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S. Salant, S. Switzer and R. Reynolds

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Losses from Merger: The Effects of a Change in Industry Structure on Cournot-Nash Equilibrium

by

S. Salant*, S. Switzer* and R. Reynolds**

I. Introduction

In the Cournot (1838) solution to the oligopoly problem, each firm's output is profit-maximizing given that the outputs of the other firms are fixed at their equilibrium levels. Patinkin (1947) extended the Cournot approach to industries with merged firms and cartels by treating the merged entities as multiplant players in a Cournot noncooperative game. The market structure in such models--whether a particular firm is assumed to act independently or in collusion with others--is specified exogenously.

The purpose of this paper is to explore an unnoticed (and, perhaps, undesirable) implication of such Cournot models: mergers may reduce the joint profits of the firms which collude. This property may seem surprising since the merged firm always has the option of producing exactly as its components did in the premerger equilibrium. But such a situation is ruled out as a postmerger equilibrium since the merged firm would then have an incentive to cut back given unchanged outputs of the other players. That the merged firm may be worse off once the new equilibrium is established is a consequence of its Cournot-

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* Staff economist, Federal Trade Commission.

** Staff economist, Department of Justice. The views expressed in this paper are those of the authors and do not necessarily represent the views of the Federal Trade Commission or the Department of Justice.

misperception of the response of other firms to variations in its own output.

The behavioral assumption underlying the Cournot model is that firms ignore the influence of their own decisions on the decisions of their rivals. This assumption is generally regarded as unrealistic; nonetheless, widespread use of the Cournot model continues--presumably because the implications of the model are thought to be realistic. Friedman (1953) has argued that a model should be judged solely by the realism of its implications, not of its assumptions. It is the contribution of our paper to spell out several implications of the Cournot model which have escaped scrutiny.

Section II of this paper illustrates graphically why the merger may be unprofitable. The change in the merged firms' profits is decomposed into two terms, one of which is negative and may outweigh the other. Section III presents a straightforward example where identical firms with constant unit costs of production sell a homogeneous product to consumers with linear demand curves. For a specified number of firms in the premerger equilibrium, conditions sufficient for merger by a subset of these firms to be unprofitable are established for this example. It turns out that unless at least 80 percent of the firms in the industry merge, collusion will result in losses! Section IV evaluates the significance of these results and suggests how they might be used to develop a theory where market structure is endogenous.

II. Graphical Decomposition of the Change in Profits Due to Merger

Consider a Cournot equilibrium in which each firm in an industry operates independently. This can be compared to the Cournot equilibrium in which a subset of the firms merge while the other firms remain independent. Such a comparison can be used to examine those cases in which the profit of the merged firms would be lower than their combined profits prior to merger. It is convenient to refer to the subset of firms which will participate in the proposed merger as "insiders" and those firms which will continue to behave independently after the merger as "outsiders".

In figure 1, R_0 is the "reaction function" of the outsiders. Given any specified quantity supplied to the market by the insiders, it indicates the total amount the independent outsiders would supply. This supply is computed by subtracting from the demand curve the given production of the insiders and then considering aggregate production in a Cournot equilibrium where the outsiders face the residual demand curve. The response of the outsiders to a given supply by the insiders is the same whether that supply is provided by insiders who are colluding or acting independently.

In figure 1, there are two reaction functions for the insiders. R_I^{NC} indicates the sum of the reactions of each of the insiders prior to the merger (the superscript stands for noncollusive) and expresses total output of the insiders as a function of any given amount produced by the outsiders when each

Figure 1

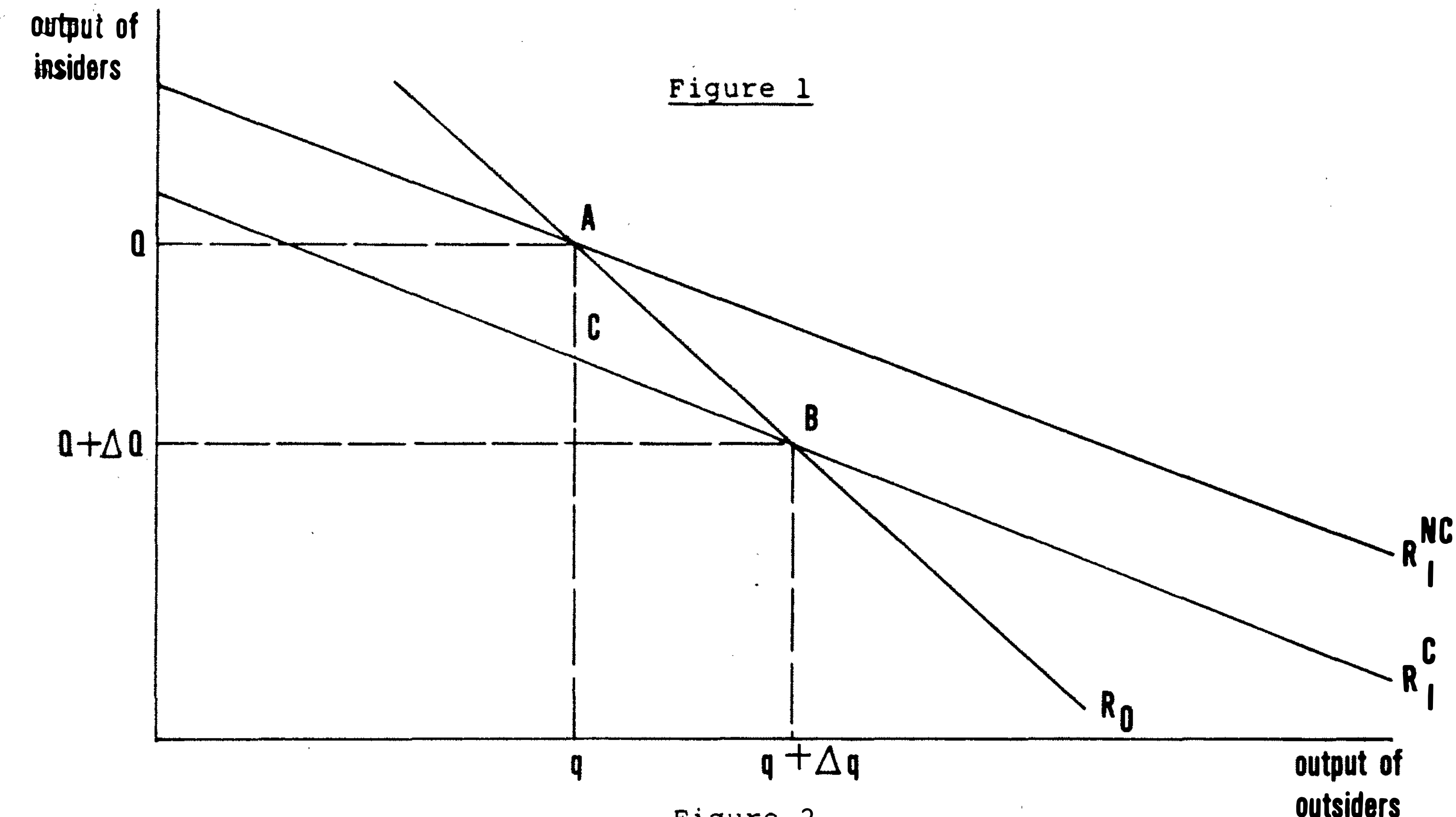
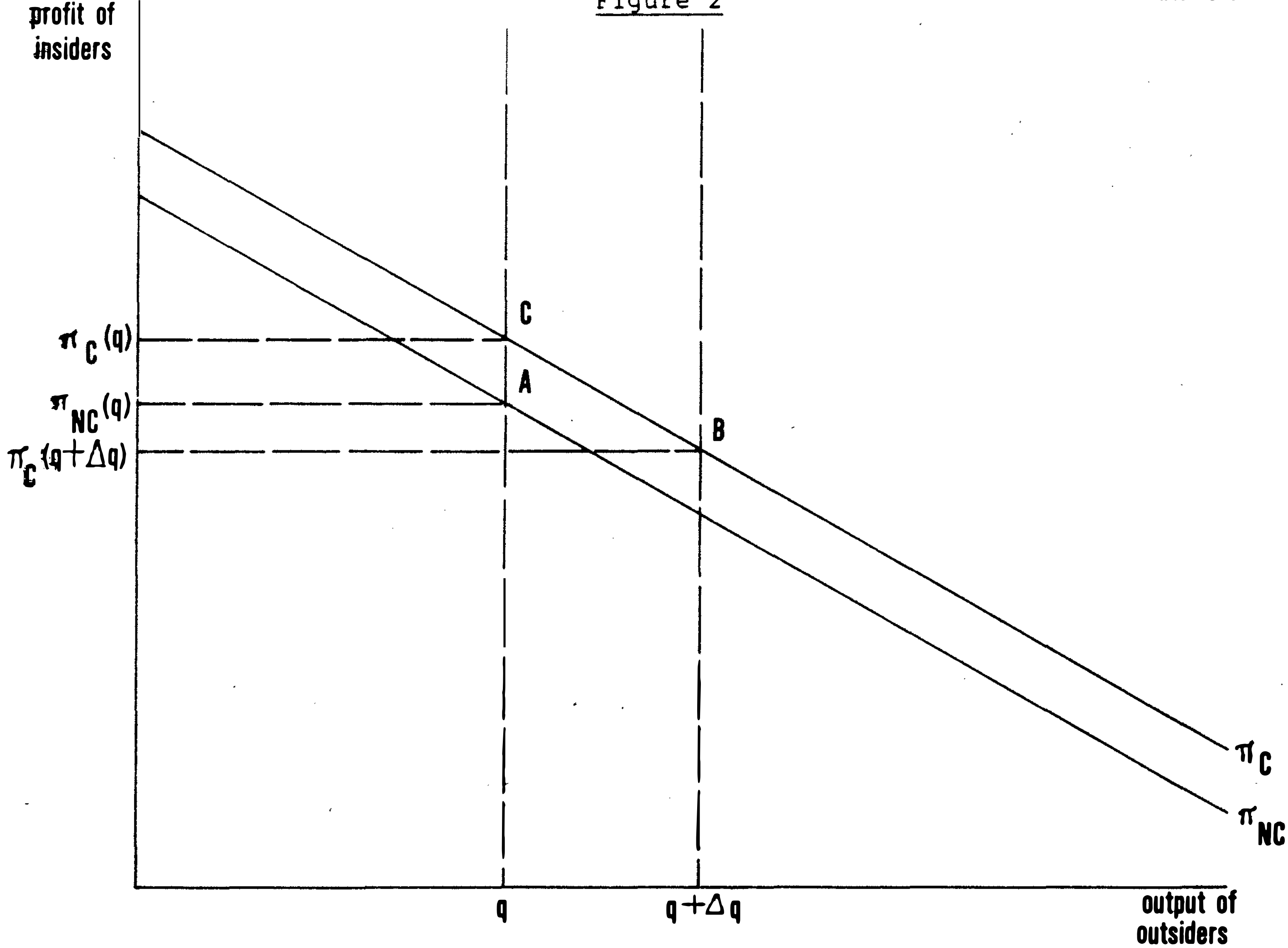


Figure 2



insider operates independently. In contrast, R_I^C indicates the response of the insiders <u>after</u> the collusion to any specified output of the noncolluding firms. Since the merged entity takes full account of the inframarginal losses which expansion at any particular plant has on its profits, the joint output of the insider plants is smaller after the merger. Consequently, R_I^C lies below R_I^{NC}.

Figure 1 indicates the equilibrium both prior to and also following the merger. The premerger equilibrium occurs at A, where R_0 intersects R_I^{NC}. The outputs of insiders and outsiders are, respectively, Q and q. The postmerger equilibrium occurs at B, where R_0 intersects R_I^C. The outputs of insiders and outsiders change to $Q+\Delta Q$ and $q+\Delta q$, respectively.

Figure 2 has the same horizontal axis as figure 1 and can be used in conjunction with it to determine the profits of the insiders. π_{NC} indicates the profits earned by the insiders prior to merger as a function of the output of the outsiders. π_C indicates the corresponding profits following the merger. To construct each curve, write profits of the insiders as a function of the output of each group ($\pi(Q,q)$) and then use the insiders' reaction function to express insiders' production as a function of the output of outsiders:

$$\pi_{NC}(q) = \pi(R_I^{NC}(q), q)$$

$$\pi_C(q) = \pi(R_I^C(q), q).$$

The change in the insiders' profits due to merger is equal to their postmerger profits, $\pi_C(q+\Delta q)$, minus their premerger profits, $\pi_{NC}(q)$. Figure 2 depicts an example in which $\pi_C(q + \Delta q) - \pi_{NC}(q) < 0$. Insight into this potential loss can be gained by decomposing the change in insiders' profits due to merger into two terms. The first term, $\pi_C(q) - \pi_{NC}(q)$, is the increase in profits that would be attained by the merging firms if the outsiders did not change their output in response to the merger. This term is represented in figures 1 and 2 by a movement from point A to point C; it is always positive since for the given output produced by the outsiders, q, the insiders will be maximizing their joint profits after merging.

The second term, $\pi_C(q+\Delta q)-\pi_C(q)$, indicates the decrease in the insiders' profits due to the increased production by the outsiders in response to the merger. This term, represented in figures 1 and 2 by a movement from point C to point B, is always negative since π_C is a decreasing function of outsiders' output. The merger of the insiders will lead to losses whenever this negative term is greater in absolute value than the positive term described above. Thus it is the output expansion of the outside firms which can in principle cause a reduction[1] in profits for the merging firms. Whether this possibility of losses from merger can in fact occur is resolved in the next section.

to

(4) $\pi^* = (m+1)\left(\frac{B-\alpha}{1+n}\right)^2.$

The gains from merger can, therefore, be expressed as a function of n and m, where

(5) $g(n,m) = \tilde{\pi}(n,m) - \pi^*(n,m) = (B-\alpha)^2\,[(1+n-m)^{-2} - (m+1)(1+n)^{-2}].$

Thus, there are losses to merger for the m+1 firms if $g<0$ ($\pi^* > \tilde{\pi}$), or equivalently if:

(6) $m+1 > \frac{1+2n+n^2}{1+2(n-m)+(n-m)^2}.$

For any specified number of firms in the premerger equilibrium (n), equation (6) can be used to determine whether a merger by m+1 of these firms would be unprofitable.

Several properties of this example are noteworthy:

(A) "Merger to monopoly" is always profitable. When all the firms in an n-firm equilibrium collude, profits must increase since joint profits will then be maximized. Formally, if $m+1=n$ and $n\geq 2$, the right-hand side of (6) exceeds the left-hand side. Hence, $\pi^* < \tilde{\pi}$.

(B) If only two firms merge, they will always be injured (provided there are other firms in the industry). The result follows by examining (6) with $m=1$ and $n\geq 3$.

(C) Collusion by a larger number of firms <u>may increase</u> the losses due to merger.

In order to demonstrate this property we can compare the joint profits of the insiders in the premerger equilibrium, π^*, with the postmerger profits of these firms, $\tilde{\pi}$, <u>when n is</u>

held fixed and m increases (in integer steps) from 0 to n-1.

In figure 3, three functions are plotted. The profit function $\tilde{\pi}(n,m)$ indicates insiders' profits following the merger of m firms with any particular firm; $\pi^*(n,m)$ indicates insiders' profits prior to the merger. g(n,m) is the change in the profits of the insiders due to merger ($\tilde{\pi} - \pi^*$). The picture illustrates the claim that the losses from merger may increase when a greater number of firms collude.

Both profit functions have the same vertical intercept (π_0) since $\pi^* = \tilde{\pi}$ (g=0) when m=0. That is, if a single firm is joined by no others in a merger, its profits will be the same before and after the merger.

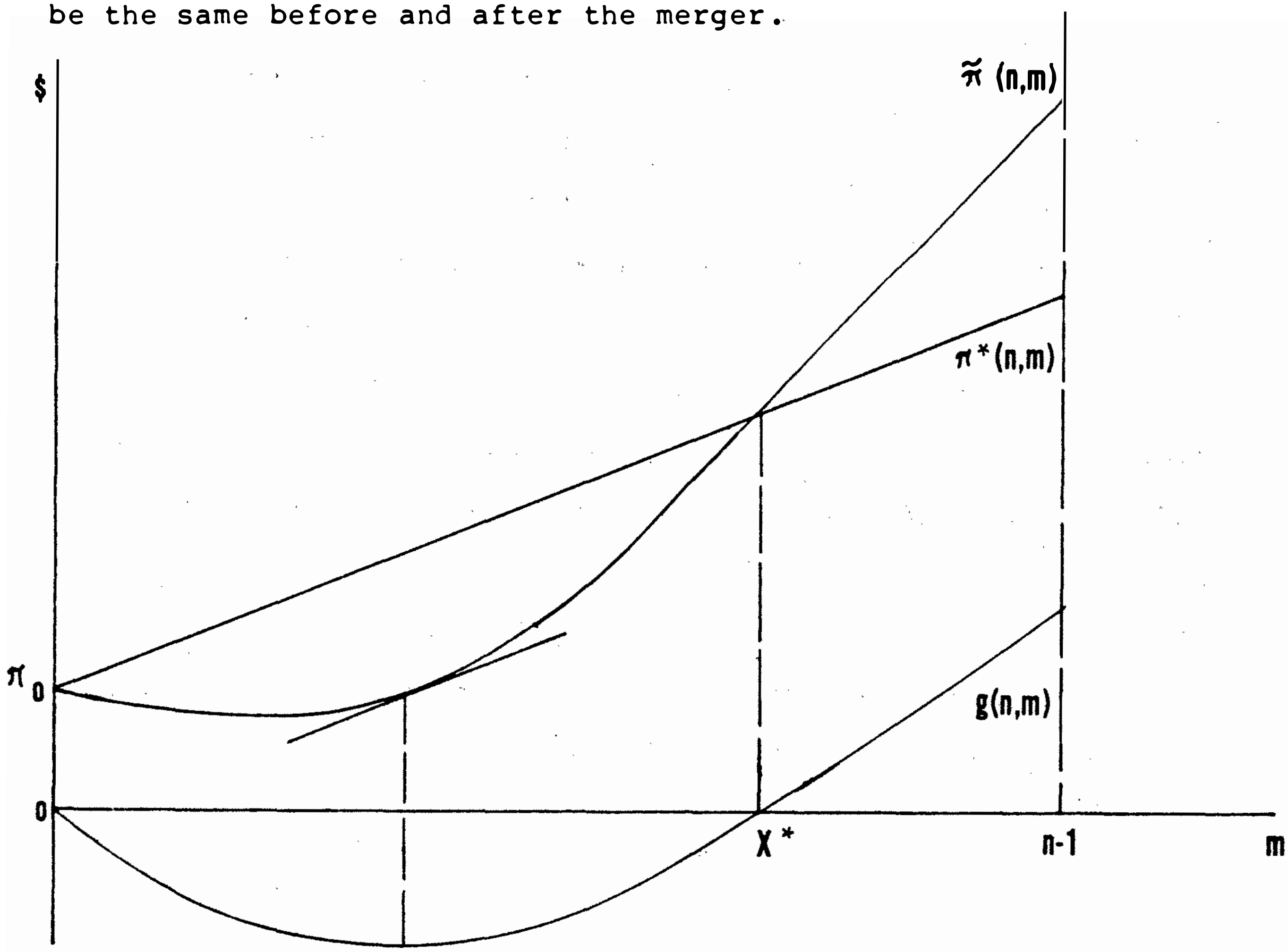


Figure 3

The slope of π^* is constant since the inclusion of an additional firm in the prospective merger increases the total profits of the merging firms by the profit per firm prior to the merger. From (4), $\frac{\partial \pi^*}{\partial m} = \left(\frac{B-\alpha}{1+n}\right)^2$.

The slope of $\tilde{\pi}$ is smaller for m=0. From (3), $\frac{\partial \tilde{\pi}}{\partial m} = \frac{2(B-\alpha)^2}{(1+n-m)^3}$.

Hence $\frac{\partial \tilde{\pi}}{\partial m} < \frac{\partial \pi^*}{\partial m}$ (or $\frac{\partial g}{\partial m} < 0$)[3] for m = 0 and n>2.

Indeed, these formulae imply that $\frac{\partial g}{\partial m} < 0$ <u>over a range of m</u>. Over this range losses from merger are <u>larger</u> the larger the number of firms in the coalition. For example, if n=12 a merger by seven firms (m=6) generates even larger losses than a merger by a smaller number of firms (m=0,1,2,3,4,5).[4]

(D) For any given number of firms in the premerger equilibrium, if a merger of k firms causes losses (gains), a merger by a smaller (larger) number of firms will cause losses (gains).

From property (A), g(n,n-1) >0. We have just noted that g(n,0)=0 and $\frac{\partial g(n,0)}{\partial m}<0$. Since $g(\cdot,\cdot)$ is continuous in its second argument there must exist at least one root $x^*>0$ such that $g(n,x^*) = 0$. Furthermore, since g(n,x) is strictly convex in[5] its second argument, g(n,x)>0 for $x>x^*$ and g(n,x)<0 for $x^*>x>0$.

(E) For any n, it is sufficient for a merger to be unprofitable that less than 80 percent of the firms collude.

Consider the gain-from-merger function g(n,m) defined above. Let $\alpha = \frac{m+1}{n}$ be the number of insiders as a proportion of all the firms in the industry. Then a merger causes neither losses nor gains if $\hat{\alpha}n-1=x^*(n)$ or $\hat{\alpha} = \frac{x^*(n)+1}{n}$. This break-even fraction reaches its minimum value of .8 when n=5.[6] Hence the break-even value for all other industry sizes exceeds 80 percent and the result follows from property (D).

(F) If any given fraction (<1) of an industry is assumed to merge there is an industry size (n) large enough for the merger to cause losses.

Let R be the ratio of the postmerger profits of the insiders to their premerger profits. From (3) and (4), R = $\frac{\tilde{\pi}}{\pi^*} = \frac{(1+n)^2}{(m+1)(n-m+1)^2} = \frac{(1+n)^2}{\alpha n(n-\alpha n+2)^2}$, where $\alpha = \frac{m+1}{n}$.

If for any α and n, R<1 then a merger by a proportion α of an industry of size n will result in losses. The result follows by noting that (for any $\alpha< 1$), $R\to 0$ as $n\to\infty$. To illustrate, even when 98 percent of the firms in an industry merge, they may incur losses if the industry is sufficiently large.

IV. Conclusion

The example of the previous section assumes that identical firms produce at constant marginal costs and without fixed costs. In such a case, the merged firms cannot produce a given combined output more cheaply than any independent outsider. Once increasing marginal costs or positive fixed costs are introduced, however, mergers do create efficiency gains.[7] But these gains may be too weak to offset the losses resulting from the insiders' Cournot-misperception of the output response of the outsiders.[8] Our example serves to isolate in its simplest context an unnoticed characteristic of Nash-Cournot equilibrium. However, our results can persist under more realistic assumptions.[9]

Since the results of this paper may be counterintuitive, it seems important that we conclude by putting their implications in perspective. Our analysis has ruled out one possibility--that firms can act like Cournot players in deciding how much to produce, can merge with anyone and can always benefit from merger. Three logical alternatives remain to be considered:

(1) Firms do not always act like Cournot players in deciding how much to produce;

(2) Firms do produce like Cournot players and some mergers may cause losses;

(3) Firms do produce like Cournot players and some specifiable mergers never occur.

The first alternative is favored not only by those who dismiss the Cournot model entirely but also by those who believe that firms sophisticated enough to merge will not subsequently be naive enough to act like Cournot players. If this latter view is correct it would have an important implication. The behavior of a multiplant player in an oligopolistic industry could not be predicted without knowing the historical circumstances under which the many plants came to be operated by that player.

Either of the remaining alternatives must be accepted logically by anyone believing that firms invariably act like Cournot players in deciding how much to produce. Historical evidence that many mergers earlier in the century turned out to be unprofitable is regarded by some as evidence supporting the second alternative.[10] The long history of useful insights[11] gained from the Cournot model and from assumptions of rationality is invoked by supporters of the third alternative.

A model where firms always act like Cournot players in deciding how much to produce but where unprofitable mergers never occur may be constructed in the following way. Suppose decisions about how much to produce are made by "managers" who act as if they were Cournot (possibly multiplant) players. However, prior decisions about the assignment of plants to managers (that is, about coalition structure) are made by "executives" who confer with each other and know how the managers--whom they cannot influence--will behave. Which coalitions will form in these circumstances falls within the province of the theory of cooperative games.[12] Any reasonable solution would exclude coalitions which generate losses for all participants.[13] Such a model would then predict both that certain coalitions would not form (or, if formed, would disintegrate[14]) and that managers act like Cournot players in deciding how much to produce.

FOOTNOTES

1 It should be noted, however, that industry profits will always increase in response to the merger. If the merger is unprofitable, the profits of the noncolluding firms will have increased by more than the loss in the profits of the insiders. This raises another issue (which is not dealt with in this paper)--that is, the "pregame" determination of which firms will merge. This will depend on not only whether a group of firms can gain by joining together but also on whether such a coalition will emerge when firms realize that they may gain even more by remaining outside the merging group. This is the cartel problem in which the nonparticipants always gain more from collusion than do the members of the cartel.

2 Any linear demand curve can be expressed in this form if the output units are defined appropriately.

3 $$\frac{\partial g(n,m)}{\partial m} = (B-\alpha)^2[2(1+n-m)^{-3} - (1+n)^{-2}]$$

4 For n=12 and m=6,5,4,3,2,1,0, $\frac{g(n,m)}{(B-\alpha)^2}$ = -.021, -.020, -.018, -.014, -.010, -.005, 0, respectively.

5 $$\frac{\partial^2 g(n,m)}{\partial m^2} = (B-\alpha)^2 \; [6(1+n-m)^{-4}] > 0$$

6 $$g(n,\alpha n-1) = (B-\alpha)^2 \left(\frac{(1+n)^2-\alpha n(2+n-\alpha n)^2}{(2+n-\alpha n)^2 \; (1+n)^2}\right).$$

$g(n,\alpha n-1)=0$ when the numerator (N) of the bracketed terms equals zero: $N = (1+n)^2 - \alpha n(2+n - \alpha n)^2 = 0$. This equation is a cubic in α and has three roots: $\alpha_1 = 1/n$, $\alpha_2 = \frac{(2n+3) - \sqrt{4n+5}}{2n}$, and $\alpha_3 = \frac{(2n+3) + \sqrt{4n+5}}{2n}$.

The third root exceeds unity and is inadmissible; the first is the root associated with the degenerate merger. The second is the root of interest and is itself a function of n:

$$\frac{d\alpha_2}{dn} = \frac{(2n+5)(4n+5)^{1/2} - 3(4n+5)}{2n^2(4n+5)}$$

$$\frac{d^2\alpha_2}{dn^2} = \frac{6(4n+5)-(2n+10)(4n+5)^{1/2}-(4n^2+10n)(4n+5)^{-1/2}}{2n^3(4n+5)}$$

$\alpha_2(n)$ reaches a relative minimum at n=5 and a relative maximum at n=-1. Hence for n>1, $\alpha_2(n) > \alpha_2(5) = .8$.

[7] The introduction of increasing marginal costs of production at each plant may also have a second effect. It may reduce the supply response of the outsiders to the merger. Such a reduction would tend to reinforce the efficiency effect in making the merger more profitable.

[8] Suppose in the example of section III that a merger of two firms results in a loss of $100. If instead each firm had positive fixed costs but the same marginal costs as before, the equilibrium would be unchanged, but the entire output of the merged firm would be produced by one of the two plants. As long as the fixed cost at each plant was under $100, however, the fixed cost saved would be too small to outweigh the loss resulting from the Cournot-misperception of the output response of the outsiders. Hence, the merger would still cause a loss.

[9] Indeed, we first observed losses from merger in a dynamic oil model where each Cournot player chooses a time-dated vector of extraction (subject to capacity and exhaustion constraints) and incurs marginal costs which are increasing functions of the rate of extraction. The parameters used in this computerized model were not intended to generate peculiar behavior, but to approximate the current world oil market.

[10] Further evidence that mergers have often been unprofitable is discussed in Scherer (1980).

[11] It is sometimes believed that the Cournot model provides a useful lower bound on prices (and an upper bound on outputs) in oligopolistic industries. However, this belief is erroneous since--as Hause [1977] has illustrated--price can in fact be <u>higher</u> in a Cournot equilibrium than in an equilibrium where one player acts as a Stackelberg dominant firm while the others behave like Cournot followers. Indeed, such an outcome is almost inevitable when mergers would cause losses in a Cournot model. To see this, note that a merger can never cause a loss if the merged firm uses the reaction function of the outsiders and acts like a Stackelberg dominant firm. But, by hypothesis, the merged firm would make larger profits replicating the output of the insiders in the premerger equilibrium than it would make replicating the restricted output of these firms in the Cournot postmerger equilibrium. Hence, provided its profit function is single-peaked in its own output, the Stackelberg dominant firm will discover that the output it would choose if it played Cournot in the postmerger equilibrium is too small, and it will produce a larger output at a lower price.

[12] For a solution to be interesting, it might also be necessary to impose a constraint on the executives which makes the grand coalition (merger to monopoly) illegal or otherwise infeasible.

[13] As we complete this paper, the new AER has arrived with a related article by Okuno, Postlewaite, and Roberts [1980]. They study a pure exchange, Nash general-equilibrium model with a continuum of traders and show in the context of that model that certain coalitions will be disadvantageous. An analogous result for cooperative games was discovered by Aumann [1973]. A review of this literature and its relation to the new results in non-cooperative settings is discussed in Okuno et al. [1980].

[14] Such a model can also be used to study the stability of cartels. It is commonly observed that each plant in a multiplant cartel could benefit from expanding production--provided either price or the output of others is held constant. This creates in all cartels an incentive for plants to chisel. It is always taken for granted that other members of the cartel would be injured by defection and would attempt to deter it. But we have found circumstances under which defection would be beneficial to all members of the cartel. If a change in the economic environment made the cartel disadvantageous to <u>all</u> parties, the cartel would be unstable in a stronger sense than is usually meant.

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