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INTERTEMPORAL ALLOCATION OF EXHAUSTIBLE RESOURCES: A SIMPLE GEOMETRIC INTERPRETATION

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I. INTRODUCTION

In recent years interest in the topic of exhaustible resources has sparked the growth of a substantial body of economic analysis drawing on the intertemporal resource allocation problem set out in Hotelling's (1931) classic article. The purpose of this note is to provide a geometric interpretation of the Hotelling problem which is simple and informative. It was suggested by the analogy between the intertemporal allocation problem facing a profit maximizing monopoly owner of an exhaustible resource and the more familiar monopoly price discrimination problem. The analogy is that in the intertemporal problem the monopolist discriminates between markets segmented by time while in the more familiar problem the monopolist discriminates between contemporaneous markets segmented by customer characteristics. From a formal point of view, however, it does not matter how markets are segmented. If in the intertemporal market segmentation problem one begins with demand and cost schedules that have been appropriately discounted (so that dollar valued magnitudes are comparable between "time-segmented" markets), the similarity between the two problems becomes apparent. This permits a novel application of familiar geometry.

Section II describes the geometric solution to the intertemporal allocation problem for a profit maximizing monopolist owner of an exhaustible resource with costless extraction. Section III indicates how extraction costs can be dealt with and how the socially optimal (competitive) solution is determined. Section IV examines the effect of price constraints in an

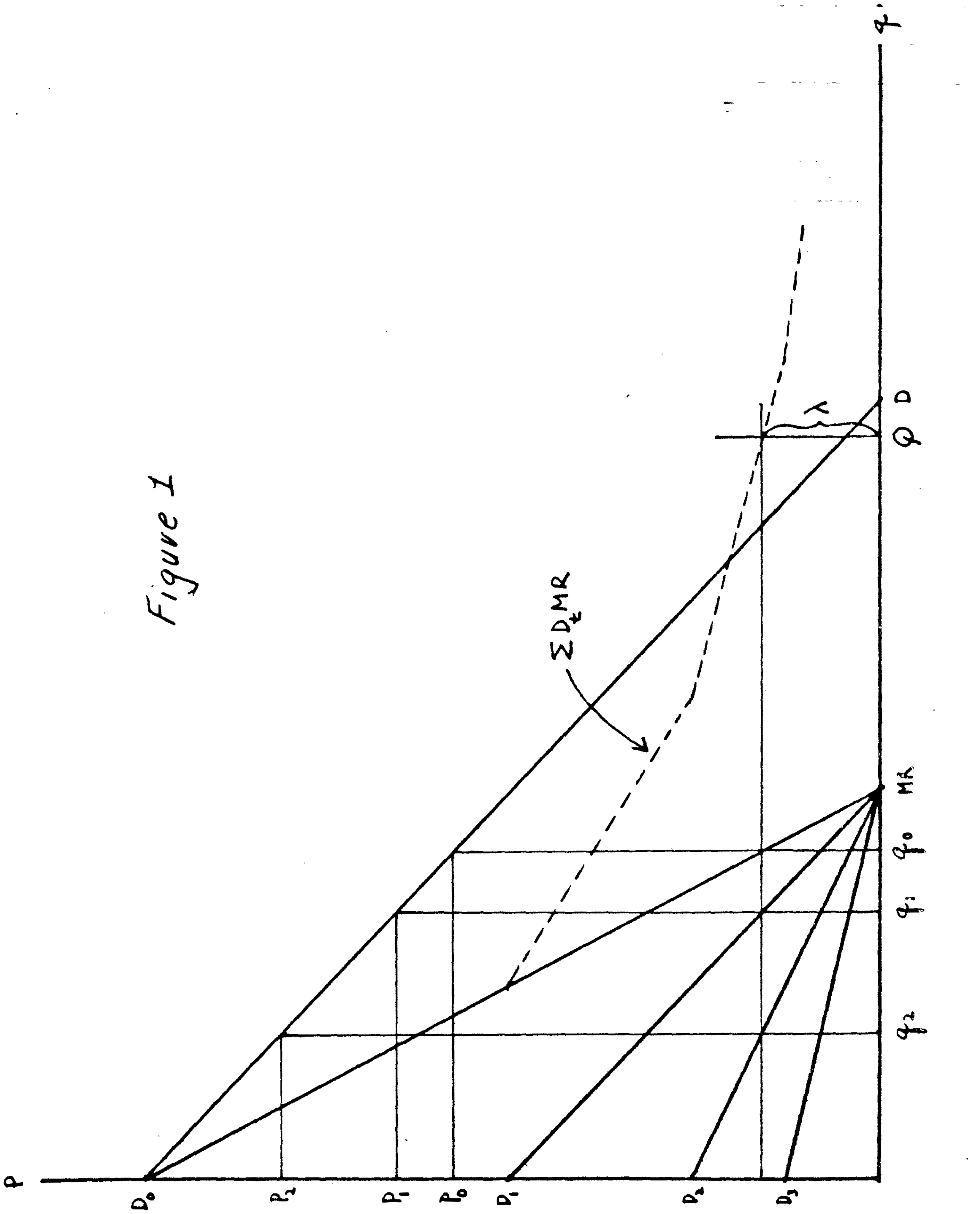

Figure 1

III. Extensions: Extraction Costs and the Competitive Solution

The geometry can easily be adapted to the case of a monopolist with costly extraction so long as extraction costs depend solely on the current rate of extraction (not cumulative extraction).[3] This is accomplished by subtracting marginal costs from marginal revenue to derive a marginal net revenue function. We would then proceed to the solution by deriving the horizontal summation of the discounted marginal net revenue schedules.

In order to determine the socially optimal intertemporal allocation--that which would result under competition given a complete set of futures markets--the only change required is to substitute the demand curve for the marginal revenue curve in deriving the schedule to be discounted (an example is given in the following section). The geometric solution for the competitive case will give another well-known result: that under stationary cost and demand conditions the net price will rise with the rate of interest. Comparing the solutions to the competitive and monopolistic problems under identical cost and demand conditions where the demand curve is linear (a comparison facilitated by the fact that the geometric solution for both problems can be found using the same diagram) will show that, relative to the competitive solution, the monopolist is a "conservationist." However, as two recent papers have shown, this plausible result holds only if elasticity of demand is a nonincreasing function of quantity,[4] a condition which is of course satisfied by linear demand schedules.

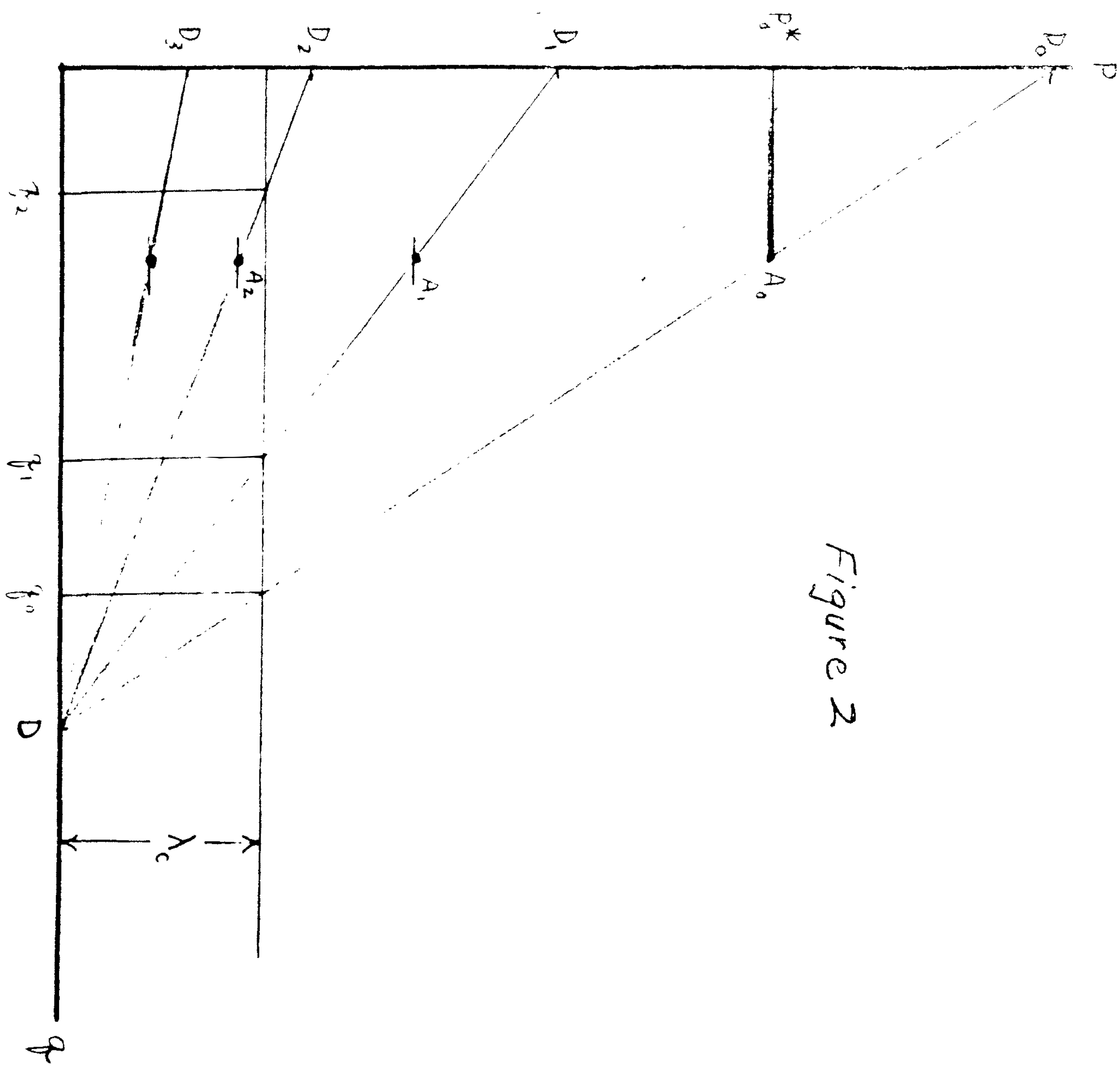

Figure 2

complications introduced by strategic considerations.) Gilbert and Goldman analyzed the effect of the constraint in a more general model (which would include linear demand functions and free extraction as special cases) employing a continuous time framework and claim an unambiguous result--that the effect of the constraint is an increase in the current period monopoly price. In the discrete time model the effect can go either way; thus, their proof is either incorrect or ambiguity results from the use of discrete rather than continuous time. Rather than provide two examples to show the ambiguous effect of the constraint, I will here provide only a clear counterexample to the Gilbert-Goldman result, i.e., one in which the current period monopoly price is decreased by the imposition of the constraint.

In Figure 3 let D_0D and D_0MR be, respectively, the monopolist's demand and marginal revenue functions for the unconstrained problem with the discounted marginal revenue functions given by D_tMR (t=1,2,...). For the sake of clarity, we set up the problem so that only two periods are involved. Let the resource stock be sufficiently small such that the solution in the unconstrained case is determined by λ_m giving the indicated quantities sold over the two-period ultilization horizon. Now let the backstop price be set at P^*_0. This gives $P^*_0A_0D$ and $P^*A_0B_0MR$, respectively, as the demand and marginal revenue functions for the constrained problem. On the discounted marginal revenue functions, the tick marks indicate the (discounted) height of the plateau, $P^*{}_0^A$ (horizontal tick) and the "kink" that occurs at point B_0 (vertical tick).

To determine the effect of $P^*{}_0$, suppose the monopolist were to follow the allocation called for in the unconstrained problem. The discounted value of marginal revenue in t_1 would be clearly less than that in t_0, since the plateau on the discounted demand function for t_1 is below λ_m. Thus profits would be increased by a revised program which would shift sales from t_1 to t_0. Hence the current period price is decreased in response to the imposition of the constraint.

V. CONCLUSION

After working through the examples presented above it should be apparent that the geometrical interpretation of the intertemporal resource allocation problems is quite flexible. Aside from convenience, there is no reason to impose the restrictions that the demand and marginal cost schedules be linear, or even that these be time-stationary. Nonlinear functions can be used,[8] and any evolution of these through time can be assumed so long as their evolution does not depend on prices or quantities in prior periods.

This qualification requires some comment. In a number of models, a lag adjustment model of demand is assumed or it is assumed that the marginal cost of extraction is a function of remaining reserves, or both.[9] The solution for such models, which I classify as "advanced" in contrast to the "simple" model examined in this note, requires the use of dynamic programming

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FOOTNOTES (CONTINUED)

4 In other words, marginal revenue is a strictly decreasing function of q. See the papers by Lewis (1976) and Lewis, Matthews, and Burness (1979).

5 The term "backstop technology" was first given prominence by Nordhaus (1976, p. 533) and was used to denote a technology with an effectively unlimited resource base such as solar or fusion power in the case of energy (Solow (1973, p. 4)). The constant cost assumption, though typically made, is not necessary.

6 Lee (1978, 1979) deals with explicit price controls, and Nordhaus (1976), Heal (1976), and Clark (1978) with backstop technology in modeling the competitive--socially optimal--depletion of exhaustible resources. The effect of backstop technology on the optimal program of a resource monopolist is the subject of papers by Stiglitz and Desgupta (1975), Gilbert and Goldman (1978), Hoel (1978), Salant (1979), Heal (1977), and Clark (1979). (Several of these papers deal with models in which marginal extraction costs are an increasing function of cumulative extraction. For reasons discussed in the following section, models employing this assumption regarding extraction costs cannot be analyzed using the geometric model developed in this paper.)