WORKING
PAPERS

Mergers of capacity-constrained firms

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WORKING PAPER NO. 338

December 2018

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December 6, 2018

Abstract

Merging firms regularly argue that mergers involving capacity-constrained firms are unlikely to be anticompetitive, because the incentive for the merged firm to raise prices and reduce quantity may not be strong enough to generate slack in the capacity constraints and lead to higher prices. We construct a modified notion of upward pricing pressure called $ccGUPPI$, or capacity constrained $GUPPI$, which accounts for the upward prices pressure from binding capacity constraints, in addition to standard merger price effects. $ccGUPPI$ is sufficient to correctly predict whether a merger of capacity-constrained firms will have positive price effects, irrespective of the functional form of demand. Further, using Monte Carlo simulation, we show that $ccGUPPI$ is generally a useful proxy for actual price effects, with lower informational requirements than full merger simulation.

1   Introduction

Firms with binding capacity constraints increase price and lower quantity relative to their optimal choices absent constraints. Merging firms often argue that this implies that mergers involving capacity-constrained firms are unlikely to increase price, even when there is significant demand substitution between the merging firms' products. The authors have heard such claims in connection with

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mergers before the FTC in a variety of industries. Merging fitness gyms recently made such arguments to the UK competition authority.[1] Penn State Hershey and PinnacleHealth hospital systems argued that capacity constraints mitigated antitrust concerns in response to the FTC's 2016 effort to block their merger in United States District Court.[2] Merging parties typically argue that since constrained firms would lower price and increase quantity but for the constraint, a merger involving capacity-constrained firms is unlikely to result in higher prices. They also argue that a capacity-constrained firm is unable to take on additional customers and thus does not impose a competitive constraint on its merger partner, meaning a merger involving a constrained firm and an unconstrained firm would not reduce competition.

While numerous studies point out that mergers involving capacity-constrained firms indeed may increase price,[3] the economics literature lacks tools both to predict which mergers will cause a price increase and to predict the magnitude of any price increase. We aim to fill this gap. Our paper constructs a version of gross upward pricing pressure ($GUPPI$) modified to account for capacity constraints, which we call $ccGUPPI$, or capacity-constrained $GUPPI$. Like other measures of upward price pressure, $ccGUPPI$ relies only on information that is local to pre-merger equilibrium (price, quantity, margins, and demand elasticities of the merging parties' products). It can qualitatively predict whether or not a merger will increase prices, and it can quantitatively predict the magnitude of merger price effects.

Specifically, we employ $ccGUPPI$ to predict whether both merging firms' constraints will continue to bind post-merger, and thus eliminate merger price effects. Used in this way, $ccGUPPI$ provides a diagnostic as to whether a proposed merger between capacity-constrained firms will likely raise prices, irrespective of the curvature of demand. We further show that $ccGUPPI$ can be used to predict the magnitude of merger price effects. We compare $ccGUPPI$'s predictions to actual price increases calculated via merger simulation, using a version of the Monte Carlo experiment of Miller

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[1]"According to the parties, the fact that they operate at or close to capacity indicates that they are not providing a significant competitive constraint on each other, as neither of them is seeking to win new customers.'' See Competition and Markets Authority (2014), "Anticipated combination of Pure Gym Limited and The Gym Limited,'' paragraphs 141 and 142, found via Neurohr (2016).

[2]Defendents' expert Bobby Willig testified as follows in FTC vs. Penn State Hershey Medical Center and Pinnacle-Health System, April 15, 2016: "But once capacity is taken into account, there can't be substantial diversion of patients from ... Hershey to Pinnacle ... because Hershey just doesn't have the capacity to take on a major influx of patients... So the practical diversion between Pinnacle and Hershey is insignificant due to Hershey's capacity constraint." One of the defendants' briefs contained the following: "...the combination will alleviate Hershey's capacity constraints and simultaneously allow both hospitals' physicians to treat more people,'' in "Defendants' Opposition to Plaintiffs' Motion For An Injunction Pending Appeal,'' May 12, 2016.

[3]See Froeb et al. (2003), Higgins et al. (2004), Sandford and Sacher (2016), Neurohr (2016), Oxera (2016), Balan et al. (2017), and Chen and Li (2018).

et al. (2016 and 2017) modified so that some firms are capacity-constrained prior to the merger. We find that $ccGUPPI$ offers excellent predictions of merger price effects when demand is linear or logit, and generally underestimates merger price effects when demand is AIDS. Across all three demand systems, $ccGUPPI$ appears to perform better than the next best alternative predictor, unmodified $GUPPI$. We further show that when used as a screen to identify mergers that will generate a specified minimum price increase, $ccGUPPI$ has a much lower false positive rate than $GUPPI$ under all three demand systems, and a roughly similar false negative rate.

Our paper adds to the somewhat sparse literature on mergers involving capacity–constrained firms. Froeb et al. (2003) simulates the effects of a hypothetical merger in a industry producing differentiated goods, subject to differing capacity constraints on the merging and non-merging firms. Based on their simulations, they argue that capacity constraints on merging firms attenuate merger effects more than capacity constraints on non-merging firms amplify them, and are critical of the 1992 Horizontal Merger Guidelines, which acknowledge the importance of the latter but not the former.

Froeb et al. (2003) is commonly cited by merging parties alleging that capacity constraints would eliminate or mitigate merger price effects. Notably, the merging firms are so tightly capacity-constrained in the paper's main example a merger does not increase price at all.[4] Our modified notion of upward pricing pressure and Monte Carlo experiment illustrate that whether and to what extent capacity constraints attenuate merger price effects depends critically on how tightly they bind.

Higgins et al. (2004) discuss a more general model in the same vein as Froeb et al. (2003), and again demonstrate via simulated results that capacity constraints on merging firms may attenuate merger price effects. Chen and Li (2018) argue that in a Bertrand-Edgeworth setting with identical firms, firms play a pure strategy if capacity constraints are low enough or high enough and a mixed strategy for the intermediate range. A merger both expands this intermediate range in both directions and shifts the distribution of prices within the mixed equilibrium to the right. Consistent with the Froeb et al. (2003) example, Chen and Li find that a merger has no effect on price outside of this intermediate range of capacity values. However, any industry that falls into the pre-merger intermediate range results in a price increase, as does any merger that causes the industry to shift from a pure to a mixed equilibrium.

Other papers discuss merger price effects when one or both merging firm is constrained in the context of a Cournot model (see Balan et al. (2017), Sacher and Sandford (2016)) or a differentiated Bertrand model (see Balan et al. (2017), Neurohr (2016), Oxera (2016)). All point out that if both merging firms are capacity-constrained pre-merger, positive price effects of the merger result if and only if at least one constraint no longer binds post-merger. Of particular relevance to our paper is

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[4]See tables 2 and 3 of Froeb et al. (2003).

Neurohr (2016), which illustrates how the tightness of pre-merger capacity constraints determines the extent to which the constraints attenuate merger price effects, and discusses a measure of tightness that can be used to modify standard notions of upward pricing pressure when both merging firms are constrained pre-merger, but neither is constrained post-merger. Our paper extends Neurohr's intuition to a full analysis of mergers involving capacity-constrained firms.

The next section presents a leading example, which shows how capacity constraints alter merger price effects. Section 3 describes the modeling framework and derives the effect on pricing incentives of a merger of one or more capacity-constrained firms. Section 4 describes how we construct $ccGUPPI$. Section 5 describes the Monte Carlo experiment and resulting data. The final section discusses our results and conclusions.

2   Leading example

We first consider a illustrative example of duopoly firms merging to monopoly. Specifically, suppose firms 1 and 2 produce differentiated but substitutable products at constant marginal cost 0, competing *a la* Bertrand by simultaneously setting price. Firms face the following demand system:

$$q_1 = 10 - p_1 + \tfrac{1}{2}p_2$$
$$q_2 = 10 - p_2 + \tfrac{1}{2}p_1$$
(1)

Absent capacity constraints, the Nash equilibrium of the Bertrand pricing game in which firm i maximizes $\Pi_i = (p_i - c_i)q_i$ is $(p_i, q_i) = (\tfrac{20}{3}, \tfrac{20}{3})$ for $i = 1, 2$. Were the two firms to merge the merged entity jointly chooses p_1 and p_2 to maximize $\Pi_1 + \Pi_2$, and post-merger prices and quantities would be $(p_i, q_i) = (10, 5)$ for $i = 1, 2$. Figure 1(a) plots pre-merger best response functions (in red) and post-merger first order conditions for the merged firm (in blue). In both cases, solid lines correspond to firm 1, and dashed lines to firm 2. Since the merged firm recaptures some of the lost sales from a price increase, it has an additional incentive to raise prices that did not exist before the merger, and thus the post-merger first order conditions are bowed out relative to the pre-merger best response functions, so that the post-merger equilibrium has higher prices.

Now, suppose that each firm has K_i units of capacity, with marginal cost constant for $q_i \leq K_i$ and prohibitively high for $q_i > K_i$. Figure 1(a) sets $K_1 = K_2 = 8$. We divide each figure into four subsets of the (p_1, p_2) space: where firms 1 and 2 are capacity-constrained, respectively, where both are constrained, and where neither is constrained. In figure 1(a), since both the pre- and post-merger equilibria lie in the region in which neither firm is constrained, the capacity constraints have no effect on either.

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The example in figure 1(b) is identical, except that $K_1 = K_2 = 4$. This expands the set of prices for which one or both firms is capacity-constrained. Since each of the firms' unconstrained profit-maximizing prices, both pre- and post-merger, would cause demand to exceed capacity, each firm raises price until its demand just equals its productive capacity. Thus, in figure 1(b), each firm optimally sets a price of 12, and sells quantity 4. Here, the constraints are severe enough that the merger has no price effect; each firm is so constrained pre-merger that the incentive to raise price from the constraint exceeds the incentive to raise price coming from the merger and consequent elimination of competition.

Figure 1(c) considers an example where $K_i = 6$, $i = 1, 2$. Here each firm's constraint binds before the merger but not after. Before the merger, we have $p_i = 8$, $i = 1, 2$ while post-merger we have $p_i = 10$, $i = 1, 2$. Hence, the capacity constraints attenuate the merger price effect by elevating premerger prices, relative to the case in which firms were not capacity-constrained.

Figure 1(d) considers a case with asymmetric capacity ($K_1 = 8$ and $K_2 = 4.5$), so that exactly one firm is constrained, both before and after the merger. Absent the constraints, the pre-merger Nash equilibrium would be located at the intersection of the red best response curves, or $(\frac{20}{3}, \frac{20}{3})$. Since firm 2 is constrained at this point (but not firm 1), firm 2 will increase its price until $q_2 = 4.5$. Since firm 1's best response to a higher p_2 is itself higher, the Nash equilibrium is located at the intersection of firm 1's pre-merger best response curve and the $q_2 = K_2$ locus, or $(p_1, p_2) = (7.3, 9.1)$, with $(q_1, q_2) = (7.3, 4.5)$.

Following the merger, an unconstrained monopolist would set prices of $(p_1, p_2) = (10, 10)$ and $(q_1, q_2) = (5, 5)$; this is the point at which the two blue lines intersect. However, this point is not feasible, as firm 2 would exceed its capacity constraint of 4.5. Hence, p_2 is set so that $10 - p_2 + \frac{1}{2}p_1 = 4.5$, while p_1 is the solution to:

$$\max_{p_1, p_2} p_1 * q_1 + p_2 * K_2 \tag{2}$$

$$\text{s.t. } q_1 = 10 - p_1 + \frac{1}{2}p_2$$

$$p_2 = 5.5 + \frac{1}{2}p_1$$

In solving (2), the merged firm is choosing the point on the $q_2 = K_2$ locus that maximizes $\pi_1 + \pi_2$. In particular, the monopolist knows an increase in p_1 will lead to an increase in p_2, since q_2 is increasing in p_1 and decreasing in p_2. The result is that the merged firm sets prices of $(p_1, p_2) = (10, 10.5)$, meaning that $(q_1, q_2) = (5.25, 4.5)$. Figure 1(e) magnifies the area surrounding the point $(10, 10.5)$ and depicts level sets of the function $\Pi_1 + \Pi_2$, with summed profits increasing in the direction of the point $(10, 10)$. Evidently, the maximum achievable profit on the $q_2 = K_2$ locus is at $(p_1, p_2) = (10, 10.5)$.

(a) unconstrained pre-and post-merger   (b) constrained pre-and post-merger   (c) constrained pre-merger, unconstrained post-merger

(d) one constrained pre- and post-merger, one unconstrained   (e) post-merger profit level sets   Legend
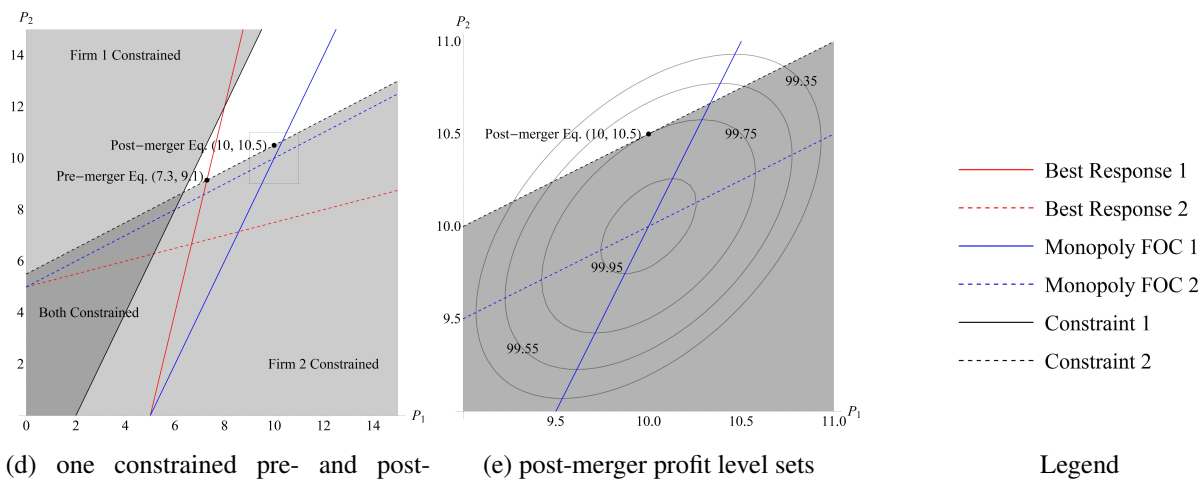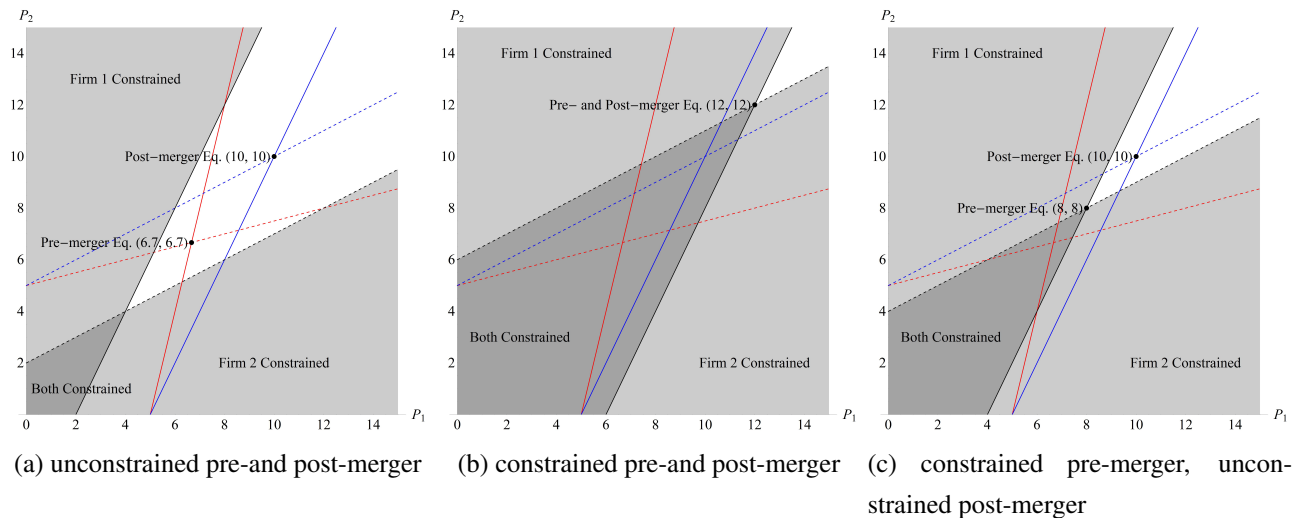
Figure 1: Capacity equals $K_1 = K_2 = 8$ in (a), $K_1 = K_2 = 6$ in (b), and $K_1 = K_2 = 4$ in (c), and $K_1 = 8$, $K_2 = 4.5$ in (d) and (e). In each case, duopolists under the demand system 1 merge to monopoly. The presence of the capacity constraints do not affect merger price effects in (a), eliminate price effects in (b), and attenuates price effects in (c)-(e).

We can take away several ideas from this example. First, absent any capacity constraints, a merger of firms 1 and 2 would have led to a 50% price increase, and capacity constraints can attenuate or eliminate the merger price effects depending on how tightly they bind. Second, while capacity constraints generally attenuate merger price effects by elevating pre-merger prices, price still increases following the merger, so long as at least one product is unconstrained post-merger. Indeed, even in figure 1(d)-(e), both p_1 and p_2 increase despite firm 2's constraint binding both before and after the merger. The optimization problem of the merged firm changes when one product is capacity

constrained and the other is not, but the merged firm still internalizes increased demand for product 2 following an increase in p_1, and this increased demand allows for a higher p_2.

The next section specifies a general model of differentiated Bertrand competition with capacity constraints. We derive the pre- and post-merger equilibrium conditions and then subsequently use those conditions to construct $ccGUPPI$.

3 Model

We study a standard model of price competition among N firms selling differentiated products. Given a vector of prices $\boldsymbol{p}$, firm i's demand is $q_i^D(\boldsymbol{p})$, while its total cost to produce quantity q is $c_i(q)$. Thus, firm i's profit is given by $\pi_i(\boldsymbol{p}) = q_i^D(\boldsymbol{p})p_i - c_i(q_i^D(\boldsymbol{p}))$. We assume that the demand function q_i^D is differentiable, decreasing in p_i, increasing in p_j for $j \neq i$, and satisfying $\frac{\partial q_i^D}{\partial p_i} + p_i \frac{\partial^2 q_i^D}{\partial p_i^2} \leq 0$ for all $p_i > 0$, so that each firm's demand becomes more elastic as price increases.

Each firm has access to a constant marginal cost production technology capable of producing K_i units (e.g., a factory). We refer to K_i as a firm's *capacity*. Each firm additionally has access to a higher cost production technology of unlimited capacity (e.g., buying or importing the product instead of producing it, or repurposing a factory producing a different good). We refer to this additional production technology as a firm's *flex capacity*. We assume that a firm's marginal cost increases by $\gamma > 1$ once it begins using its flex capacity.[5] Thus, equation (3) describes firm i's total cost:

$$c_i(q) = \begin{cases} c_i q & \text{if } q \leq K_i \\ c_i K_i + \gamma_i c_i (q - K_i) & \text{if } q > K_i \end{cases} \tag{3}$$

Firms simultaneously choose price, with each firm maximizing profit taking as given its rivals' prices. We make two simplifying assumptions, one to guarantee the existence of a pure-strategy Nash equilibrium in prices, and one to simplify discussion of capacity.

Assumption 1: Each firm sells $q_i^D(\boldsymbol{p})$ (no chance to reoptimize over quantity once prices are set).
Assumption 2: $\gamma_i > p_i^m$, where p_i^m denotes i's monopoly price (flex capacity is unprofitable).

Assumption 1 dictates that once prices $\boldsymbol{p}$ are set, a firm's demand $q_i^D(\boldsymbol{p})$ determines its quantity sold, so that firms cannot choose to supply less than their quantity demanded. The alternative, allowing firms to re-optimize over quantity once all prices are set, leads to non-existence of pure strategy

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[5]Dixit (1980) is the earliest example we know of to include a stepped cost function to model capacity constraints. See also Maggi (1996) and Boccard and Wauthy (2000), each of which uses the same cost function we do.

Nash equilibria, and mixed equilibria that depend on an assumed rationing rule.[6] By Dastidar (1997), assumption 1 is justified when prices are set by sealed bid tenders, or when there are large costs to turning away customers. Assumption 1 is ubiquitous in the literature on oligopolies, and appears in models with and without capacity constraints.[7]

Assumption 2 ensures that no firm will use its flex capacity in equilibrium. While this assumption is not necessary, it simplifies language: in equilibrium, a firm is either *capacity-constrained* ($q_i = K_i$) or *unconstrained* ($q_i < K_i$).[8]

We proceed by solving the model both before and after a merger of firms 1 and 2. Then, we study how the change in incentives generated by the merger vary in whether or not each merging firm is capacity-constrained prior to the merger.

3.1 Pre-merger equilibrium

If the N firms are separately owned, each firm i takes other prices p_{-i} as given, and chooses p_i to maximize profits. Under assumption 1, firm i's profits are given by $q_i^D(\boldsymbol{p})p_i - c_i(q_i^D(\boldsymbol{p}))$, and under assumption 2 profits are decreasing for for $p_i > p_i^m$. Let $q_i^{-1}(K_i, \boldsymbol{p}_{-i})$ denote the price p_i at which $q_i^D = K_i$, and below which $q_i^D > K_i$. Under assumption 2, all firms optimally set price $p_i \geq q_i^{-1}(K_i, \boldsymbol{p}_{-i})$ and so each firm has constant marginal cost of c_i . Thus, firm i's pre-merger maximization problem is:

$$\max_{p_i} q_i^D(\boldsymbol{p})(p_i - c_i) \tag{4}$$

$$\text{s.t. } p_i \geq q_i^{-1}(K_i, \boldsymbol{p}_{-i})$$

By definition, under any price vector $\boldsymbol{p}$ which solves (4) for all N firms, each firm i is either

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[6]Suppose there were a pure strategy equilibrium under this alternative assumption. Then, each unconstrained firm would set price so that demand equals both marginal revenue and marginal cost, while constrained firms would price so that demand equals capacity. But then any one unconstrained firm would have an incentive to increase price slightly, causing all other firms to choose a quantity less than demand. Some of this quantity would then be reallocated towards the firm who increased price, according to the assumed rationing rule. As a marginal price increase would have no direct effect on an optimizing firm's profit, the total effect on profit of the price increase plus the additional quantity must be positive. Thus, there can be no pure strategy equilibrium under the alternative assumption. See Shapley and Shubik (1969) for a fuller discussion of potential non-existence of equilibrium.

[7]In addition to Dastidar (1997), see Bulow et al. (1985), Vives (1990), Dixon (1990), Dastidar (1995), and Chen (2009).

[8]A firm with $q_i > K_i$ (absent assumption 2) could also be said to be unconstrained with marginal cost $\gamma_i c_i$. However, a merger involving this firm could lower its equilibrium quantity to be less than or equal to K_i. It is this nuisance case we avoid with assumption 2.

unconstrained or constrained. If firm i is unconstrained, its first-order condition for (4) is:

$$\frac{\partial \pi_i}{\partial p_i} = q_i(p) + \frac{\partial q_i}{\partial p_i}(p_i - c_i) = 0$$

$$\Rightarrow \frac{p_i - c_i}{p_i} = -\frac{1}{\epsilon_{ii}} \tag{5}$$

where $\epsilon_{ii} = \frac{\partial q_i}{\partial p_i}\frac{p_i}{q_i}$ denotes firm i's own-price elasticity. Equation (5), relating firm i's margin over cost to its elasticity of demand, is the well-known Lerner condition.

If firm i is capacity-constrained, so that the constraint in (4) binds with equality, its margin is greater than its inverse elasticity ($\frac{p_i - c_i}{p_i} > -\frac{1}{\epsilon_{ii}}$), so the Lerner condition no longer holds. Let $\lambda_i = \frac{p_i - c_i}{p_i} + \frac{1}{\epsilon_{ii}} > 0$ be the difference between margin and inverse elasticity, or the "wedge'' between the two sides of the Lerner condition. Then, for any constrained firm, we have:

$$\frac{p_i - c_i}{p_i} = -\frac{1}{\epsilon_{ii}} + \lambda_i \tag{6}$$

The quantity λ_i is a measure of upward pricing pressure due to the capacity constraint K_i binding; a greater value of λ_i implies greater upward pricing pressure from the constraint. The quantity λ_i is directly comparable to upward pricing pressure resulting from a merger with another firm (caused by each firm internalizing the effect of its own price increase on its former rival's profits). In particular, we show in the following section that a merger involving two capacity constrained firms results in a price increase if and only if λ_i is less than the pricing pressure resulting from the merger for at least one merging firm.

Finally, we formally define Nash equilibrium in light of the optimality conditions (5) and (6). A pre-merger Nash equilibrium is a price vector p such that the first order condition (5) holds for all firms for which $q_i^D(p) < K_i$, such that no firm's quantity demanded exceeds its capacity (i.e. $q_i^D(p) \leq K_i$ for all i), and such that $\lambda_i > 0$ in equation (6) for all firms at capacity ($q_i^D(p) = K_i$).

3.2 Post-merger equilibrium

We now consider a merger of firms 1 and 2, and derive post-merger analogues of pricing equations (5) and (6). Under assumptions 1 and 2, the merged firm jointly chooses prices for products 1 and 2 to maximize $q_1(p)(p_1 - c_1) + q_2(p)(p_2 - c_2)$, subject to the constraint that neither product exceed its capacity. We first solve for pricing equations when the merging firm is capacity-constrained in choosing zero, one, or both of its prices. The next section develops a prediction for which products will be capacity-constrained post-merger, as a function of pre-merger information.

First, suppose that neither product is capacity-constrained post-merger. Then, the merged firm's

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profits are maximized for prices p_1 and p_2 satisfying the first order conditions below:

$$\frac{\partial \pi_i}{\partial p_i} = q_i(p) + \frac{\partial q_i}{\partial p_i}(p_i - c_i) + \frac{\partial q_j}{\partial p_i}(p_j - c_j) = 0$$

$$\Rightarrow \frac{p_i - c_i}{p_i} = -\left(\frac{1}{\epsilon_{ii}} + \frac{\frac{\partial q_j}{\partial p_i} p_j}{\frac{\partial q_i}{\partial p_i} p_i} \frac{p_j - c_j}{p_j} \right) \text{ for } i = 1, 2 \tag{7}$$

Let $D_{ij} = -\frac{\partial q_j}{\partial p_i} / \frac{\partial q_i}{\partial p_i}$ denote the diversion ratio between firms i and j, or the fraction of firm i's marginal customers who view firm j as their next-best option. Then, let $GUPPI_i = D_{ij} \frac{p_j}{p_i} \frac{p_j - c_j}{p_j}$. $GUPPI_i$, well known in the antitrust literature and commonly used by antitrust practitioners,[9] is a measure of upward pricing pressure due to a merger. Its terms are intuitive: following a merger, firm i has an incentive to increase price because some of the customers it loses will be recaptured by its former rival, and the value of these customers depends on relative prices and the former rival's margin. We rewrite equation (7) below. If neither of the merged firm's capacity constraints are binding, it sets prices p_1 and p_2 according to:

$$\frac{p_i - c_i}{p_i} = -\frac{1}{\epsilon_{ii}} + GUPPI_i \text{ for } i = 1, 2 \tag{8}$$

Next, assume product 2 is capacity-constrained following a merger, while product 1 is unconstrained. In this case, the merged firm sets $p_2 = q_2^{-1}(K_2, \boldsymbol{p})$, meaning that the merger does not directly alter pricing incentives for product 2, which has upward pricing pressure of λ_i both before and after the merger.

Contrary to arguments made by merging parties, and discussed in the introduction, the fact that firm 2 is constrained both before and after the merger does not imply that firm 1's incentives are unaffected by the merger. Instead, an increase in p_1 diverts some of product 1's customers to product 2. Since firm 2 is constrained, the merged firm is unable to capture these diverted customers in the form of a greater quantity q_2. Instead, customers diverted to product 2 bid up the price at which product 2 is exactly at capacity, enabling the merged firm to charge a higher price for product 2 to sell the same quantity. Some of product 2's marginal customers will divert to product 1 in response to this price increase, further increasing the merged firm's profits. Alternatively, if q_2 is fixed at K_2, the merged firm sets p_1 so that the value of a reduction in q_1 equals the value of an increase in $p_2 = q_2^{-1}(K_2, \boldsymbol{p}_{-2})$ caused by increasing p_1.

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[9]For example, "As a general matter, Dollar Tree and Family Dollar stores with relatively low GUPPIs suggested that the transaction was unlikely to harm competition... Conversely, Dollar Tree and Family Dollar stores with relatively high GUPPIs suggested that the transaction was likely to harm competition," from "Statement of the Federal Trade Commission In the Matter of Dollar Tree, Inc. and Family Dollar Stores, Inc.," July 13, 2015, accessed on October 2, 2017 from www.ftc.gov/public-statements/2015/07/statement-federal-trade-commission-matter-dollar-tree-inc-family-dollar.

This effect — a merged firm increasing price when exactly one of its two products is constrained — generates a new first-order condition, distinct from (8). In this case, the merged firm sets $p_j = q_j^{-1}(K_j, \boldsymbol{p}_{-j})$ for the constrained product and sets price for its remaining product to satisfy:

$$\max_{p_i} q_i^D(\boldsymbol{p})(p_i - c_i) + K_j(q_j^{-1}(K_j, \boldsymbol{p}_{-j}) - c_j) \tag{9}$$

Equation (9) has the following first-order condition:

$$q_i^D(\boldsymbol{p}) + \left(\frac{\partial q_i^D}{\partial p_i} + \frac{\partial q_i^D}{\partial p_j}\frac{\partial p_j}{\partial p_i} \right)(p_i - c_i) + K_j \frac{\partial p_j}{\partial p_i} = 0 \tag{10}$$

where $\frac{\partial p_j}{\partial p_i}$ reflects how p_j responds to a small change in p_i along the locus of points satisfying $p_j = q_j^{-1}(K_j, \boldsymbol{p}_{-j})$. Applying the implicit function theorem, $\frac{\partial p_j}{\partial p_i} = -\frac{\frac{\partial q_j^D}{\partial p_i}}{\frac{\partial q_j^D}{\partial p_j}}$. We can then rewrite (10) as:

$$\frac{p_i - c_i}{p_i} = -\frac{1}{\epsilon_{ii}} + \theta_i, \text{ where } \theta_i = m_i D_{ij} D_{ji} - \frac{p_j}{p_i} D_{ij} \frac{1}{\epsilon_{jj}} \tag{11}$$

The term θ_i defines a third source of upward pricing pressure, describing the change in incentive for a firm which is not capacity-constrained post-merger, but whose former rival is. θ_i consists of two terms, both positive (as $\epsilon_{jj} < 0$), meaning that both increase i's margin and thus its price relative to its pre-merger first order condition (5). The first term captures the value of customers diverted from j to i following an increase in p_i and a consequent increase in p_j. The second term captures the value of the increase in p_j caused by the increase in p_i, holding fixed j's quantity at K_j. Note that the second term is smaller the more elastic j's demand is, reflecting the fact that a smaller increase in p_j would be needed to sell out capacity the more elastic its demand is.

If both of the merged firm's products are capacity constrained, it simply sets price for each such that quantity demanded equals capacity, or $p_i = q_i^{-1}(K_i, \boldsymbol{p}_{-1})$ for $i = 1, 2$. In this case, each firm has post-merger upward pricing pressure equal to λ_i. It is direct that a merged firm can be capacity-constrained in both products post-merger only if both firms were capacity-constrained pre-merger. In this case, the merger does not increase the upward pricing pressure of either product.

In summary, while prior to a merger there is one possible source of upward pricing pressure — λ_i from binding capacity constraints — following a merger there are three — $GUPPI$ if both merging firms are unconstrained post-merger, θ if i's former rival is constrained but i is unconstrained, and λ_i if both merging firms continue to be capacity constrained. We define post-merger Nash equilibrium in terms of these three sources of pricing pressure. If firms 1 and 2 are commonly owned, a price vector

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p comprises a Nash equilibrium if and only if:

$$\text{(i)} \quad \frac{p_i - c_i}{p_i} = -\frac{1}{\epsilon_{ii}} + GUPPI_i \ \text{ if } \ q_i^D(\boldsymbol{p}) < K_i \text{ for } i = 1, 2$$

$$\text{(ii)} \quad \frac{p_i - c_i}{p_i} = -\frac{1}{\epsilon_{ii}} + \theta_i \ \text{ if } \ q_j^D(\boldsymbol{p}) = K_j, i, j \in \{1, 2\}$$

$$\text{(iii)} \quad \frac{p_i - c_i}{p_i} = -\frac{1}{\epsilon_{ii}} \ \text{ if } \ q_i^D(\boldsymbol{p}) < K_i \ \text{ for } \ i = 3, ..., N$$

$$\text{(iv)} \quad q_i^D(\boldsymbol{p}) \leq K_i \ \text{ for } \ i = 3, ..., N$$

Calculating Nash equilibrium given knowledge of the demand system is straightforward. First, compute a price vector $\boldsymbol{p}$ satisfying $\frac{p_i - c_i}{p_i} = -\frac{1}{\epsilon_{ii}} + GUPPI_i$ for each merging firm and $\frac{p_i - c_i}{p_i} = -\frac{1}{\epsilon_{ii}}$ for each non-merging firm. Then, for any firm i such that $q_i^D(\boldsymbol{p}) > K_i$ replace firm i's first-order condition with $q_i^D(\boldsymbol{p}) \leq K_i$, and recompute the price vector that satisfies all N first-order conditions. Iterate as necessary until a price vector satisfying (i)-(iv) above is reached.

Of course, merger review generally takes place absent knowledge of the demand function. Following a burgeoning literature on first-order approximations of merger price effects, in the next section we evaluate post-merger pricing pressure terms λ, $GUPPI$, and θ at the pre-merger equilibrium, and argue that these terms can be compared to one another to make both qualitative and quantitative predictions about post-merger behavior, even in the absence of information about the form of demand. The result, $ccGUPPI$ is obtainable using only pre-merger information on price, quantity, merging firms' margins and elasticities, and diversion between merging firms, and, when paired with a pass-through matrix, can be employed to estimate merger price effects involving capacity-constrained firms.

4 $ccGUPPI$: Upward pricing pressure with capacity constraints

If a firm is constrained prior to merging, under what conditions should we expect it to continue to be constrained following a merger? How should we map those conditions to estimates of merger price effects? This section develops both qualitative and quantitative estimates of the effects of a merger involving capacity-constrained firms, and includes our main theoretical result, proposition 2, which provides precise conditions under which capacity constraints overwhelm merger price effects, and post-merger pricing coincides with pre-merger pricing. First we predict whether each constrained merging firm's capacity constraint will continue to bind following a merger. Then, given these qualitative predictions about the post-merger status of capacity constraints, we calculate net pricing pressure as the difference between post-merger pricing pressure (evaluated at pre-merger equilibrium, and equal to λ, $GUPPI$, or θ depending on the qualitative prediction) and pre-merger pricing pressure

(equal to 0 or λ if pre-merger constraint is nonbinding or binding, respectively). We use the term $ccGUPPI$, or capacity-constrained $GUPPI$, to refer to the pricing pressure due to a merger. As with other measures of pricing pressure, when multiplied by a suitable pass-through matrix $ccGUPPI$ provides a useful approximation for price increases from a merger of one or more capacity-constrained firms, based only on pre-merger information.

The previous section defined λ, $GUPPI$, and θ, respectively, as the pricing pressure resulting from a binding capacity constraint, a merger with an unconstrained firm, and a merger with a constrained firm. From here forward, we evaluate each source of pricing pressure at the pre-merger equilibrium, so that all three measures can be calculated using only pre-merger data on prices, quantities, margins, and elasticities, and without knowledge of the demand curve. Then, for example, if two constrained firms merge with $\theta_1 > \lambda_1$ and $\lambda_2 > GUPPI_2$, firm 1's pricing pressure resulting from the merger would be greater than its pricing pressure from the capacity constraint, causing it to increase price and lower quantity. Firm 2, on the other hand, sees greater pricing pressure from its capacity constraint binding than from the merger, meaning that post-merger it will continue to sell out its capacity.

Since the three pricing pressure terms, λ, $GUPPI$ and θ are directly comparable to one another, in that pricing pressure (of any type) is equivalent to an increase in the marginal cost c_i equal to the magnitude of the pricing pressure, it follows that should the pricing pressure from a merger ($GUPPI$ or θ) exceed pre-merger pricing pressure from a binding capacity constraint (λ), the difference between the two ($GUPPI - \lambda$ or $\theta - \lambda$) represents the increase in pricing pressure due to the merger. Thus, we define $ccGUPPI$ to be the difference between pre-merger and post-merger pricing pressure, incorporating the qualitative predictions as to which constraints will bind post-merger described above. $ccGUPPI$ has identical motivation to $GUPPI$ for non-constrained firms (e.g. Farrell and Shapiro (2010)).

Both the qualitative and quantitative predictions are generally inexact. To see why, return to the example in which $\theta_1 > \lambda_1$ and $\lambda_2 > GUPPI_2$ at pre-merger prices, so that firm 1 has an incentive to increase price, but firm 2's pricing pressure from its constraint is greater than its pricing pressure from the merger. However, the increase in p_1 gives an additional incentive to increase p_2 if the products are strategic complements, and this feedback effect is not captured by λ or θ when evaluated at pre-merger equilibrium.

Importantly, the feedback effects causing our qualitative predictions to be inexact are only present when at least one firm increases price following the merger. Thus, we can characterize exactly when a merger of two capacity-constrained firms will result in both constraints continuing to bind post-merger, and thus no price increase. As proposition 2 shows, a merger of two firms with binding capacity constraints will result in a price increase if and only if at least one firm has a unilateral

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incentive to increase price following the merger, meaning that $\theta_i > \lambda_i$. Proposition 2 is our paper's main theoretical result.

First, lemma 1 establishes results on the ordering of λ, $GUPPI$, and θ. While the lemma is mainly used in the proof of proposition 2, the relationships established in the lemma are useful for understanding the qualitative and quantitative predictions of $ccGUPPI$.

Lemma 1. *The pricing pressure terms* λ, $GUPPI$ *and* θ *are ordered as follows:*

1. $GUPPI_i > \theta_i \iff \lambda_j > GUPPI_j$ *for* $i, j \in \{1, 2\}$

2. $\lambda_i > \theta_i$ *and* $\theta_j > \lambda_j \Rightarrow \lambda_i > GUPPI_i$ *for* $i, j \in \{1, 2\}$

Proof See appendix.

We use lemma 1 to prove proposition 2 by exhaustion, considering the universe of possible orderings of λ, $GUPPI$, and θ for each firm, eliminating those orderings inconsistent with lemma 1, and showing that a subset of the remaining orderings are necessary and sufficient for the existence of a post-merger equilibrium that is unchanged from the pre-merger equilibrium.

Proposition 2. *Suppose that firms 1 and 2 are both capacity-constrained pre-merger, at price vector* $\boldsymbol{p}^*$. *Following a merger of firms 1 and 2,* $\boldsymbol{p}^*$ *remains an equilibrium if and only if the following condition holds:*

$$\lambda_i > \theta_i, \text{ for } i = 1, 2 \tag{12}$$

Proof See appendix.

Proposition 2 precisely characterizes conditions under which merging firms do not increase price in equilibrium. These conditions have a simple interpretation: a merger of two capacity-constrained firms does not result in a price increase if and only if the pricing pressure from the pre-merger capacity constraint (λ_i) exceeds the pricing pressure resulting from the merger for each firm, given that its former rival remains constrained. If we instead had $\theta_i > \lambda_i$, then a unilateral price increase would be profitable for firm 1. Under the condition of proposition 2, neither firm has such a unilateral incentive, and hence pre-merger pricing remains an equilibrium outcome following a merger. Proposition 2 is a valuable result for an antitrust agencies charged with predicting whether a particular merger between capacity constrained firms would raise prices.

When proposition 2 does not apply, meaning either $\theta_i > \lambda_i$ for at least one i or when one or both merging firms is unconstrained pre-merger, we construct approximate measures for the merger's pricing pressure using $ccGUPPI$. First, suppose that both firms are capacity-constrained pre-merger, but the ordering of λ, $GUPPI$ and θ do not satisfy the condition of proposition 2. For example,

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suppose $GUPPI_1 > \theta_1 > \lambda_1$, and $\lambda_2 > \theta_2 > GUPPI_2$. In this case, firm 1's pricing pressure from the merger is greater than its pricing pressure from its capacity constraint, and so we predict that firm 1 will increase price, regardless of whether firm 2 does so as well. Firm 2, on the other hand, has greater pricing pressure from its capacity constraint than from the merger (in the sense that $\lambda_2 > GUPPI_2$, when evaluated at the pre-merger equilibrium). Thus, we predict that firm 2 will remain constrained following the merger. Then, we calculate $ccGUPPI_i$ as the difference in pricing pressure from after the merger to before, so that $ccGUPPI_1 = \theta_1 - \lambda_1$, and $ccGUPPI_2 = 0$.

To define $ccGUPPI$, let $\left(\frac{p_i - c_i}{p_i} + \frac{1}{\epsilon_{ii}} \right)^{pre}\Big|_{p_i^{pre}}$, which equals either 0 or λ_i, depending on whether i is constrained or not, denote i's pre-merger pricing pressure. Then, let $\left(\frac{p_i - c_i}{p_i} + \frac{1}{\epsilon_{ii}} \right)^{post}\Big|_{p_i^{pre}}$, which equals λ_i, $GUPPI_i$, or θ_i, depending on whether i and i's former rival are constrained, denote i's post-merger pricing pressure, evaluated at the pre-merger equilibrium. Then, we have:

$$ccGUPPI_i = \left(\frac{p_i - c_i}{p_i} + \frac{1}{\epsilon_{ii}} \right)^{post}\Bigg|_{p_i^{pre}} - \left(\frac{p_i - c_i}{p_i} + \frac{1}{\epsilon_{ii}} \right)^{pre}\Bigg|_{p_i^{pre}} \tag{13}$$

Definition 3 explicitly defines $ccGUPPI$ in terms of $GUPPI$, θ, λ, and pre-merger constraints, all of which can be calculated using only the merging firms' pre-merger margins, prices, diversion ratios, and demand elasticities. In so defining $ccGUPPI$ we note that there are nine possible cases (e.g., both firms constrained before, neither constrained after), which we separate by row. We number the cases arbitrarily, and further name them by a 2×2 matrix, whose element $(i, 1)$ equals 1 if firm i is constrained prior to the merger and 0 otherwise, and whose element $(i, 2)$ equals 1 if firm i is constrained post-merger and 0 otherwise, for $i = 1, 2$. The first two columns of the table in definition 3 define criteria that, if met, imply the given qualitative (column 3) and quantitative (columns 4 and 5) predictions. By lemma 1, the list of criteria is comprehensive, although for simplicity we ignore the zero measure event of two or more of λ, θ, and $GUPPI$ being equal to one another.

Definition 3. *$ccGUPPI$ is defined in table 1. The first two columns describe our qualitative prediction of which pre-merger capacity constraints continue to bind post-merger, while the third and fourth column describe $ccGUPPI_1$ and $ccGUPPI_2$, respectively.*

Like other measures of upward pricing pressure, $ccGUPPI$ can be compared to the magnitude of any expected cost-saving efficiencies that would result from the merger to determine if the merger's net upward pricing pressure is positive or negative (see Werden (1996) for a discussion of comparing upward pricing pressure to efficiencies). We say that a merger of one or more capacity-constrained firms has positive pricing pressure net of efficiencies if $ccGUPPI_i > \frac{\Delta c_i}{p_i}$ for merging firm i. Such mergers will generate a price increase, regardless of the particulars of how cost increases are passed through to consumers.

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Pre-merger constraints	Pricing pressure criteria	Case		$ccGUPPI_1$	$ccGUPPI_2$
$q_1 = K_1$ $q_2 < K_2$	$\lambda_1 > GUPPI_1$	1	$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$	0	θ
$q_1 = K_1$ $q_2 < K_2$	$GUPPI_1 > \lambda_1$	2	$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$	$GUPPI_1 - \lambda_1$	$GUPPI_2$
$q_1 = K_1$ $q_2 = K_2$	$\lambda_1 > GUPPI_1$ $GUPPI_2 > \theta_2 > \lambda_2$	3	$\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$	0	$\theta_2 - \lambda_2$
$q_1 = K_1$ $q_2 = K_2$	$\theta_1 > GUPPI_1 > \lambda_1$ $\theta_2 > GUPPI_2 > \lambda_2$	4	$\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$	$GUPPI_1 - \lambda_1$	$GUPPI_2 - \lambda_2$
$q_1 < K_1$ $q_2 = K_2$	$GUPPI_2 > \lambda_2$	5	$\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$	$GUPPI_1$	$GUPPI_2 - \lambda_2$
$q_1 = K_1$ $q_2 = K_2$	$\lambda_i > \theta_i > GUPPI_i$ $GUPPI_j > \lambda_j > \theta_j$ for $i, j \in \{1, 2\}$	6	$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$	0	0
$q_1 = K_1$ $q_2 = K_2$	$GUPPI_1 > \theta_1 > \lambda_1$ $\lambda_2 > GUPPI_2$	7	$\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$	$\theta_1 - \lambda_1$	0
$q_1 < K_1$ $q_2 = K_2$	$\lambda_2 > GUPPI_2$	8	$\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$	θ_1	0
$q_1 < K_1$ $q_2 < K_2$	none	9	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$GUPPI_1$	$GUPPI_2$

Table 1: Qualitative (column 3) and quantitative (columns 4-5) predictions of upward pricing pressure for each of a comprehensive set of criteria (columns 1-2). If we assume the identity pass-through matrix describes how firms pass cost through to price, columns 4-5 also describe predicted $\frac{\Delta p_i}{p_i}$ for merging firms $i = 1, 2$. The pricing pressure terms are defined as follows: $\lambda_i = \frac{p_i - c_i}{p_i} + \frac{1}{\epsilon_{ii}}$, $GUPPI_i = D_{ij} \frac{p_j}{p_i} \frac{p_j - c_j}{p_j}$, and $\theta_i = m_i D_{ij} D_{ji} - \frac{p_j}{p_i} D_{ij} \frac{1}{\epsilon_{jj}}$

To employ $ccGUPPI$ as a predictor of merger price effects, we pre-multiply the vector of $ccGUPPI$ terms by an $N \times N$ pass-through matrix describing how changes in each firm's costs are passed through to each firm's price. While Jaffe and Weyl (2013) usefully characterize the optimal pass-through matrix as a function of the demand curve, the Jaffe and Weyl pass-through matrix is difficult to implement in practice. For this reason, we follow Miller et al. (2017) in approximating the true pass-through matrix with the identity, irrespective of the demand system. Using an identity pass-through, our prediction for $\frac{\Delta p_i}{p_i}$ is simply $ccGUPPI_i$. Section 5 conducts a series of Monte Carlo experiments in which we compare the identity times $ccGUPPI$ to true merger price effects across a variety of simulated industries. In doing so, we test the usefulness of three approximations: evaluating pricing pressure terms λ, $GUPPI$, θ using pre-merger information, qualitative predictions about which pricing constraints continue to bind post-merger, and the identity as a proxy for pass-through. Overall, the simulations appear to offer strong support for the usefulness of $ccGUPPI$ in predicting merger price effects.

One unrealistic result of using the identity as a pass-through matrix is that firms with zero pricing pressure have no predicted price increase. Absent capacity constraints, this takes the form of a prediction that non-merging firms will not increase price, even if their newly-merged rivals do so. Here, we encounter the additional problem that when exactly one merging firm (say, firm 2) is predicted to be constrained following the merger, that firm too will have $ccGUPPI = 0$. Despite this, firm 2 would clearly increase price in our example, as some of firm 1's lost customers would divert to firm 2, driving up demand and causing it to increase its price to maintain demand equal to capacity. Unlike in the case of non-merging firms, it is straightforward to describe by how much firm 2's price will increase, given information on firm 1's price increase, demand elasticities, and relative prices. Specifically, if pass-through of costs to non-merging firms is zero (as assumed when using the identity pass-through), we have that a constrained firm 2 would increase price in response to an unconstrained firm 1 raising price by approximately:

$$\frac{\Delta p_2}{p_2} \approx -\frac{\frac{\partial q_2}{\partial p_1}}{\frac{\partial q_2}{\partial p_2}} ccGUPPI_1 \frac{p_1}{p_2} \tag{14}$$

The derivation is given in a footnote.[10] Thus, we can adjust pass-through when firm 1 is predicted to be unconstrained and firm 2 constrained post-merger, so that the entry in column 2 row 1 is $-\frac{\frac{\partial q_2}{\partial p_1}}{\frac{\partial q_2}{\partial p_2}} \frac{p_1}{p_2}$, with similar adjustments in other relevant regions. Using this adjusted pass-through matrix, table 8,

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[10]In our example, firm 2 sets p_2 equal to $q_2^{-1}(K_2, \boldsymbol{p})$, or

$$q_2(p_1, p_2, ...) \equiv K_2 \tag{15}$$

Setting second order terms $\frac{\partial p_j}{\partial p_1}$ to zero for $j > 2$, and taking the derivative of both sides of (15) with respect to p_1, we

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displayed in the appendix, describes revised predictions for $\frac{\Delta p}{p}$ based on $ccGUPPI$ and the pass-through matrix implied by equation (14).

In our Monte Carlo simulations, we do not find that results differ drastically depending on whether we use predicted price increases from table 1 or table 8. Hence, in the next section, we present results using only the identity pass-through, and the predictions of table 1. The appropriate choice of pass-through in an antitrust setting is an important and understudied question (with Jaffe and Weyl (2013) and Miller et al. (2017) being the principal exceptions).

5   Monte Carlo Experiment

The previous section developed analytical results balancing upward price pressure from a merger (which occurs because the merged firm internalizes the pricing externality between substitute goods that were previously owned by different firms) and upward price pressure from capacity constraints (which occurs because constrained firms are incentivized to increase price until quantity demanded equals capacity). In this section, we develop Monte Carlo experiments that provide numerical evidence on the extent to which capacity constraints on merging firms attenuate merger price effects and demonstrate that $ccGUPPI$ is a useful tool for predicting the price effects of mergers between capacity-constrained firms.

The experiments generate a dataset where each observation, or random draw of data, represents an industry consisting of four firms. We calibrate three different demand systems (linear, logit, and AIDS) with each draw of data or industry. While the demand systems differ in functional form, firms have the same pre-merger prices, quantities, margins, and demand elasticities under each demand system. We also randomly assign capacity constraints to each firm in a given industry. The resulting dataset allows us to examine the price effect of mergers between capacity constrained firms and the accuracy of $ccGUPPI$ in predicting those effects under a wide range of market conditions.

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have:

$$\frac{\partial q_2}{\partial p_2}\frac{\partial p_2}{\partial p_1} + \frac{\partial q_2}{\partial p_1} = 0$$

$$\Rightarrow \frac{\partial p_2}{\partial p_1} = -\frac{\frac{\partial q_2}{\partial p_1}}{\frac{\partial q_2}{\partial p_2}} \tag{16}$$

We then multiply $\frac{\partial p_2}{\partial p_1}$ by the level of the change in p_1, $ccGUPPI_1 * p_1$ (assuming each element of the diagonal of the pass-through matrix is 1). $-\frac{\frac{\partial q_2}{\partial p_1}}{\frac{\partial q_2}{\partial p_2}} * ccGUPPI_1/p_1$ gives the level of price change for firm 2, and dividing by p_2 yields the percent price increase. Symmetric calculations apply to settings in which firm 2 is unconstrained and firm 1 constrained post-merger.

5.1 Data generating process

Our data generating processes is adapted from that of Miller et al. (2016 and 2017). First, we randomly draw market shares for four firms and an outside good. Next, we randomly assign each firm to be either capacity-constrained or not, excluding draws where all firms are constrained. Then, we draw the margin of a single unconstrained firm.

The margin of the unconstrained firm, market shares, and price (which we normalize to one) are sufficient to calibrate a logit demand system. Then, we calibrate linear and AIDS demand systems using the market shares, prices, and demand slopes from the logit calibration. Each unconstrained firm has marginal costs implied by marginal revenue (just as it would in a model without capacity constraints) and each constrained firm has marginal costs drawn randomly to exceed marginal revenue.

Finally, we assume a merger between firms 1 and 2, and compute the optimal post-merger price vectors using the calibrated demand systems and marginal cost vector. Thus, each industry, or draw of data, has three post-merger price vectors: one for each demand system.

The specific steps of the data generating process are as follows:

1. Draw shares s_i for 4 firms and the outside good by drawing $x_i \sim U[0,1]$ for $i = 1, ..., 5$ and setting $s_i = \frac{x_i}{\sum_j x_j}$. Normalize each firm's price to $p_i = 1$.

2. Label each firm as being unconstrained or constrained via 4 independent draws from a Bernoulli distribution with parameter $\frac{1}{2}$. If a firm is constrained, set $K_i = s_i$. If a firm is unconstrained, set $K_i > 1$. If all four firms are capacity-constrained, discard observation.

3. If firm $\tilde{i}$ is the lowest-numbered firm that is unconstrained, draw firm $\tilde{i}$'s margin $m_{\tilde{i}}$ from a $U[.2, .8]$ distribution.

4. Shares s_i, $i = 1, ..., 4$ and margin $m_{\tilde{i}}$ are sufficient to calculate the five parameters of a logit demand system. These parameters determine own and cross elasticities of demand, ϵ_{ii} and ϵ_{ij} for all firms.

5. If firm i is unconstrained, profit maximization implies its margin is given by $m_i = -\frac{1}{\epsilon_{ii}}$. If firm i is constrained, by definition its margin exceeds $\frac{1}{\epsilon_{ii}}$. In this case, draw margin as follows: $m_i \sim U[-\frac{1}{\epsilon_{ii}}, 1]$. Note that $\lambda_i = m_i + \frac{1}{\epsilon_{ii}}$ for each constrained firm, as defined in section 3.1.

6. The shares s_i and logit elasticities ϵ_{ij} imply unique parameterizations of AIDS and linear demand systems, following Appendix A of Miller et al. (2017). Pre-merger price, quantity, own and cross elasticities, capacity constraints, and margins are identical across all three demand systems.

7. Calculate profit-maximizing prices following a merger of firms 1 and 2 under each of three demand systems by applying the first-order conditions discussed in section 3.2.

8. Calculate $ccGUPPI$ and $GUPPI$ using pre-merger information on margins, shares, capacities, and elasticities (but not demand parameters).

9. Repeat until 10,000 industries are generated, discarding the small number of industries that fail to calibrate.

The resulting dataset has 10,000 observations, or industries. Per step 7, each observation or industry also has three predicted merger price effects specified by a functional form of demand. Per step 8, each industry has one value of $ccGUPPI$ and one value of $GUPPI$.

Table 2 summarizes the simulated data. It reports order statistics for firm 1's market share, margin, and demand elasticity. The distributions of these variables for the other firms are essentially the same, because the data generating process is the same for all firms. The median market share, 20.2 percent, reflects that there are four firms and an outside good. The median margin is 56 percent and the median elasticity is 2.1. Note that the traditional Lerner index relationship between margins and elasticity does not hold for capacity-constrained firms, as these firms have marginal cost below marginal revenue. The median diversion ratio from firm 1 to 2 is 25.2%.

	p50	p10	p25	p75	p90
Market Share	0.202	0.050	0.115	0.277	0.340
Own-Price Elasticity	2.139	1.382	1.621	2.987	3.978
Diversion Ratio	0.252	0.065	0.148	0.338	0.419
Margin	0.557	0.295	0.408	0.678	0.769
ccGUPPI	0.071	0.000	0.003	0.155	0.235
GUPPI	0.126	0.027	0.064	0.197	0.270
Logit Price Effect	0.070	0.003	0.022	0.143	0.221
Linear Price Effect	0.060	0.002	0.018	0.122	0.195
AIDS Price Effect	0.127	0.004	0.034	0.359	0.843

Table 2: Order statistics

Table 2 also shows the upward price pressure and simulated price effect for firm 1. The median $GUPPI$ is 12.6 percent, while the median $ccGUPPI$ is only 7.1 percent. The latter is smaller than the
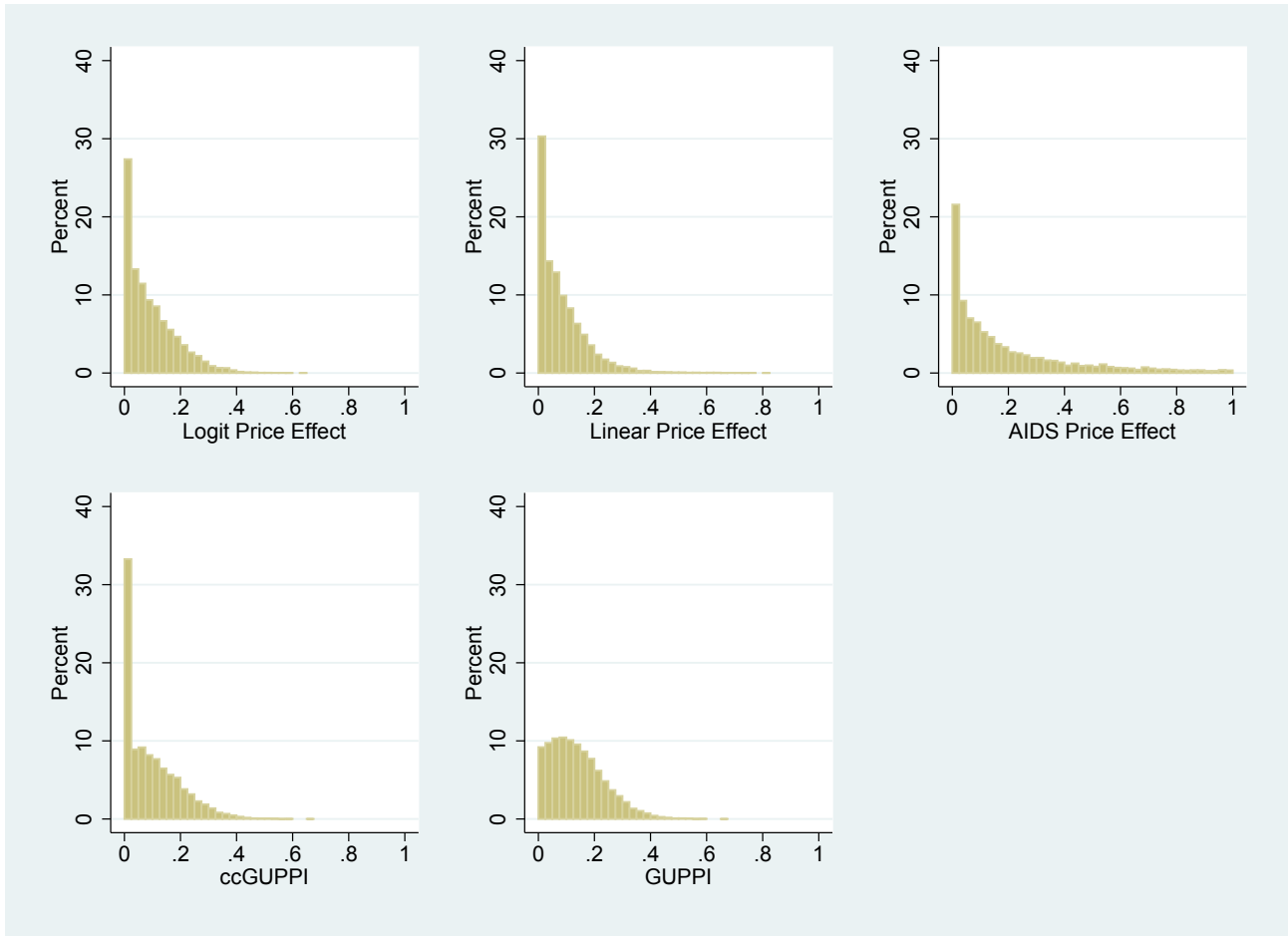
Figure 2: Distribution of $ccGUPPI$, $GUPPI$, and actual merger price effects

former whenever capacity constraints bind before the merger and put upward pressure on prices. The median price effects are 7.0, 6.0, and 12.7 percent under logit, linear, and AIDS demand respectively. The relative size of these simulated price effects are consistent with those found by Miller et al. (2016 and 2017) and Crooke et al. (1999). The greater curvature of the AIDS system generates larger price effects, all else equal.

Figure 2 illustrates the empirical distribution of $ccGUPPI$, $GUPPI$, and the simulated price effects. The graphs are standard histograms with fixed bin widths of 0.025 over the range 0 to 1. They show the full distribution of the linear and logit price effects, as well as $ccGUPPI$ and $GUPPI$ values. The right tail of the AIDS price effect histogram, which includes about 5 percent of observations, does not appear on the graph.

The histograms confirm that the distribution of $ccGUPPI$ is similar to those of the linear and logit simulated price effects. The distribution of the AIDS price effects has the same general shape but a much longer and thicker right tail. The $GUPPI$ distribution has a fundamentally different shape

because $GUPPI$ is strictly positive for every industry, whereas $ccGUPPI$ is 0 for industries with tightly-binding capacity constraints. The center of mass is also further to the right because none of the values are attenuated by the pre-merger upward price pressure from binding capacity constraints.

Not apparent from the histograms, is the fact that the linear, logit, and AIDS price effects all equal zero for the same 724 observations. These observations represent the industries where both of the merging firms are capacity constrained before and after the merger, and proposition 2 implies neither firm will raise prices. Firm one's $ccGUPPI$ is also zero for those 724 observations. There are an additional 1,681 observations where $ccGUPPI$ is zero because firm one is constrained before and after the merger and firm two is not. In these instances, there is no direct upward price pressure, because firm one continues to set a price where demand equals capacity after the merger, but firm one's price increases because its demand curve shifts out as firm two raises price and reduces output.[11]

5.2 Descriptive Analysis

Table 3 shows predicted merger price effects using $ccGUPPI$ and $GUPPI$ (and multiplying each by an identity pass-through matrix, as described in table 1) and simulated price effects for firm 1 in each of the nine possible cases defined by the pre- and post-merger capacity constraints. In cases 9 and 5, where firm 1 is unconstrained, $ccGUPPI$ and $GUPPI$ (and thus price predictions under each) are identical. Consistent with the results of Miller et al. (2017), the median upward price pressure is close to the median linear and logit price effects and smaller than the median AIDS price effect.

In cases 1 and 3, where product 1 is constrained before and after the merger, $ccGUPPI$ is zero, reflecting the fact that the merger does not change the price-setting equation of product 1. There are, however, simulated price increases, as the merged firm has an incentive to raise p_2 because of the upward price pressure represented by θ_2. A higher price on product 2 then shifts demand for product 1, resulting in a higher equilibrium price for product 1.[12]

In case 6, both firms are constrained before and after the merger, and by proposition 2 $ccGUPPI$ accurately predicts zero price effect for each of the three demand systems. Standard $GUPPI$, which ignores the capacity constraints, is positive for all industries under case 6. In the other cases (2, 4, 7, and 8) the median $ccGUPPI$ is close to those of the logit and linear price effects, and noticeably less than the median AIDS price effect. In addition, the median $ccGUPPI$ is less than that of standard $GUPPI$ because from former accounts for capacity constraints.

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[11]Note that were we to apply the pass-through matrix described in table 8, the value of pass-through multiplied by $ccGUPPI$ would equal 0 in the 724 industries with zero simulated price effect, and only these industries.

[12]Again, this "feedback'' effect is included when using the pass-through matrix described in table 8, but not when using that described in table 1.

Case		ccGUPPI	GUPPI	Simulated Price Increase		
				Linear	Logit	AIDS
1	$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$	0.000	0.074	0.018	0.015	0.027
2	$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$	0.084	0.167	0.096	0.090	0.264
3	$\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$	0.000	0.069	0.011	0.008	0.018
4	$\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$	0.126	0.216	0.121	0.113	0.404
5	$\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$	0.163	0.163	0.139	0.118	0.361
6	$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$	0.000	0.089	0.000	0.000	0.000
7	$\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$	0.069	0.178	0.060	0.040	0.084
8	$\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$	0.101	0.130	0.086	0.058	0.108
9	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	0.104	0.104	0.097	0.083	0.182

Table 3: Median ccGUPPI, GUPPI, and simulated price effects, for each of nine cases of capacity constraints binding pre- and post-merger.

	Demand System		
	Linear	Logit	AIDS
Constraints on Merging Firms			
Neither	0.083	0.097	0.182
One	0.063	0.075	0.137
Both	0.020	0.025	0.043
Constraints on Merging Firms' Rivals			
Neither	0.056	0.064	0.111
One	0.059	0.068	0.126
Both	0.067	0.080	0.145

Table 4: Median simulated price effect under constraints on merging firms and constraints on rivals.

Next, we consider the practical importance of accounting for capacity constraints during merger reviews. Our motivation is the fact that Froeb et al. (2003) argue that capacity constraints on merging firms attenuate merger effects more than capacity constraints on non-merging firms amplify them, and thus policy makers should be particularly concerned about the former.

Table 4 lists the median price effects for firm 1 based on which firms are capacity constrained before the merger. The top half of the table lists the median price effects under each demand system after separating the data into three groups of observations based on whether both, one, or neither of the merging firms are constrained. [13] The bottom half of table 4 lists the median price effects when both, one, or neither of the merging firms rivals are constrained. Clearly, constraints on the merging firms tend to attenuate merger price effects, while constraints on the merging firms' rivals tend to amplify them. The relative importance of constraints on merging firms versus those on rivals, however, is entirely case-specific.

In our dataset, capacity constraints on merging firms indeed do lower merger price effects more than capacity constraints on non-merging firms raise them, yet this is only an average effect, and not true for each individual industry. Further, even the average effect is entirely dependent on our data generating process. If we changed the data generating process, so that the merging firms were less tightly constrained (i.e., smaller values of λ_i, the difference between margin and inverse elasticity), then capacity constraints on merging firms would have a smaller attenuating effect.

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[13]The row labeled as one merging firm constrained includes instances where firm1 is constrained and firm 2 is not, as well as instances where firm 2 is constrained to firm 1 is not.

	Demand System		
λ	Linear	Logit	AIDS
$\lambda_i < 0.3$	0.027	0.032	0.062
$\lambda_i < 0.2$	0.051	0.060	0.138
$\lambda_i < 0.1$	0.101	0.123	0.365

Table 5: Merger Price Effects and the Tightness of Capacity Constraints. $\lambda_i = m_i + \frac{1}{\epsilon_{ii}}$ refers to how tightly a capacity constraint binds. Each row displays median price effect under each demand systems for the subset of data with λ_i less than .3, .2, and .1, respectively, for $i = 1, 2$.
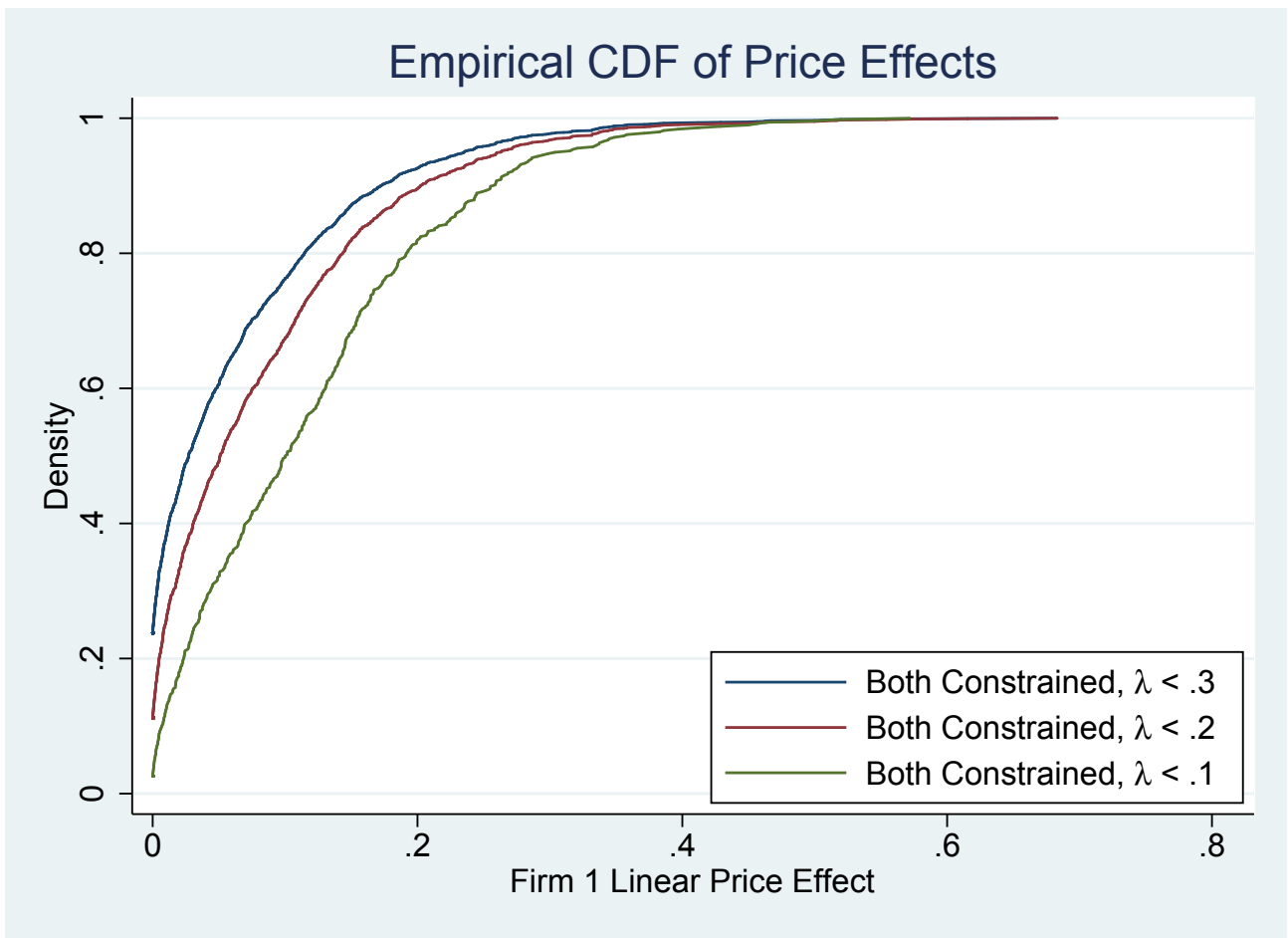

Figure 3: CDF of price effects when both merging firms are constrained.

Table 5 shows how the median price effects change when we restrict the data based on the values of λ_1 and λ_2, measuring how tightly firms 1 and 2 are constrained. The first row of table 5 shows the median price effects when we restrict the dataset to observations where both of the merging firms are constrained before the merger and $\lambda_i < .3$ for $i = 1, 2$. In the second row we restrict the dataset further, to otherwise similar observations where λ_i is less than .2, and we see that the median price effects increase. In the third row, the median price effects increase further because the data are restricted to observations where λ_i is less than .1. Figure 3 shows the empirical CDF of the linear demand price effects for each subset of data shown in table 5.

Table 5 and figure 3 both illustrate the logic behind $ccGUPPI$: capacity-constrained firms have pre-merger prices that are elevated relative to what they would be absent the constraints. Constraints can be more or less tightly binding, depending on the magnitude of the price increase required so that quantity demanded equals capacity, with tighter constraints causing prices that are elevated to a greater degree. As merger price effects reflect the difference between incentive to increase price stemming from the merger ($GUPPI$ or θ) and the price increase caused by a capacity constraint, these price effects are decreasing in the latter. In an applied setting, measuring net pricing pressure depends on measuring the difference between marginal costs and marginal revenue (λ).

5.3 Accuracy of $ccGUPPI$

This section addresses how well $ccGUPPI$ predicts actual merger effects under different functional forms of demand. We find that it does a better job predicting merger effects when demand is linear or logit than it does when demand is AIDS, where $ccGUPPI$ tends to under predict the magnitude of price effects. Overall, we find that $ccGUPPI$ offers more accurate and precise predictions of merger price effects than $GUPPI$, based on our simulated dataset. We assess accuracy based on median absolute prediction error and precision based on the standard deviation of prediction errors.

First, we evaluate $ccGUPPI$'s accuracy as a predictor of merger price effects graphically, using the identity pass-through. The graphs in figure 4 each plot either $ccGUPPI$ or $GUPPI$ on the vertical axis and the simulated merger price effect under a specific demand systems on the horizontal axis. A 45-degree reference line indicates exact predictions, where $ccGUPPI$ or $GUPPI$ equals the simulated price increase.

Under logit and linear demand, $ccGUPPI$ is quite accurate with the dots tightly dispersed around the 45-degree line. [14] In contrast, standard $GUPPI$ is systematically biased upward, with the dots

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[14]The line of dots clustered along the horizontal axis represent observations where firm 1 is constrained before and after the merger but firm 2 is not. These dots shift up closer to the 45-degree line if we use the alternative pass through matrix contemplated in table 8 of the appendix.
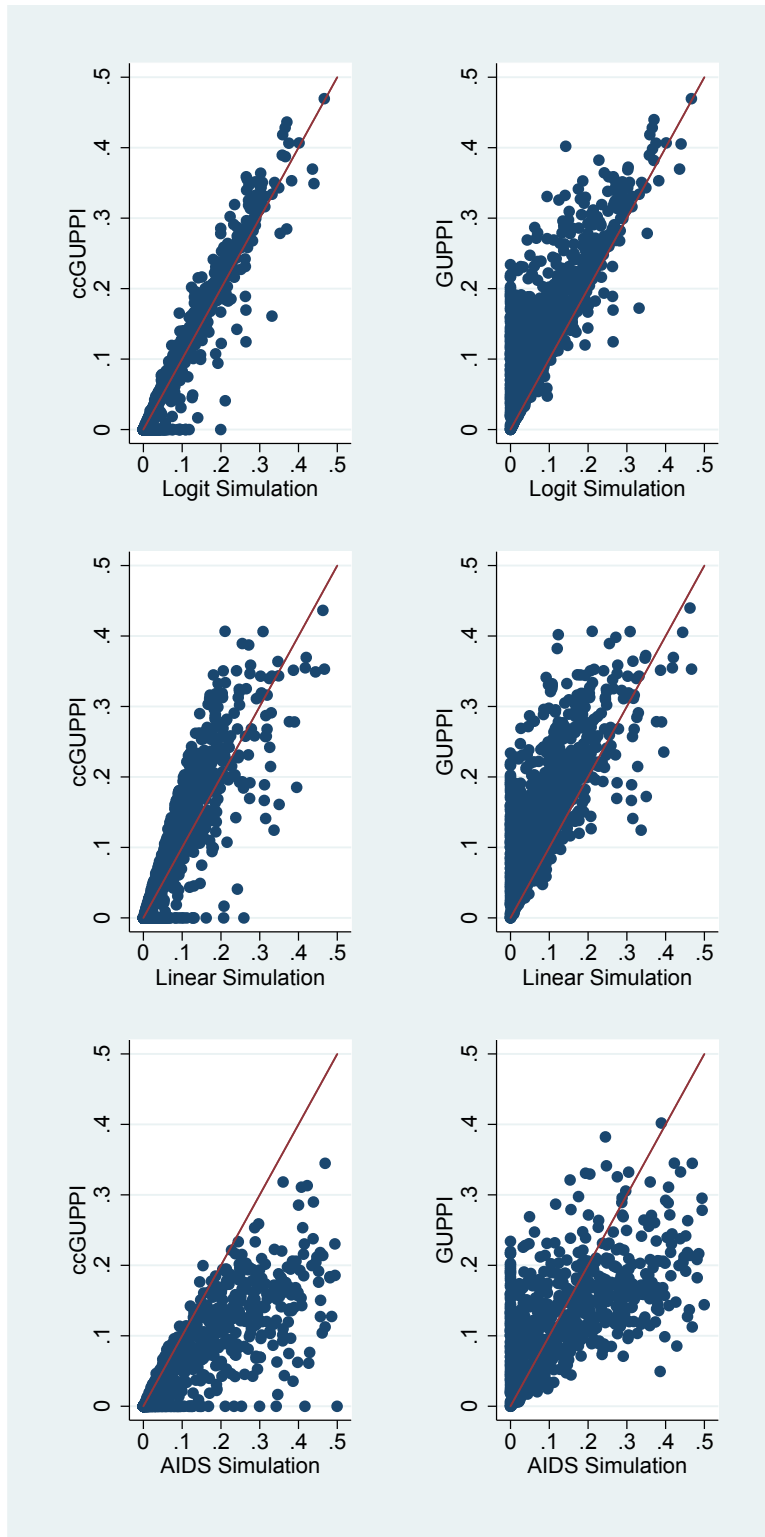
Figure 4: $ccGUPPI$ and $GUPPI$ price predictions (y-axis) versus simulated price effect (x-axis).

	Demand System		
	Linear	Logit	AIDS
Median Prediction Error			
ccGUPPI	0.007	0.003	-0.051
GUPPI	0.046	0.027	-0.019
Standard Deviation of Prediction Error			
ccGUPPI	0.053	0.028	5.019
GUPPI	0.064	0.054	5.018
Median Absolute Prediction Error			
ccGUPPI	0.021	0.009	0.051
GUPPI	0.050	0.029	0.069

Table 6: Prediction Error of $ccGUPPI$ and $GUPPI$ relative to merger simulation under linear, logit, and AIDS demand.

dispersed above the reference line. The line of dots along the vertical axis represents observations where the merging firms are constrained before and after the merger. Under AIDS demand, $ccGUPPI$ under-predicts simulated price effects, consistent with Miller et al. (2016 and 2017) who find that standard $GUPPI$ and the identity pass-through matrix tend to under-predict price increases with AIDS demand absent capacity constraints. Under AIDS demand, $GUPPI$ predictions are widely dispersed, as $GUPPI$ over-predicts the actual price increase when capacity constraints bind tightly and under-predicts prices effects capacity constraints do not bind.

Next, we evaluate $ccGUPPI$ and $GUPPI$'s accuracy as a predictor of merger price effects numerically. Define the prediction error as $ccGUPPI$ or $GUPPI$ minus the analytically derived price increase under a specific demand system. Table 6 shows the median prediction error, the standard deviation of the prediction error, and the median absolute prediction error of $ccGUPPI$ and $GUPPI$ relative to each demand system.

The median prediction error confirms that $ccGUPPI$ is a good predictor of price effects under linear and logit demand, but tends to under-predict price increases under AIDS demand. In addition, the prediction error of $GUPPI$ has a higher standard deviation than $ccGUPPI$ under linear of logit demand, and is roughly the same under AIDS.[15] Perhaps more importantly, if the underlying demand

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[15]The standard deviation of the $ccGUPPI$ prediction error under AIDS demand is 0.40 if we exclude the top 1 percent of prediction error values. The standard deviation of the $GUPPI$ prediction error under AIDS is 0.41 if we exclude the

is either linear, logit, or AIDS, one can be confident that $ccGUPPI$ is either relatively accurate (under linear or logit) or under-predicts price effects (under AIDS). By comparison, there is no way to predict the likely sign of the prediction error with standard $GUPPI$. Finally, the median absolute prediction error statistics again suggest that $ccGUPPI$ out performs $GUPPI$. Under all demand systems, $ccGUPPI$ has a lower median absolute error than standard $GUPPI$.

Antitrust agencies may also want to flag mergers whose price effects will likely be greater than a specified threshold. Following Miller et al. (2017) we consider a test to screen out all mergers likely to generate a price increase greater than 10 percent, as predicted by $ccGUPPI$ or standard $GUPPI$. For each observation in the simulated data, we determine whether $ccGUPPI$ and standard $GUPPI$ exceed ten percent. A false positive, or Type II error, means that the $ccGUPPI$ or $GUPPI$ of at least one of the merging products is greater than ten percent while the actual price effect of both merging products is less than ten percent. A false negative, or Type I error, means that the $ccGUPPI$ or $GUPPI$ of both products is less than ten percent and the actual price effect of at least one product is greater than ten percent.

Table 7 summarizes the frequency of type I and type II errors. The prevalence of type I errors is clearly lower for $ccGUPPI$ than standard $GUPPI$. This is obviously because standard $GUPPI$ over-predicts price effects when firms are capacity constrained. The prevalence of type II errors is similar for $GUPPI$ and $ccGUPPI$ under linear and logit demand, and lower for standard $GUPPI$ under AIDS. This is in part explained by instances where standard $GUPPI$ generates larger price effects because it does not account for capacity constraints. In essence, standard $GUPPI$ generates fewer false positives by mistake, because it does not account for capacity constraints. Overall, $ccGUPPI$ generates substantially fewer total type I and II errors under logit, linear, and AIDS demand.

6 Conclusion

This paper provides antitrust practitioners with a simple tool to evaluate mergers involving one or more capacity-constrained firms. Simulated data from our Monte Carlo experiments suggest that $ccGUPPI$ performs better than standard $GUPPI$, and is a quite accurate predictor of merger price effects when demand is linear or logit, and a lower bound on price effects under AIDS demand. We now briefly discuss two caveats.

First, capacity constraints are necessarily transitory. It is entirely appropriate for antitrust agencies to consider additional capacity that is about to become available. If the merging firms are deemed likely to build additional capacity in the near future, $ccGUPPI$ can be used as part of a broader

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top 1 percent of the prediction error values.

	Demand System		
	Linear	Logit	AIDS
ccGUPPI			
Type 1 Error (False Positive)	0.190	0.046	0.010
Type 2 Error (False Negative)	0.001	0.002	0.073
GUPPI			
Type 1 Error (False Positive)	0.391	0.247	0.166
Type 2 Error (False Negative)	0.001	0.001	0.027

Table 7: Threshold Merger Screen Accuracy. An industry generates a false positive if $ccGUPPI$ or $GUPPI$ exceeds .1, while the simulated price increase is below 10%. An industry generates a false negative if $ccGUPPI$ of $GUPPI$ is below 10%, while the simulated price increase exceeds 10%.

analysis that considers both short- and long-run effects. Notably, so long as the putative capacity expansion is not merger-specific, an increase in capacity exacerbates merger price effects for capacity-constrained firms. This is because such an increase would lower the constrained firm's (long run) pre-merger price. In the language of the paper, this would decrease λ without affecting $GUPPI$ or θ.

Second, implementing $ccGUPPI$ requires one additional piece of information not required for a traditional $GUPPI$ calculation. Specifically, one needs to know the price elasticity of demand, or, equivalently, the difference between marginal revenue or marginal costs (λ). Identifying the price elasticity of demand econometrically is difficult given it requires exogenous variation in price. Doing so when firms are capacity constrained can be even more challenging given some consumers may face a truncated choice set (see for example Conlon and Mortimer (2013)). Nevertheless, antitrust agencies can supplement econometric evidence with deposition testimony, data, or documents from industry participants regarding the price sensitivity of demand. Consumers' stated preferences and profit-maximizing firms' understanding of consumer preferences may provide valuable supplementary evidence. Finally, natural experiments such as unexpected supply disruptions that generate exogenous variation in the merging firms' prices might allow the agencies to identify the price elasticity of demand short of full demand system estimation.

Appendix

This appendix contains four items, a proof of lemma 1 (describing conditions on the ordering of pricing pressure terms λ, θ, and $GUPPI$), a proof of proposition 2 (giving exact conditions under which a merger of two capacity-constrained firms results in a price increase), table 8 (containing estimates of $\frac{\Delta p}{p}$ based on an alternative pass-through matrix implied by equation (14) and $ccGUPPI$), and a revised version of figure 4 using the predictions of table 1. We begin with the proof of lemma 1.

Proof of lemma 1

Item 1:

$$GUPPI_i > \theta_i$$

$$\iff \frac{p_j}{p_i} m_j D_{ij} > m_i D_{ij} D_{ji} - \frac{p_j}{p_i} D_{ij} \frac{1}{\epsilon_{jj}} \text{ (using the definitions of } GUPPI \text{ and } \theta)$$

$$\iff m_j > \frac{p_i}{p_j} m_i D_{ji} - \frac{1}{\epsilon_{jj}}$$

$$\iff \lambda_j > GUPPI_j \text{ (using the definitions of } \lambda \text{ and } GUPPI)$$

Item 2:

$$\lambda_i > \theta_i$$

$$\iff \lambda_i > m_i D_{ij} D_{ji} - \frac{p_j}{p_i} D_{ij} (\lambda_j - m_j) \text{ (using definitions of } \theta, \lambda)$$

$$\Rightarrow \lambda_i > m_i D_{ij} D_{ji} - \frac{p_j}{p_i} D_{ij} (\theta_j - m_j) \text{ (given } \theta_j > \lambda_j)$$

$$\iff \lambda_i > m_i D_{ij} D_{ji} + GUPPI_i - \frac{p_j}{p_i} D_{ij} \left(m_j D_{ij} D_{ji} - \frac{p_i}{p_j} D_{ji} \frac{1}{\epsilon_{ii}} \right) \text{ (using definitions of } \theta, GUPPI)$$

$$\iff \lambda_i > m_i D_{ij} D_{ji} + GUPPI_i - \frac{p_j}{p_i} m_j D_{ij}^2 D_{ji} + D_{ij} D_{ji} (\lambda_i - m_i) \text{ (using definition of } \lambda)$$

$$\iff \lambda_i (1 - D_{ij} D_{ji}) > GUPPI_i (1 - D_{ij} D_{ji}) \text{ (using definition of } GUPPI)$$

$$\iff \lambda_i > GUPPI_i$$

∎

Next, we turn to the proof of proposition 2:

Proof of proposition 2 Tautologically, there are six possible ways of ordering the quantities λ_i, θ_i, and $GUPPI_i$ for $i = 1, 2$, meaning there are 36 combinations of orders across the two merging firms. Lemma 1.1 rules out 26 of these combinations, while lemma 1.2 rules out an additional two. The

remaining eight possible combinations of orders are:

1. $\lambda_1 > \theta_1 > GUPPI_1$ and $GUPPI_2 > \lambda_2 > \theta_2$
2. $GUPPI_1 > \lambda_1 > \theta_1$ and $\lambda_2 > \theta_2 > GUPPI_2$
3. $\lambda_1 > GUPPI_1 > \theta_1$ and $\lambda_2 > GUPPI_2 > \theta_2$
4. $GUPPI_1 > \theta_1 > \lambda_1$ and $\lambda_2 > \theta_2 > GUPPI_2$
5. $\lambda_1 > \theta_1 > GUPPI_1$ and $GUPPI_2 > \theta_2 > \lambda_2$
6. $\theta_1 > \lambda_1 > GUPPI_1$ and $GUPPI_2 > \theta_2 > \lambda_2$
7. $\theta_1 > GUPPI_1 > \lambda_1$ and $\theta_2 > GUPPI_2 > \lambda_2$
8. $GUPPI_1 > \theta_1 > \lambda_1$ and $\theta_2 > \lambda_2 > GUPPI_2$

Thus, given lemma 1, the proposition states that the pre-merger price vector, p^*, remains an equilibrium following a merger of firms 1 and 2 under combinations 1, 2, or 3 (all of which have $\lambda_i > \theta_i$ for $i = 1, 2$), but not under combinations 4-8 (all of which have $\theta_i > \lambda_i$ for at least one i).

First, to show the "if" part of the claim, suppose that $\lambda_1 > \theta_1$ and $\lambda_1 > GUPPI_1$, as in combinations 1 and 3 (and no other feasible combinations). Suppose further, for the sake of argument, that firm 1 is unconstrained following the merger. Then, its first order condition would be either $m_1 = -\frac{1}{\epsilon_{11}} + GUPPI_1$ or $m_1 = -\frac{1}{\epsilon_{11}} + \theta_1$, depending on whether firm 2 is unconstrained or constrained post-merger. As margin $m_1 = \frac{p_1-c_1}{p_1}$ is increasing in p_1 and $-\frac{1}{\epsilon_{11}}$ decreasing in p_1 by assumption, the price that would satisfy either equation is less than firm 1's pre-merger price, p_1^*, meaning firm 1 would be capacity-constrained, absent an increase in p_2. Given that firm 1 prefers not to unilaterally lower its price, were firm 2 unconstrained post-merger, its first-order condition would be $m_2 = -\frac{1}{\epsilon_{22}} + \theta_2$. But, since $\theta_2 < \lambda_2$ in both cases 1 and 3, by the same logic as used above firm 2 could only satisfy this first-order condition at a price $p_2 < p_2^*$, but at this price firm 2 would be constrained post-merger. Thus, it follows that under combinations 1 and 3, both firms must continue to be constrained post-merger, so that post-merger prices are identical to pre-merger prices. A symmetric argument applies to combination 2, under which $\lambda_2 > \theta_2$, $\lambda_2 > GUPPI_2$, and $\lambda_1 > \theta_1$ (an ordering which appears in no other feasible combination).

Next, to show the "only if" part of the claim, under combinations 4-8, $\theta_i > \lambda_i$ for at least one $i \in \{1, 2\}$. Given that margin m_i is increasing in p_i and inverse elasticity $-\frac{1}{\epsilon_{ii}}$ decreasing, i can set a post-merger price $p_i > p_i^*$ satisfying (17) below, at which firm i is unconstrained:

$$\frac{p_i^{post} - c_i}{p_i^{post}} = -\frac{1}{\epsilon_{ii}} + \theta_i \tag{17}$$

Given the assumptions on demand, such a price maximizes firm i's profit, contradicting the premise that firm i is constrained following the merger. ∎

Table 8 describes our alternative estimates of $\frac{\Delta p}{p}$ using $ccGUPPI$ and the pass-through matrix implied by equation (14). Finally, figure 5 reproduces figure 4 using the predictions for $\frac{\Delta p}{p}$ from table 8. This removes the cluster of points along the horizontal axis in figure 4 where $ccGUPPI$ is zero because firm 1 is constrained before and after the merger, yet still raises price because firm 2 is not constrained after the merger (see footnote 14). Otherwise the joint distributions of $ccGUPPI$, $GUPPI$, and the three simulated price increases appear to be substantively identical to those in the main body of the paper, which use an identity pass-through.

Pre-merger constraints	Pricing pressure criteria	Case		pass-through $*$ $ccGUPPI_1$	pass-through $*$ $ccGUPPI_2$
$q_1 = K_1$ $q_2 < K_2$	$\lambda_1 > GUPPI_1$	1	$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$	$-\dfrac{\frac{\partial q_1}{\partial p_2}}{\frac{\partial q_1}{\partial p_1}}\dfrac{p_2}{p_1}\theta_2$	θ_2
$q_1 = K_1$ $q_2 < K_2$	$GUPPI_1 > \lambda_1$	2	$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$	$GUPPI_1 - \lambda_1$	$GUPPI_2$
$q_1 = K_1$ $q_2 = K_2$	$\lambda_1 > GUPPI_1$ $mathitGUPPI_2 > \theta_2 > \lambda_2$	3	$\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$	$-\dfrac{\frac{\partial q_1}{\partial p_2}}{\frac{\partial q_1}{\partial p_1}}\dfrac{p_2}{p_1}(\theta_2 - \lambda_2)$	$\theta_2 - \lambda_2$
$q_1 = K_1$ $q_2 = K_2$	$\theta_1 > GUPPI_1 > \lambda_1$ $\theta_2 > GUPPI_2 > \lambda_2$	4	$\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$	$GUPPI_1 - \lambda_1$	$GUPPI_2 - \lambda_2$
$q_1 < K_1$ $q_2 = K_2$	$GUPPI_2 > \lambda_2$	5	$\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$	$GUPPI_1$	$GUPPI_2 - \lambda_2$
$q_1 = K_1$ $q_2 = K_2$	$\lambda_i > \theta_i > GUPPI_i$ $GUPPI_j > \lambda_j > \theta_j$ for $i, j \in \{1, 2\}$	6	$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$	0	0
$q_1 = K_1$ $q_2 = K_2$	$GUPPI_1 > \theta_1 > \lambda_1$ $\lambda_2 > GUPPI_2$	7	$\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$	$\theta_1 - \lambda_1$	$-\dfrac{\frac{\partial q_2}{\partial p_1}}{\frac{\partial q_2}{\partial p_2}}\dfrac{p_1}{p_2}(\theta_1 - \lambda_1)$
$q_1 < K_1$ $q_2 = K_2$	$\lambda_2 > GUPPI_2$	8	$\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$	θ_1	$-\dfrac{\frac{\partial q_2}{\partial p_1}}{\frac{\partial q_2}{\partial p_2}}\dfrac{p_1}{p_2}\theta_1$
$q_1 < K_1$ $q_2 < K_2$	none	9	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$GUPPI_1$	$GUPPI_2$

Table 8: Predictions of $\frac{\Delta p}{p}$ using a revised pass-through matrix that accounts for a constrained merging firm increasing its price in response to an unconstrained former rival's price increase, as described in equation (14).
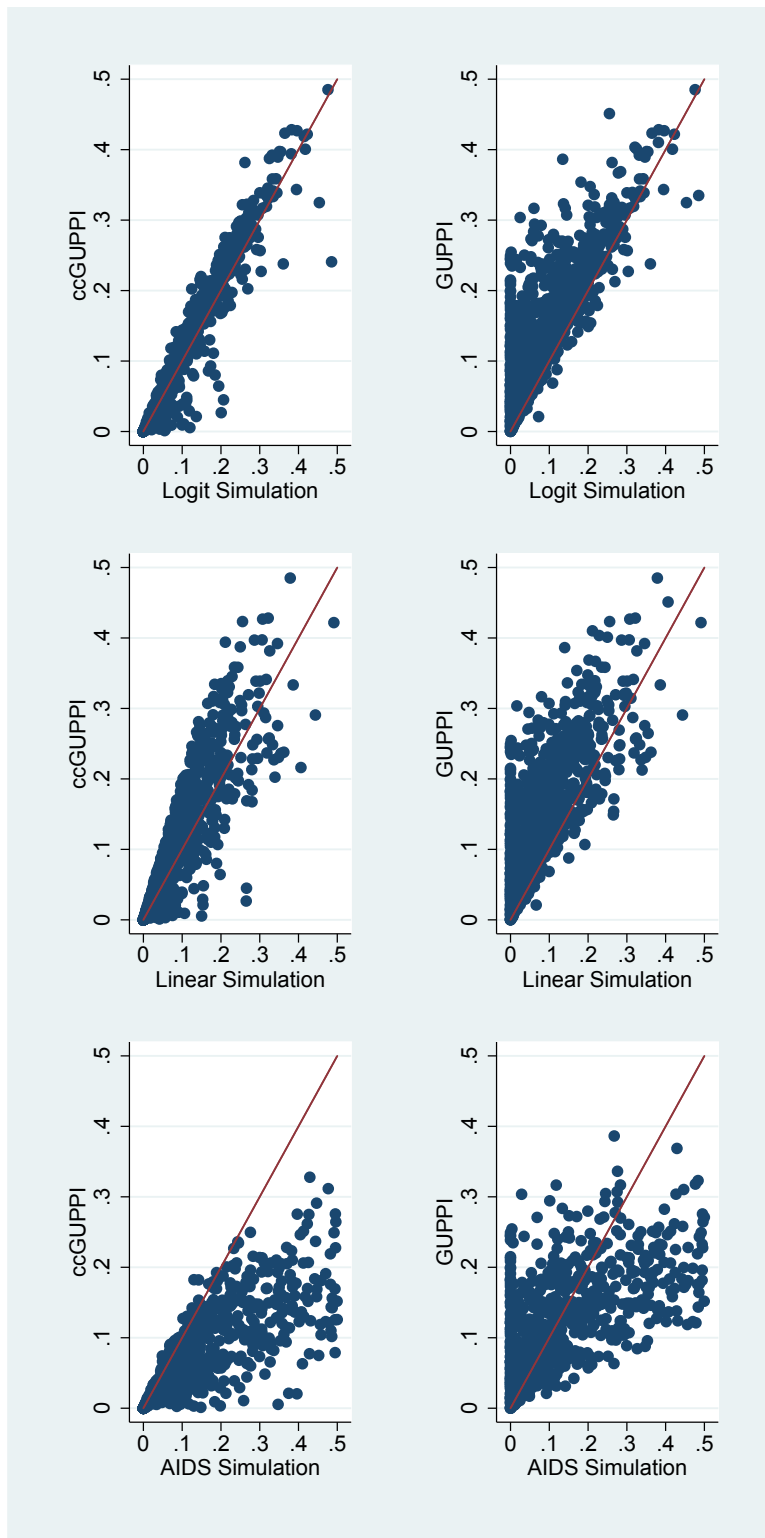
Figure 5: $ccGUPPI$ and $GUPPI$ price predictions using the revised pass-through matrix described in table 8 (y-axis) versus simulated price effect (x-axis).

References

[1] Balan, D., P. Degraba, and J. O'Connor, 2017. "Horizontal mergers with one capacity constrained firm increase prices," *mimeo*

[2] Bocard, N. and X. Wauthy, 2000. "Bertrand competition and Cournot outcomes: further results," *Economics Letters*, 68, pp. 279-285

[3] Bulow, J., J. Geanakoplos, and P. Klemperer, 1985. "Multimarket oligopoly: strategic substitutes and complements," *Journal of Political Economy*, 93:3, pp. 488-511

[4] Chen, J., 2009. "The effects of mergers with dynamic capacity accumulation," *International Journal of Industrial Organization*, 27, pp. 92-109

[5] Chen, Z. and G. Li, 2018. "Horizontal mergers in the presence of capacity constraints," *Economic Inquiry*, forthcoming

[6] Conlon, C. and J. Mortimer, 2013. "Demand estimation under incomplete product availability," *American Economic Journal: Microeconomics*, 5:4, pp. 1-30

[7] Dastidar, K., 1995. "On the existence of pure strategy Bertrand equilibrium," *Economic Theory*, 5:1, pp. 19-32

[8] Dastidar, K., 1997. "Comparing Cournot and Bertrand in a homogeneous product market," *Journal of Economic Theory*, 75, pp. 205-212

[9] Dixit, A., 1980. "The role of investment in entry-deterrence," *The Economic Journal*, 90, pp. 95-106

[10] Dixon, H., 1990. "Bertrand-Edgeworth equilibria when firms avoid turning customers away," *The Journal of Industrial Economics*, 39:2, pp. 131-146

[11] Farrell, J., and C. Shapiro, 2010. "Antitrust Evaluation of Horizontal Mergers: An Economic Alternative to Market Definition," *The B.E. Journal of Theoretical Economics*, 10:1 -355, Article 9

[12] Froeb, L., S. Tschantz, and P. Crooke, 2003. "Bertrand competition with capacity constraints: mergers among parking lots" *Journal of Econometrics*, 113, pp. 49-67

[13] Higgins, R., P. Johnson, and J. Sullivan, 2004. "Spatial competition and merger," *Topics in Economic Analysis & Policy*, 4:1, pp. 1-37

[14] Jaffe, S. and G. Weyl, 2013. "The First-Order Approach to Merger Analysis," *American Economic Journal: Microeconomics* 5:4, pp. 188-218

[15] Maggi, G., 1996. "Strategic trade policies with endogenous model of competition,'' *The American Economic Review* 86:1, oo. 237-258

[16] McElroy, F.W., 1993. "The effects of mergers in markets for differentiated products'' *Review of Industrial Organization* 8, pp. 69-81

[17] Miller, N., M. Remer, C. Ryan, and G. Sheu, 2016. "Pass-through and the prediction of merger price effects,'' *Journal of Industrial Economics*, 64, pp. 684-709

[18] Miller, N., M. Remer, C. Ryan, and G. Sheu, 2017. "Upward pricing pressure as a predictor of merger price effects,'' *International Journal of Industrial Organization*, 52, pp. 216-247

[19] Neurohr, B., 2016. "Upward pricing pressure under capacity constraints, kinked demand and other cases of a constrained pre-merger equilibrium," *Economics Letters*, 139, pp. 49-51

[20] Oxera, 2016. "Constrained by capacity or competition? Capacity constraints in merger analysis,'' *Agenda*, https://www.oxera.com/Latest-Thinking/Agenda/2016/Constrained-by-capacity-or-competition-Capacity-co.aspx

[21] Sacher, S. and J. Sandford, 2016. "The role of capacity in antitrust analysis,'' *Journal of Competition Law and Economics*, 12:4, pp. 661-700

[22] Shapley, L. and M. Shubik, 1969. "Price strategy oligopoly with product variation,'' *Kyklos*, 22:1, pp. 30-44

[23] Vives, X., 1990. "Nash equilibrium with strategic complementarities,'' *Journal of Mathematical Economics*, 19, pp. 305-321

[24] Werden, G., 1996. "A robust test for consumer welfare enhancing mergers among sellers of differentiated products,'' *The Journal of Industrial Economics*, 64:4, pp. 409-413