combined with (26) means (24) is satisfied.

Since the real parts of the characteristic roots of $\bar{C}$ are negative, the real parts of the characteristic roots of $-\bar{C}$ are positive. Since $\bar{M}$ is a MR matrix, repeating the argument used in the first paragraph of this theorem's proof establishes that $\bar{s} > \lambda(\bar{M})$. By invoking Lemma 1 of the Appendix we derive that $-\bar{C}$ has a p.d.d. Moreover, by Lemma 2 of the Appendix, there exists a permutation matrix P and an I* such that

(27) $-I^*P'\bar{C}PI^* = \bar{s}I - I^*P'\bar{M}PI^*$ has a p.d.d., where

(28) $I^*P'\bar{M}PI^* \geqq 0$.

Since the matrix in (27) has nonpositive off diagonal elements, (27) implies that there exists an n component vector $x = (x_i)$ such that

(29) $-I^*P'\bar{C}PI^*x > 0$.

First suppose P = I in (27).

Taking off diagonal terms and the b_{ii} to the right hand side and recognizing the sign pattern in (29) we get

$$(30) \quad x_i \, |a_{ii}| > \sum_{\substack{j=1 \\ j\neq 1}}^{n} x_j \, |a_{ij}| + \sum_{j=1}^{n} x_j \, |b_{ij}| \qquad i = 1,\ldots,n$$

In addition we have that:

$$(31) \quad x_i \, |-r_i| = x_i \, |r_i| \qquad i = 1,\ldots,n.$$

Define the 2n component positive vector $y' = (x', x')$. (30) and (31) combined imply that C satisfies

$$(32) \quad y_i \, |c_{ii}| \geqq \sum_{\substack{j=1 \\ j\neq i}}^{2n} y_j \, |c_{ij}| \qquad i = 1,\ldots,2n$$

with strict inequality for $i = 1,\ldots,n$.

Define $D = [d_{ij}]$ where $d_{ii} = |c_{ii}|$ and $d_{ij} = -|c_{ij}|$ $i\neq j$, $i, j=1,\ldots,2n$.

Then (32) implies

$$(33) \quad Dy \geq 0 \quad \text{with } y > 0$$

First suppose C is indecomposable: we have that D must be indecomposable;

Then from [11, Th. 7.4. (i)], (33) implies that:

$$(34)\quad \begin{vmatrix} d_{11} \cdots d_{1k} \\ \\ d_{k1} \cdots d_{kk} \end{vmatrix} > 0 \qquad k = 1, \ldots n \text{ holds.}$$

However (34) implies that C has a d.d. Since the diagonal of C is negative, (7) is stable. 5/

The theorem has established that stability in unaffected by the choice of adaptive expectations over the more "naive" static expectations hypothesis provided the substitute-complement relationship of commodities, both present and future, satisfy the Morishima pattern. The work of Sato [12] would suggest that an extension to the class of power positive matrices might be possible.

APPENDIX

Lemma 1. Let M be a MR matrix and define $A = sI-M$, $s > 0$. Then A has a positive dominant diagonal (denoted p.d.d.) if and only if $s > \lambda(M)$.

Proof: First suppose A has a p.d.d. Then from [7, Theorem 2] the real parts of the characteristic roots of A are positive. Using the method of proof exhibited in the first paragraph of the proof of this paper's theorem, it follows that $s > \lambda(M)$ when M is a MR matrix.

Now suppose $s > \lambda(M)$. Define $B = [b_{ij}]$ $b_{ii} = |s-m_{ii}|$ $b_{ij} \underset{i \neq j}{=} -|m_{ij}|$

Then

(a) A has a d.d. $\Leftrightarrow$ B satisfies

$$(b)\quad \begin{vmatrix} b_{11}, \cdots b_{1k} \\ \\ b_{k1} \cdots b_{kk} \end{vmatrix} > 0 \qquad k = 1, 2, \ldots, n$$

i.e. B satisfies H.-S. 6/

As $b_{ij} \leqq 0$ $i \neq j$, (b) $\Leftrightarrow$

(c) P'BP satisfies H.-S. 7/ where P is the permutation matrix which transforms M into M*, i.e. P'MP = M*.

Define $M^+ = [|m_{ij}|]$.

Since $s > \lambda(M)$ by hypothesis and $\lambda(M) \geqq m_{ii}$ from the Frobenius Theorem and $m_{ii} \geqq 0$ since M is a MR matrix we may rewrite $B = [sI-M^+]$.

Thus A has a d.d. $\Leftrightarrow$

(c) $P'BP = P'[sI-M^+]P = [sI-P'M^+P]$

$$= sI - \begin{bmatrix} M_{11}^+ & M_{12}^+ \\ M_{21}^+ & M_{22}^+ \end{bmatrix}$$ satisfies H.-S., where we define

$$\begin{bmatrix} M_{11}^+ & M_{12}^+ \\ M_{21}^+ & M_{22}^+ \end{bmatrix} = P'M^+P \geqq 0.$$

However $M^{**} = I^*P'MPI^* = \begin{bmatrix} M_{11} & -M_{12} \\ -M_{21} & M_{22} \end{bmatrix} = \begin{bmatrix} M_{11}^+ & M_{12}^+ \\ M_{21}^+ & M_{22}^+ \end{bmatrix}$.

Consequently we have (c) $\Leftrightarrow$

(d) $[sI-M^{**}]$ satisfies H.-S.

Since M^{**} is obtained by a similarity transformation from M we know that

(e) $s > \lambda(M) = \lambda(M^{**})$ obtains.

However (e) implies (d) is satisfied. 8/ $\therefore$ A has a d.d. and the diagonal must be positive since $s > \lambda(M) \geqq m_{ii}$.

Lemma 1 is an extention of Theorem 1 in [3]. Lemma 1 removes the indecomposability requirement assumed in [3].

<u>Lemma 2</u> Let M be a MR matrix and let S be an invertible matrix such that $S^{-1}MS \geqq 0$. Then $[sI-M]$ has a p.d.d if and only if $[sI-S^{-1}MS]$ has a p.d.d.

Proof: $\lambda(M) = \lambda[S^{-1}MS]$.

Footnotes

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1/ Unless otherwise indicated all variables are functions of the continuous variable time.

2/ Note that the Morishima pattern requires only that the complement-substitute pattern be satisfied "weakly," i.e. $a_{ij} = 0$ means that we may consider i and j either weak gross substitutes or weak gross complements.

3/ An nxn matrix is said to have a dominant diagonal (d.d.) if there exist $x_i > 0$ $i = 1,\ldots,n$ such that:

$$x_j |a_{jj}| > \sum_{i \neq j} x_i |a_{ij}| \qquad j = 1,\ldots,n$$

If in addition $a_{jj} > 0$ for all j, the matrix is said to have a positive dominant diagonal, (p.d.d.).

4/ If $a_{ij} \geqq 0$ and $b_{ij} \geqq 0$ we use the notation sgn a_{ij} = sgn b_{ij} even if one of the terms is zero and the other positive and similarly for nonpositive terms.

5/ If P of (27) becomes $P = [e_{i_1},\ldots,e_{i_n}]$, it is necessary to define the n component vector $z = (z_i)$ where $z_{i_j} = x_j$ and the x_j are those of equation (29). Then equation (30) holds with the z_i instead of the x_i and the proof follows.

If C is decomposable a permutation of the rows and columns will transform it into a block triangular matrix with square indecomposable blocks on the diagonal [5, p. 75]; a set of equations analogous to (33) will hold for each indecomposable matrix on the diagonal. The proof follows since the characteristic equation of C equals the product of the characteristic equations of the matrices along the diagonal.

6/ A matrix with nonpositive off diagonal elements which satisfies (b) is said to satisfy the Hawkins-Simon condition and it will be denoted as B satisfies H.-S. See [6] or [9, Th. 4'] for a proof of the equivalence of (a) and (b).

7/ See Nikaido [11, Th. 6.1].

8/ See Debreu and Hernstein [4, Th. IV].

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