

# WORKING PAPERS



**PRODUCT RELIABILITY, WARRANTIES AND PRODUCER  
LIABILITY, AND ADVERTISING**

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PRODUCT RELIABILITY, WARRANTIES AND  
PRODUCER LIABILITY, AND ADVERTISING\*

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ABSTRACT

Product Reliability, Warranties and Producer  
Liability, and Advertising

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This paper presents static and dynamic models which develop the implications of the assumption that consumers are likely to have imperfect information about product reliability, and in particular, that consumer perceptions of product "reliability" can be influenced by producer actions, particularly the terms of the warranty offered and by advertising about reliability. It is shown that "misleading" advertising about product reliability is a likely equilibrium outcome, and that "informative" advertising about product reliability will not generally be an equilibrium outcome. These results are shown to be independent of market structure.

Two plausible concepts of efficiency are discussed and the efficiency of the market allocation (in terms of these criteria) is analyzed. The efficacy of various piecemeal policy remedies is examined. We argue that many of the commonly used piecemeal policies may be ineffective because they are not incentive compatible. Finally, simple dynamic models are developed which address the question of the long run efficiency of the unregulated market. These models suggest that a monopoly market may be more likely to attain long run efficiency than a competitive market.

## I. Introduction

In recent years, governments at all levels have been increasingly concerned with the "quality" of consumer goods and the protection afforded consumers against deficient quality. Various policy instruments have been used, including setting quality standards, mandating warranty protection and the provision of information about product quality.<sup>1</sup> It is well known that market imperfections, informational imperfections being among the most important in this context, may impair the efficiency of consumer goods markets, particularly markets for consumer durables. However, the policy implications of this fact have not been fully explored. In this paper we will provide a more thorough explanation of how the market might deal with informational imperfections and we examine the efficacy of various policy instruments.

Our point of departure will be the model of Spence (1977), which was developed to analyze the allocative implications of the assumption that consumers may misperceive the "reliability" of products for which health, safety or durability characteristics are important. Spence shows that in a static framework these misperceptions will generally lead to a market failure which under some circumstances may be difficult to remedy.

One major weakness of the Spence model, as pointed out in the concluding section of the Spence paper, is the simplistic

modeling of consumer perceptions. In this paper, we will argue that consumer perceptions of product reliability are influenced not only by product reliability (as assumed in the Spence model), but also by other actions of producers. In particular, we consider the implications of the assumption that consumer perceptions about product reliability are influenced by the warranty or guarantee offered by producers and by expenditures by producers aimed at directly influencing perceptions about product reliability (such as packaging, labeling, and other cosmetic design, and advertising), which we will summarize under the term "advertising."

We begin by deriving the properties of the competitive market allocation for a static model. We show that expenditures on "advertising" may be consistent with equilibrium, with such expenditures likely to be designed to misinform consumers. In addition, the monopoly allocation is derived and is shown to have the same qualitative properties as the competitive allocation.

We turn our attention next to the welfare implications of the equilibria. The efficiency of the market allocation is examined, using both the efficiency criterion proposed by Spence and a "Common Law" criterion. After demonstrating that the static market allocation will generally be inefficient (in either sense), we consider the desirability of various policy instruments. Among the important points revealed by this analysis is that many of the obvious policy instruments

are not incentive compatible, and so the use of such instruments would be expected to create incentives to vitiate such policies. A review of the success of such policies in the U.S. suggests that this is an important problem.<sup>2</sup>

Finally, we develop some dynamic models in which consumers learn about product reliability over time, in order to ascertain the likelihood that the market will correct the market failure in the long run. We show that the market may be more likely to correct the market failure if the market structure is a monopoly than if it is competitive.

## II. Assumptions and Notation

The basic framework of our model and our notation will be similar to that found in Spence [1977]. Consumers are assumed to have identical tastes and perceptions, and the uncertainty about the reliability of the product is assumed to be over only two states: failure and no failure. Consumers have an inelastic demand for one unit of the product.<sup>3</sup>

As in Spence, the notation is:

$y$  = ex ante income of consumers

$s$  = probability the product does not fail

$c(s)$  = marginal cost of the product

$p$  = equilibrium price of the product

$r$  = consumers' perception of  $s$

$m$  = producer liability in the event of failure

$u(x)$  = consumer's von Neumann-Morgenstern utility of income if product doesn't fail

$v(x)$  = consumer's utility function of income if the product fails

### A. Consumer Perceptions

As in the Spence model it is assumed that  $r = r(s)$ , i.e., consumer perceptions are influenced by the actual reliability of the product, so that  $\partial r / \partial s > 0$ . Consider a product where  $m$  (producer liability) is determined by the market (i.e., chosen by firms in response to perceived consumer demands) rather than determined by recourse to the legal system. Producer liability, in the form of guarantees and warranties, for most consumer



durables except with respect to safety hazards, is of this type. For such products, at least in the short run, the terms of the warranty ( $m$  in our simple model) presumably influence consumers' perception of reliability, so that  $\partial r / \partial m > 0$ . (We will show that if consumers don't have good direct information on reliability,  $r_s = 0$ , this is rational.)<sup>4</sup>

Finally, in a short-run situation it is also reasonable to assume that consumer perceptions can be influenced by expenditures by producers of the product on what we will call "advertising," which is any "information" (true or false) providing activity of the firm or industry. This "advertising" may take many forms: design and content of labeling and packaging, the usual booklet which explains the warranty on consumer durables, or more general forms of advertising expenditures. We assume that such advertising expenditure only influences consumers' perceptions of reliability, not consumers' tastes.

We will distinguish two types of advertising expenditure: (1) "informative" advertising, where an expenditure of  $\$A$  per customer reduces  $|r - s|$ ; and (2) misleading advertising, where an expenditure of  $\$a$  per customer increases  $|r - s|$ . The reader can probably easily furnish his own examples of the two types of advertising. Although for economy of notation we will write consumer perceptions as  $r(s, m, A)$  or  $r(s, m, a)$ ,  $r(s, m, A)$  for example would be two different functions, depending on the sign of  $(r - s)$ .

### III. Products for Which Safety Hazards Are Not an Important Adjunct to Product Failure

For products for which injury to person or property is not an important adjunct to product failure it is reasonable to assume that  $v(x)$  (utility-of-income function in the event of product failure) is the same as the utility-of-income function in the absence of product failure. In this case product failure causes a dollar loss of  $\$L$ , representing the costs required to have the product repaired, plus perhaps a dollar cost of foregone leisure involved with repairing the product. (The utility function can reasonably be assumed to be invariant if the amount of foregone leisure is small). The (nonsafety hazard) losses arising from the failure of most consumer durables can probably be modeled this way. In the Spence paper, this type of product is not given prominence, because he is evidently primarily concerned with issues of product safety.

In this section we assume that  $v(x) = u(x - L)$ , and for notational clarity we will write  $u(y - p) = u_1$ ,  $u(y - p - L + m) = u_2$ .

#### A. Unregulated (Short-Run) Competitive Market Equilibrium with Misleading Advertising

With advertising expenditures aimed at misleading consumers perceptions of reliability, consumers' perceptions can be written  $r(s, m, a)$  where  $r_s > 0$ ,  $r_m > 0$ , and  $\partial |r(s, m, a) - s| / \partial a >$

0. The representative consumer's expected utility can be written  
(1)  $\bar{U} = r(s, m, a) u(y - p) + (1 - r(s, m, a)) u(y - p - L + m)$ .<sup>5</sup>

Under competitive conditions in (short run) equilibrium the market will "choose"  $(s, m, a)$  so as to maximize (1) subject to the zero expected profit condition,

$$(2) \quad p = c(s) + (1 - s)m + a.$$

The first order conditions for the maximization of (1) with respect to  $(s, m, a)$  subject to (2) can be written

$$(3) \quad (a) \quad r_s(u_1 - u_2) + r u'_1(m - c') + (1 - r) u'_2(m - c') = 0$$

$$(b) \quad r_m(u_1 - u_2) - r(1 - s)u'_1 + (1 - r)s u'_2 < 0, \quad m > 0$$

$$(c) \quad r_a(u_1 - u_2) - r u'_1 - (1 - r)u'_2 < 0, \quad a > 0,$$

where  $\text{sign } r_a = \text{sign}(r - s)$  (by the definition of misleading advertising).

Let us first consider (3)(c). To show that misleading advertising may be profitable in equilibrium we will show that the first (and second) order conditions can hold for  $a > 0$ . There are two cases to consider.

(a) Case 1:  $r_a > 0$ , which means that  $r > s$ .

If  $r_a > 0$ , then (assuming  $a > 0$ ) (3)(c) requires  $u_1 - u_2 > 0$ , (which requires  $L > m$ , so that consumers are less than fully insured), and so by concavity of  $u(x)$ ,  $u'_1 - u'_2 < 0$ . Since (3)(b) must be nonpositive, if  $u_1 - u_2 > 0$  we must have  $-r(1 - s)u'_1 + (1 - r)s u'_2 < 0$ , which is consistent with  $u'_1 - u'_2 < 0$  and  $r > s$ . Finally, if  $u_1 - u_2 > 0$ , (3)(a) requires  $r' > m$ . Therefore,  $a > 0$  is consistent with

the first order conditions. (It can also be shown that  $a > 0$  is consistent with the second order conditions. The reader can easily verify that  $a > 0$  is possible using the risk neutral case which is considered later in the paper). Thus it may be profitable for (competitive) producers to influence consumers to overestimate reliability. The profitability of misleading advertising under competitive conditions is perhaps a surprising conclusion.

b) Case 2:  $r_a < 0$ , which means that  $r < s$ .

If  $r_a < 0$ , then (assuming  $a > 0$ ), (3)(c) requires  $u_1 - u_2 < 0$ . The concavity of  $u(x)$  then requires  $u_2' - u_1' < 0$ . Since  $u_1 - u_2 < 0$ , we must have  $m > L$ , i.e., warranty reimbursement is greater than actual loss!<sup>6</sup> (This occurs of course because  $r < s$ ). Since  $m > 0$ , (3)(b) holds with equality, so that  $-r(1-s)u_1' + (1-r)s u_2' > 0$ , which is consistent with  $u_1 - u_1' > 0$ ,  $r < s$ . Finally, with  $u_2' - u_1' < 0$ , (3)(a) requires  $c' < m$ . Therefore  $a > 0$  is consistent with the first order conditions. (It can also be shown that  $a > 0$  is consistent with the second order conditions. The reader can easily verify that  $a > 0$  is possible using the risk neutral case which is considered later in the paper.) Thus it may be profitable for (competitive) producers to influence consumers to underestimate reliability! This of course is a more surprising conclusion than the previous one.

The reason for this surprising conclusion can perhaps be seen more clearly by considering risk neutral consumers. With

risk neutral consumers  $\bar{U} = r u(y-p) + (1-r) u(y-p+m-L)$  can be written  $y - c(s) + s m + r(L-m) - L - a$ . Consider an equilibrium without advertising ( $a=0$ ) in which  $s > r$ . Using (3)(b) it can easily be shown that an equilibrium with  $s > r$  requires  $u_1 - u_2 < 0$ , so that  $L - m < 0$ . Now differentiate  $\bar{U}$  (assuming risk neutrality) partially with respect to  $a$ , which gives  $\partial U/\partial a = r_a(L - m) - 1$ . Since  $r_a < 0$  ( $s > r$ ), and  $L - m < 0$ ,  $\partial \bar{U}/\partial a$  may be positive for  $a = 0$ .

Although the possibility of the profitability of expenditures which influence consumers to underestimate reliability is surprising, the practical importance of the result appears limited, since the profitability of such advertising requires an equilibrium with  $L - m < 0$ , which would seem to be an unusual occurrence. Notice however that such an occurrence would be more likely in markets with "cynical" consumers ( $r(s, m, a) < s$ , for all  $s > 0, m, a$ )!

Therefore misleading advertising about reliability may arise in (competitive) markets, and such advertising may be designed to influence consumers to either overestimate or underestimate reliability.

B. Unregulated (Short-Run) Competitive Market Equilibrium with "Informative" Advertising

With informative advertising consumer perceptions are  $r(s, m, A)$ , with  $\partial|r-s|/\partial A < 0$ ,  $r \neq s$ , and  $p = c(s) + (1 - s) m + A$ . The (short-run) competitive market equilibrium now is described as the maximization of (1) with respect to  $(s, m, A)$  subject to the price equation. The first order conditions

are the same as

(3), except (3)c) now becomes

$$(3')c) \quad r_A(u_1 - u_2) - r u_1' - (1 - r)u_2' \leq 0, \quad A > 0,$$

where  $\text{sign } r_A = \text{sign}(s - r)$  (by the definition of informative advertising).

As before, we will examine the consistency of  $A > 0$  with (3)(a), (3)(b) and (3')(c). There are two cases to consider.

a) Case 1:  $r_A > 0$ , which means that  $r < s$ .

If  $A > 0$  and  $r_A > 0$ , (3')(c) requires  $(u_1 - u_2) > 0$ , and so by the concavity of  $u$ ,  $(u_1' - u_2') < 0$ . But (3)(b) requires  $-r(1 - s)u_1' + (1 - r)s u_2' < 0$ , which is inconsistent with  $r < s$  and  $(u_1' - u_2') < 0$ . Therefore  $A > 0$  is not possible, so if  $r_A > 0$  ( $r < s$ ), informative advertising is never profitable.

b) Case 2:  $r_A < 0$ , which means that  $r > s$ .

If  $A > 0$  and  $r_A < 0$ , (3')(c) requires  $(u_1 - u_2) < 0$ , and so by the concavity of  $u$ ,  $(u_1' - u_2') > 0$ . But if  $(u_1 - u_2) < 0$ ,  $m > L > 0$ , so that (3)(b) requires  $-r(1 - s)u_1' + (1 - r)s u_2' > 0$ , which is inconsistent with  $r > s$  and  $(u_1' - u_2') > 0$ . Therefore  $A > 0$  is not possible, so if  $r_A < 0$  ( $r > s$ ), informative advertising is never profitable.

Therefore, informative advertising is never profitable in competitive markets! This is perhaps surprising, since intuition may suggest that competitive conditions would create incentives for producers to be truthful.

To understand the advertising results it must first be realized that even in this simple model, consumers are purchasing a complex product, comprised of the physical product itself, the warranty protection offered, and the "aura" of reliability (affected by  $(s,m,A,a)$ ). The more reliable the consumer believes the product is, the lower will be his demand for insurance against product failure and the higher will be his valuation of the product for any level of warranty protection (if  $m < L$ ), ceteris paribus. Consider the possible actions by a producer if his customers overestimate the reliability of his product. If the consumer overestimates reliability and the producer can incur costs to remedy this misperception, however, besides these costs the producer will now have to offer greater warranty protection or lower his price to make a sale to the newly informed consumer. On the other hand, if the producer incurs costs to further distort the consumer's perceptions and this action is successful, the consumer will now be willing to pay a higher price for the same actual level of reliability and warranty coverage.

False or deceptive advertising is illegal under both federal and state statutes, so there are legal disincentives to deception. Furthermore, our model gives producer reputation no role, and reputation, if important, also provides disincentives to

deception. On the other hand, a lot of advertising is clearly directed to enhancing consumers' perceptions of product reliability (although this need not be deceptive, if in the absence of the advertising consumers underestimate reliability).

Of more importance, in our view, than the fact that producers may deceive their customers, is our conclusion that producers will not have an incentive to provide (truthful) information to their customers. Although the result is also likely to be tempered if producer reputation is important, it is clear that producers often have significant information about product reliability which they do not make available to producers. Recent examples such as the Firestone 500 and Ford Pinto cases provide supporting evidence for this assertion, but such dramatic examples are not needed to make the point. A more pedestrian illustration is that home appliance (e.g., freezers, refrigerators) manufacturers have generally not provided information on the cost of running their products, even though they have this information or could obtain it very easily (uncertain operating costs could easily be incorporated in our model in a manner analogous to uncertain reliability). Our model captures the disincentives for producers to provide such information.

Finally, it should be noted that if consumers do not have good direct information on reliability ( $r_s \approx 0$ ), (3)(a) becomes  $c' = m$ . Therefore " $ds/dm > 0$ ", which means that it is rational for consumers to use the terms of the warranty as a signal of reliability.



## C. Efficiency and Regulation in Competitive Markets

### 1. The Definition of Efficiency

The market allocation described by (3) is ex ante efficient, since by definition, consumers' well being as measured by their ex ante preferences is maximized in the market allocation. However, one cannot feel completely comfortable with the market allocation, since consumers may misperceive reliability, and resources may be devoted to misleading their perceptions. Therefore, it seems useful, as in Spence, to characterize ex post efficiency and to also measure the efficiency of the market allocation by this criterion.

However, even in this simple model ex post efficiency doesn't have a unique a priori definition. This is because although consumers are identical ex ante, in the absence of full warranty coverage they are heterogeneous ex post. Denote those consumers who purchase nondefective units as being in the G group and those who purchase defective units as being in the F group. The Pareto methodology suggests than an efficient allocation should be defined as one which maximizes the utility of a representative member of the F group, given some exogenous utility level of a representative member of the G group (or vice versa). However, this is not possible, since it can't be known, ex ante, which group a consumer will be in, ex post, or how many consumers will be in each group ( which is determined by  $s$ , a choice variable).

Therefore some explicit value judgements must be made-- i.e., a particular social welfare criterion must be defined. Since there are only two types of consumers ex post, there are two obvious criteria: (i) a Common Law criterion (absolute preference for equality),<sup>7</sup> and (ii) the Spence criterion<sup>8</sup> (maximization of average ex post utility).

a. Common Law criterion

The Common Law efficient allocation is given as the solution of the problem:

$$(4) \max_{(s,m)} u(y - c(s) - (1-s)m),$$

subject to  $u(y - c(s) - (1-s)m) = u(y - c(s) + sm - L)$ ,<sup>9</sup>

and it is easily seen that this allocation requires

$$(5) \text{ a) } m = L$$

$$\text{ b) } c' = L^{10}$$

b. Spence criterion

The Spence-efficient allocation is given as the solution of the problem:

$$(6) \max_{(s,m)} U^* = s u(y-c(s) - (1-s)m) + (1-s)u(y-c(s) + sm - L),$$

and this solution requires

$$(7) \quad \begin{array}{l} \text{a) } m = L \\ \text{b) } c' = L \end{array}$$

Therefore, the Spence and Common Law criteria are equivalent for the case  $v(y) = u(y - L)$ . This result occurs of course because the Spence criterion will always require equalization of marginal utilities, and for the case  $v(y) = u(y - L)$ , this requires equalization of utilities. However, we will see below that second best versions of the criteria are not equivalent, and of course they aren't equivalent for general  $v(y)$ .

It is easily seen that if misleading advertising is profitable, the unregulated market equilibrium is not (first best) efficient (in either sense). If  $r_a > 0$ ,  $L - m > 0$ , and if  $r_a < 0$ ,  $L - m < 0$ , although in either case it may be that  $c' = L$ . Even if misleading advertising is not profitable, the unregulated equilibrium will not generally be (first best) efficient (in either sense). If consumer perceptions are accurate ( $r=s$ ), the equilibrium is efficient (in either sense).

## 2. Imposition of Producer Liability

For the efficiency criteria we have described, imposition of full producer liability ( $m = L$ ) results in (first best) efficiency (in either sense). This can easily be seen by examining (3). If  $m = L$  is imposed,  $u_1 = u_2$ ,  $u_1' = u_2'$  and (3)(a) then requires  $c' = m = L$ . Imposition of  $m = L$  also makes misleading advertising unprofitable, as can be seen from (3)(c).

It should be noted however that efficiency may not be attainable by such regulation without enforcement costs because it is in the interest of both consumers and producers to deviate from the regulated equilibrium. This is because the ex post efficient allocation is not ex ante efficient (unless  $r=s$ ). Since the unregulated market-allocation is ex ante efficient (i.e. it is best for consumers and producers given the information they have, forced deviations from this allocation create incentives for both consumers and producers to evade the regulations). One method by which the market can circumvent a mandated warranty is for producers to adjust the quality of service given under the warranty. As an illustration, one explanation for the seeming variation in warranty service performance by auto dealers is that this is an example of a market with a mandated (by the manufacturer) warranty. This mandated warranty may not be optimal, ex ante, for all consumers and dealers. Thus we have the apparent phenomenon of "low overhead", low price, low service quality (?) dealers, and high price, high service quality (?) dealers, and high price, high service quality (?) dealers. Therefore such regulation may not be effective without enforcement costs. Finally, attempting to define product failure at all, let alone in a manner that deals effectively with moral hazard problems is a very formidable task.

The likelihood of enforcement costs (and the associated unhappiness of political constituencies), the rare use of such regulation for products of the type we have been considering (where possible injury to person or property is not an important product characteristic), and the transactions costs involved with recourse to the courts (common law liability), lead us to consider alternative policy instruments.

### 3. Direct Regulation of Reliability

#### a. Common Law criterion

Another possible policy instrument is the direct regulation of reliability. To satisfy the Common Law criterion, consumers must be equally well off, ex post. Therefore the second best Common Law efficient policy requires: a) setting  $s$  so that the market provides full warranty coverage, or b) setting  $s = 1$ . If both policies are feasible, the one which results in highest utility is chosen.

Since setting  $s = 1$  is not likely to be feasible, a policy of indirectly setting  $m$  by choosing  $s$  is unnecessarily complicated if  $m$  itself can be set. Therefore direct regulation of reliability seems a particularly inappropriate piecemeal policy for satisfying a Common Law criterion. Indeed, all piecemeal

policies, other than liability rules, are particularly inappropriate for the Common Law criterion<sup>11</sup>, so that our discussion of other piecemeal policies will focus on the Spence criterion.

b. Spence criterion

The (second best) Spence-optimal regulation of reliability maximizes  $U^*$  (defined in (6)) with respect to  $s$ , subject to (3)(b) and (c). The first order conditions require:

$$(8) \frac{dU^*}{ds} = (u_1' - u_2') + (m - c')(su_1' + (1-s)u_2') \\ + (1-s)s(u_2' - u_1')m_s - (su_1' + (1-s)u_2')a_s = 0,$$

where  $m_s = \partial m / \partial s$  and  $a_s = \partial a / \partial s$  ( $a > 0$ ), are derived from the equilibrium conditions (3)(b) and (c).

Not surprisingly perhaps, with only  $s$  as a policy instrument, first best optimality cannot generally be attained. Furthermore second best optimal reliability will not generally coincide with first best reliability ( $c' = L$ ), even if misleading advertising is unprofitable.

Since the signs of  $m_s$  and  $a_s$  are ambiguous, further assumptions are necessary to obtain more results from (8). Therefore we will assume consumers are risk neutral. This is a reasonable approximation for goods for which the amount of expenditure and possible loss due to product failure is small (e.g., a toaster). We will also assume that  $r(s, m, a)$  is additively separable in its arguments, which resolves the problem of the ambiguity of the signs and magnitudes of the cross partials of  $r$ .

(i) Risk neutrality and additively separable r

Under these assumptions  $U^* = y - c(s) - (1-s)L - a$ , and

(8) becomes:

$$(9) \quad dU^*/ds = (L - c') - a_s = 0,$$

and (3)(b), (c) becomes

$$(10) \quad a) \quad r_m(L-m) + s - r < 0, \quad m > 0$$

$$b) \quad r_a(L - m) - l < 0, \quad a > 0.$$

From (9) we see that if misleading advertising is not profitable, regulation of reliability achieves first best optimality. (This result of course does not depend on our assumption that  $r(s, m, a)$  is additively separable.) Using (10) and our separability assumptions, it can be shown that if  $m = 0$ ,  $a_s = 0$ , so that  $c' = L$  is also (second best) optimal.

If  $m$  and  $a$  are both positive,  $a_s$  can be determined from (10) (using the second order conditions), and it can be shown that:

$$(11) \quad \text{sign } a_s = -\text{sign}(1-r_s)m a,$$

Then, returning to (9), second best optimality requires

$$(12) \quad \text{sign } (L-c') = \text{sign } (1-r_s) ma,$$

$$\text{since } \text{sign } r_a = \text{sign } (r-s).$$

Now let us consider the unregulated equilibrium for  $s$  (which we will denote  $(\hat{s}, \hat{m}, \hat{a})$ ), under our assumptions. Equilibrium condition (3)a becomes:

$$(13) \quad \hat{r}_s (L - \hat{m}) - \hat{c}' + \hat{m} = 0,$$

$$\text{which can be written } \hat{c}' = \hat{r}_s L + (1 - r_s)\hat{m}.$$

If in unregulated equilibrium  $\hat{r} > \hat{s}$ , then by (10)(a),  $L > \hat{m}$ . In this case using (13) it is easily seen that  $\text{sign}(L - \hat{c}') = \text{sign}(1 - \hat{r}_s)$ . If in unregulated equilibrium  $\hat{r} < \hat{s}$ , then  $L < \hat{m}$ , and so  $\text{sign}(L - \hat{c}') = -\text{sign}(1 - \hat{r}_s)$ . Now if we denote the (second best) optimum by  $(\bar{s}, \bar{m}, \bar{a})$ ,  $(L - \hat{c}')(L - \bar{c}') > 0$ , and it may be the case that  $\hat{c}' = \bar{c}'$ .

Therefore regulation of reliability may not be required for second best optimality (even if  $a > 0$ ), and  $(L - c')(L - \bar{c}') > 0$ . If we denote the first best optimum by  $(s^*, m^*)$ , and assume  $c'' > 0$ , then  $(s^* - \hat{s})(s^* - \bar{s}) > 0$ , i.e., the divergences from first best reliability of the regulated and unregulated equilibria are in the same direction.

Finally, as in the case of imposition of producer liability, if the regulated and unregulated equilibria do not coincide, enforcement costs can be anticipated since it is then in the interests of both consumers and producers to deviate from the regulated reliability.

#### 4. Regulation of Misleading Advertising

One obvious piecemeal policy alternative is the banning of misleading advertising (since in our model this is the only type of advertising which producers will voluntarily use, banning all advertising will have an equivalent effect). However, it will not be surprising to students of the second best that such a policy is not necessarily (second best) optimal. To see why this



is the case consider an equilibrium in which advertising is banned. Then the first order conditions for producers are the same as (3)a) and b)):

$$(14) \text{ a) } r_s(u_1 - u_2) + ru_1'(m - c') + (1 - r)u_2'(m - c') = 0$$

$$\text{b) } r_m(u_1 - u_2) - r(1 - s)u_1' + (1 - r)su_2' < 0, m > 0$$

Differentiating the efficiency criterion  $U^*$ :

$$(15) \frac{dU^*}{da} = [(u_1 - u_2) + (m - c')su_1' + (1 - s)u_2']s_a \\ + (1 - s)s(u_2' - u_1')m_a \\ - (su_1' + (1 - s)u_2'),$$

where  $s_a = \partial s / \partial a$ , and  $m_a = \partial m / \partial a$ , are determined from (12).

The sign of (15) can be shown to be ambiguous (except of course at the first best optimum, where  $u_1 = u_2$ ,  $u_1' = u_2'$ ,  $c' = L = m$ ). Therefore banning misleading advertising is not necessarily (second best) optimal.

a. Risk neutrality and separable  $r$ .

For illustrative purposes we will now consider a particular case where the ambiguity is resolved. We will again assume that consumers are risk neutral and that  $r(s, m, a)$  is additively separable in its arguments. Equations (3)(a) and (b) become:

$$(16) \text{ a) } r_s(L - m) - c' + m = 0$$

$$\text{b) } s - r + r_m(L - m) < 0, m > 0,$$

and (15) can be written

$$(17) \frac{dU^*}{da} = (L - c') s_a - 1$$

If  $m = 0$ ,  $s_a = 0$ , so that  $dU^*/da < 0$ . If  $m > 0$ , it can be shown (using (16) and the second order conditions) that (18)  $\text{sign } s_a = -\text{sign } r_a(1-r_s)$ .

Rewriting (16)a) as  $c' = r_s L + (1 - r_s)m$ , it can easily be shown that

$$(19) \text{ sign } (L - c') = \text{sign } (L - m)(1 - r_s)$$

If  $r_a < 0$ ,  $r < s$  and  $(L - m) < 0$ . Then, using (18), (19),

$$(20) (L - c')s_A - 1 < 0, \text{ if } r_a < 0.$$

If  $r_a > 0$ ,  $r > s$  and  $(L - m) > 0$ . Then, using (18), (19), (21)  $(L - c')s_A - 1 < 0$ , if  $r_a > 0$ .

Therefore (using (17), (20), (21) in the special case of risk neutral consumers and additively separable  $r$ , banning misleading advertising is always (second best) optimal). The ambiguity in the general case arises because of the ambiguity of the signs and magnitudes of expressions such as  $(u_i + u''_i)$  and  $r_{jk}$ , and because with risk aversion  $U^*$  has  $m$  as an argument.

##### 5. Provision of Information

A commonly advocated form of government intervention for problems of the type examined in this paper is the provision of information (about reliability) either by some government agency directly, or by producers under government regulation. Obvious examples are "truth in lending" regulations, and health warning labels on cigarette packages. As we saw earlier, informative

advertising is never profitable, so that a possible efficiency-enhancing role for the government is suggested.

It is naive to suppose that consumers' estimates of reliability can be costlessly influenced (either through direct government action or regulation or that consumers will necessarily believe such information. Rather, we would argue that a more reasonable modeling of such intervention would be similar to our earlier modeling of informative advertising. Thus we will assume that an expenditure of \$A per consumer affects consumers' estimates of reliability through  $r(s, m, A)$ ,  $\partial |r - s| / \partial A < 0$  for  $r \neq s$ . We will assume that such information-providing activity, if done directly by the government, is financed by a specific tax on the good, so that it is immaterial whether the information is provided directly by the government or by producers under government regulation.

The "optimal" expenditure (per customer) on information provision maximizes  $U^* = su_1 + (1-s)u_2$  with respect to A, subject to the equilibrium conditions (3). If misleading advertising is also banned as part of the policy, (3)(c) is no longer included in the equilibrium conditions. The first order conditions for the maximization of  $U^*$  require:

$$\begin{aligned}
 (22) \quad dU^*/dA &= [(u_1' - u_2') + (m - c')(su_1' + (1-s)u_2')]s_A \\
 &+ (1-s)s(u_2' - u_1')m_A - (su_1' + (1-s)u_2')a_A \\
 &- (su_1' + (1-s)u_2')y_2' < 0, \quad A > 0,
 \end{aligned}$$

where  $s_A = \partial s / \partial A$ ,  $m_A = \partial m / \partial A$ , and  $a_A = \partial a / \partial A$  are determined from (3), and  $a_A \equiv 0$  if misleading advertising is banned.

In light of the analysis of the preceding section it is probably not surprising that (22) does not necessarily have an interior solution, so that it may not be (second best) optimal to engage in expenditures on the provision of information.

a. Risk neutrality and separable  $r$

As in the preceding sections for illustrative purposes we will consider the particular case of risk neutral consumers and additively separable  $r(s, m, a, A)$ . In that case (22) can be written

$$(23) \quad dU^* / dA = (L - c')s_A - a_A - 1 < 0, \quad A > 0,$$

from which it can be seen that (second best) optimal provision of information will not generally achieve first best optimality, since  $c' = L$  is not required by (16). Under our assumptions (3) becomes

$$(24) \quad \begin{aligned} (a) \quad & r_s(L-m) - c' + m = 0 \\ (b) \quad & s - r + r_m(L-m) < 0, \quad m > 0 \\ (c) \quad & r_a(L-m) - 1 < 0, \quad a > 0 \end{aligned}$$

If  $m = 0$ , it can be shown, (using (23) and the second order conditions), that  $s_A = a_A = 0$ , so that  $A = 0$  is (second best) optimal. Notice however that  $m = 0$  is not (first best) optimal unless  $r_s = 1$ , because of (24)(a). If  $m > 0$  it can be shown that

$$(25) \text{ (a) } \text{sign } s_A = -\text{sign } r_A(1-r_s)$$

$$\text{ (b) } \text{sign } a_A = \text{sign } r_A r_a < 0, \text{ if } a > 0$$

From (24) a) it can be shown that

$$(26) \quad \text{sign } (L - c') = \text{sign } (L - m)(1 - r_s)$$

and (24)(b) requires:

$$(27) \quad \text{sign } (L - m) = -\text{sign } r_A,$$

since  $\text{sign } r_A = -\text{sign } (r-s)$ .

Finally, using (25)(a), (26), and (27),

$$(28) \quad \text{sign } s_A = \text{sign } (L - c'),$$

so that

$$(29) \quad (L - c')s_A - a_A > 0,$$

which is consistent with (23) having an interior solution.

Therefore expenditure on the provision of information may be (second best) optimal.

Since  $\text{sign } s_A = \text{sign } (L - c')$ , assuming risk neutrality and separable  $r$  and  $m > 0$ , if  $c'' > 0$ , it is easily seen that the provision of information reduces the divergence between equilibrium reliability and (first best) optimal reliability. The provision of information also reduces expenditures on misleading advertising and reduces  $L - m$ .

#### IV. Products for Which Safety Hazards are an Important Adjunct to Product Failure

For products for which injury to persons or property is an important adjunct to product failure, it will usually not be reasonable to assume  $v(x) = u(x - L)$ . This more general case is considered extensively in Spence [1977], focusing on producer

liability as an instrument for achieving optimality. However, the role of warranties and "advertising" in influencing consumer perceptions of reliability and Common Law efficiency are not considered in the Spence paper.

A. Unregulated (Short Run) Competitive Market Equilibrium with Misleading Advertising

With misleading advertising, the equilibrium conditions corresponding to (3) are:

$$(30) \quad \begin{aligned} (a) \quad & r_s(u-v) + ru'(m-c') + (1-r)v'(m-c') = 0 \\ (b) \quad & r_m(u-v) - r(1-s)u' + (1-r)s v' \leq 0, m > 0 \\ (c) \quad & r_a(u-v) - ru' - (1-r)v' \leq 0, a > 0, \text{ where} \\ & \text{sign } r_a = \text{sign } (r-s) \end{aligned}$$

It can easily be seen that if in equilibrium  $\text{sign}(u-v) = -\text{sign}(u' - v')$ , which is a sort of generalized concavity condition, the analysis is identical to that of Section III.A. Even if  $\text{sign}(u-v) = \text{sign}(u' - v')$  in equilibrium,  $a > 0$  is consistent with the first and second order conditions, so that it may be profitable for (competitive) producers to influence consumers to either over- or underestimate reliability. However, unlike Section III.A., if advertising is unprofitable and  $m > 0$ , it is no longer necessarily the case that  $(u-v)(r-s) > 0$ .

B. Unregulated (Short Run) Competitive Market Equilibrium with Informative Advertising

With informative advertising (30)(c) is replaced by

$$(31) \quad r_A(u-v) - ru' - (1-r)v' \leq 0, A > 0, \text{ where } \text{sign } r_A = \text{sign } (s-r).$$

If in equilibrium  $\text{sign}(u-v) = -\text{sign}(u'-v')$ , the analysis is the same as in Section III.B., so that in this case misleading advertising is never profitable. However, if  $\text{sign}(u-v) = \text{sign}(u'-v')$  in equilibrium, the first and second order conditions are consistent with  $A > 0$ , so that informative advertising may be profitable for (competitive) producers, and a necessary condition for this to occur is  $\text{sign}(u-v) = \text{sign}(u'-v')$  (in equilibrium).

### C. Efficiency and Regulation in Competitive Markets

#### 1. Common Law Criterion

The (first best) Common Law efficient allocation is given as the solution of

$$(32) \max_{s,m} u(y-c(s)-(1-s)n)$$

subject to  $u(y-c(s)-(1-s)n) = v(y-c(s) + sm)$ , and the first order conditions for an interior solution for this problem are:

$$(33) \quad (a) \quad (1+\lambda)u'(m-c') - \lambda v'(m-c') = 0$$

$$(b) \quad -(1+\lambda) u'(1-s) - \lambda v's = 0,$$

where  $\lambda$  is the LaGrangean multiplier corresponding to the constraint. This multiplier is nonpositive, and by (33)(b),  $(1+\lambda) > 0$ .

Therefore (33) can be rewritten

$$(34) \quad (a) \quad c' = m$$

$$(b) \quad u' = \{-\lambda s / ((1+\lambda)(1-s))\} v'$$

#### 2. Spence criterion

The efficiency criterion  $U^*$  now becomes

$$(35) \quad U^* = s u + (1-s)v,$$

so that (first best) optimality requires:

$$(36) \quad (a) \quad u' = v'$$

$$(b) \quad c' = m + (u - v)/(su' + (1 - s)v')$$

Unlike Section III, it is possible for advertising to be profitable in equilibrium ( $a$  or  $A > 0$ ), and (36) to hold.

### 3. Comparison of the Two Efficiency Criteria

It is easily seen that the two efficiency criteria are no longer generally equivalent, making policy choices particularly troublesome. Furthermore, the market allocation will not usually be efficient in either sense, so that policy action may be called for.

Of the two criteria, the spirit of the common law treatment of product defect liability is closer to the Common Law criterion. As we will see, this criterion is also probably easier to achieve.

### 4. Imposition of Producer Liability

#### a. Common Law criterion

First best Common Law efficiency can be attained simply by specifying the liability rule  $v(y - p + m) = u(y - p)$ . Given this rule the market will then produce an allocation that maximizes  $u$ , consistent with the rule. It could be argued that common law liability is in fact this rule, so that in the absence of frictions and transactions costs the market allocation will be Common Law-efficient.

#### b. Spence criterion

It is shown in the Spence paper that imposition of producer liability will not generally achieve first best (Spence)



optimality unless the liability structure is supplemented by a tax-liability scheme. In this scheme producers incur a liability to the customer and to the state in the event of product failure, and the liability payments to the state are paid to consumers in the form of a specific subsidy on the good. However, with the possibility of advertising, such a scheme will not necessarily achieve first best optimality unless advertising is prohibited.

With liability to the state of  $f$  and a specific subsidy of  $k$  (with  $k = (1 - s)f$ ), the arguments of  $u$  and  $v$  can be written  $u(y - c(s) - (1 - s)m - a)$ ,  $v(y - c(s) + s m - a)$ , and the equilibrium conditions for  $s$  and  $a$ , given  $m$  and  $f$ , are

$$(37) \quad (a) \quad c' = m + f + r_s(u - v) / (s u' + (1 - s)v')$$

$$(b) \quad r_a(u - v) - r u' - (1 - r)v' \leq 0, \quad a > 0.$$

Since  $u$  and  $v$  are not functions of  $f$ ,  $f$  can be chosen arbitrarily to satisfy (37)(a), such a solution may not require  $a$  (or  $A$ ) = 0. Therefore the Spence liability rule should be accompanied by an advertising ban.

#### c. Comments on producer liability

The efficiency implications of common law liability rules are quite different, depending on whether the Common Law or Spence criteria for efficiency are used. The Spence (1977) view is apparently that for products without safety hazard problems, the market will operate efficiently (through recourse to common law liability), but for products for which safety hazards are important, even recourse to common law liability will not generally produce an efficient allocation.

We do not agree with this view. Attempting to deal with product failures which do not involve personal injury through the courts will often be precluded by transactions costs. Attempting to deal with such failures through direct regulation of warranty terms is cumbersome because of the problems of defining product failure and moral hazard.

On the other hand, product failures which do involve personal injury may be more amenable to resolution by the courts. Furthermore, we feel more comfortable with the Common Law criterion in cases involving personal injuries.

### 3. Other Forms of Regulation

For the other forms of regulation considered in Section III (direct regulation of reliability, regulation of misleading advertising, provision of information), the conclusions are similar: the desirability of these piecemeal policies is ambiguous.

### V. Long Run Equilibrium

We will define long-run equilibrium by the condition  $r = s$ . With the complexity introduced by the many instruments influencing consumer perceptions of reliability, the likelihood that the market will attain such a long-run equilibrium deserves attention, and we will consider this problem below. In this section we will consider the properties of such long-run equilibrium, assuming it is attained.

If  $v(x) = u(x - L)$  equilibrium conditions (3) in long-run equilibrium can be written:

$$(38) \quad (a) \quad r_s(u_1 - u_2) + ru_1'(m - c') + (1 - r)u_2'(m - c') = 0$$

$$(b) \quad r_m(u_1 - u_2) + r(1 - r)(u_2' - u_1') < 0, \quad m > 0$$

$$(c) \quad r_a(u_1 - u_2) - ru_1' - (1 - r)u_2' < 0, \quad a > 0$$

Equilibrium condition (38)(b) requires  $(u_1 - u_2) =$

$(u_1' - u_2') = 0$  in long-run equilibrium, since

$\text{sign}(u_1 - u_2) = \text{sign}(u_2' - u_1')$ . (If  $(u_1 - u_2) <$

$0, m > L$ , which is inconsistent with (33)(b), as is

$(u_1 - u_2) > 0$ ). Therefore long-run equilibrium requires  $L$

$= m, c' = L, a = 0$ , which is (first best) optimal in both the

Rawlsian and Spence senses.

For general  $v(x)$ , the long-run equilibrium conditions are:

$$(39) \quad (a) \quad r_s(u - v) + ru'(m - c') + (1 - r)v'(m - c') = 0$$

$$(b) \quad r_m(u - v) + r(1 - r)(v' - u') < 0, \quad m > 0$$

$$(c) \quad r_a(u - v) - ru' - (1 - r)v' < 0, \quad a > 0,$$

(in long-run equilibrium  $r_A = 0$ , so informative

advertising is never profitable.

These equilibrium conditions do not necessarily require  $u = v$  or  $u' = v'$ , so that long run equilibrium will not generally be first best optimal in either sense. Furthermore, long-run equilibrium is consistent with the profitability of misleading advertising.

Therefore in markets where product failure may result in injury to person or property, even if consumers correctly perceive reliability ( $r = s$ ) in equilibrium, the market may fail to allocate reliability and insurance efficiently (in either sense).

However, if in the long-run consumer perceptions of reliability are not influenced by warranties and advertising ( $r_m = r_a = r_A = 0$ ), and  $r = s$ , so that  $r_s = 1$ , the long-run equilibrium will be Spence-efficient.

## VI. Monopoly

It is easily seen that the previous analysis is essentially unchanged if the market structure is assumed to be a monopoly.<sup>12</sup>

The monopolist's problem is

$$(40) \max p - c(s) - (1 - s)m - a$$
$$\text{subject to: } ru + (1 - r)v > \hat{U}.^{13}$$

The first order condition for this problem can be written (with  $\lambda$  the Lagrange multiplier):

$$(41) \text{ (a) } 1 - \lambda [ru' + (1 - r)v'] = 0$$
$$\text{ (b) } (-c' + m) + \lambda r_s(u - v) = 0$$
$$\text{ (c) } -(1 - s) + \lambda [r_m(u - v) + (1 - r)v'] \leq 0, m > 0$$
$$\text{ (d) } -1 + \lambda [r_a(u - v)] \leq 0, a > 0^{14}$$
$$\text{ (e) } ru(y - p) + (1 - r)v(y - p + m) > \hat{U}.$$

From (41)(a),  $\lambda > 0$ , so that the constraint is binding. Using (41)(a) to substitute for  $\lambda$  in (36)(b - d), it is easily seen that (41)(b - d) are equivalent to (30)(a - c).

## VII. Dynamics

Perhaps the major shortcoming of the model we have developed here is that in a static, one period context there is no scope for consumer learning. In this section we will remedy this deficiency. The main goal of our analysis will be to determine the conditions under which the long-run equilibrium defined in Section V will be attained. Not surprisingly it will be shown that the producer signals of reliability are critical determinants of the stability (or lack thereof) of long-run equilibrium. Because of the complexity of long-run equilibrium

for general  $v(x)$  described in Section V, we will restrict our analysis to the case  $v(x) = u(x-L)$ .

#### A. Competition

Our dynamic version of the competitive model will assume that a temporary equilibrium characterized by (3) is attained in each period, but that consumer perceptions of product reliability are influenced by past experience and current producer signals. For technical simplicity we will model consumer perceptions as:

$$(42) \quad r_t = r_{t-1} - \gamma(r_{t-1} - s_{t-1}) + g(m_t - m_{t-1}) + f(a_t),$$

where  $0 < \gamma < 1$ ,  $g(0), f(0) = 0$ ;  $g', f' > 0$ ;  $g'', f'' < 0$ .

The "story" behind this specification is as follows. By the end of each period consumers learn whether they have over- or underestimated reliability, and adjust their perceptions in the next period accordingly, ceteris paribus. They also react to changes in the terms of the warranty offered. (If the terms of the warranty do not change, no new information about reliability is obtained from the warranty). Finally, we will confine our analysis to the case  $r_t > s_t$ , since the equilibrium described by (3) requires overinsurance ( $m_t > L$ ) for the case  $r_t < s_t$ , which would be ruled out by moral hazard. (Thus our assumption that  $f' > 0$ ).

Our dynamic model consists of (3) and (42). Although simplistic, we believe the model is reasonably characteristic of many consumer durables markets. In such markets (e.g., automobiles, toasters), by the time that consumers begin to develop precise estimates of reliability, the producer typically changes

the product (at least in terms of consumer perceptions). It is possible that this occurs because the producer finds it in his interest to trick or confuse his customers. However, in our competitive model these model changes will occur simply in response to changing consumer perceptions. (In our dynamic version of the monopoly model which follows, the monopolist has scope to confuse his customers by model changes). As with the static model it is important to realize that consumers are not getting "ripped off" by producers. Producers are providing the combination of product, reliability, and warranty which best satisfies consumers' demands, given their perceptions.

Substituting (42) into (3), we have:

$$(43) \text{ (a) } (r_t u'_{1t} + (1-r_t) u'_{2t}) (-c'_t + m_t) = 0$$

$$\text{(b) } g'_t (u_{1t} - u_{2t}) - r_t (1-s_t) u'_{1t} + (1-r_t) s_t u'_{2t} < 0, m > 0$$

$$\text{(c) } f'_t (u_{1t} - u_{2t}) - r_t u'_{1t} - (1-r_t) u'_{2t} < 0, m > 0$$

$$\text{(d) } r_t - r_{t-1} + \gamma (r_{t-1} - s_{t-1}) - g(m_t - m_{t-1}) + f(a_t) = 0,$$

where  $g'_t = g'(m_t - m_{t-1})$ ,  $f'_t = f'_t(a_t)$ , etc.

This is a system of nonlinear first-order difference equations in the variables  $(s_t, m_t, a_t, r_t)$ , with a stationary solution  $(s^*, L, 0, s^*)$ , where  $s^*$  is such that  $c'(s^*) = L$  (this is the long-run equilibrium defined in Section V).

Tractability forces us to limit our attention to the local stability of the stationary equilibrium. Initially, we will assume that (43) (c) holds with equality at the stationary equilibrium. (This can be shown to be the case if  $f(a_t) = \sqrt{a_t}$  and  $u'' = 0$ ). Linearizing the system (43) around the stationary equilibrium, we have

$$(44) \quad \begin{bmatrix} -u' c'' & u' & 0 & 0 \\ u' & (-u' g' + s^*(1-s^*)u'' & 0 & -u' \\ 0 & -u' f & A & 0 \\ 0 & -g' & -f' & -(1-\gamma) \end{bmatrix} \begin{bmatrix} \bar{s}_t \\ \bar{m}_t \\ \bar{a}_t \\ \bar{r}_t \end{bmatrix} \\ + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \gamma & g' & 0 & -(1-\gamma) \end{bmatrix} \begin{bmatrix} \bar{s}_{t-1} \\ \bar{m}_{t-1} \\ \bar{a}_{t-1} \\ \bar{r}_{t-1} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

where  $s_t = (s_t - s^*)$ , etc., and  $A = \lim_{r \rightarrow s^*} f''(u_1 - u_2)$ .

The functions are evaluated at the stationary solution, so that  $c'' = c''(s^*)$ ,  $g' = g'(0)$ , etc.

We now have a system of linear first order difference equations, the solution of which is of the form

$$(45) \quad \begin{bmatrix} \bar{s}_t \\ \bar{m}_t \\ \bar{a}_t \\ \bar{r}_t \end{bmatrix} = \begin{bmatrix} b_s \\ b_m \\ b_a \\ b_r \end{bmatrix} \cdot x^t, \text{ for some } x.$$

The necessary and sufficient condition for local stability of the stationary solution is  $0 < x < 1$ . ( $-1 < x < 0$  is ruled out by our assumption that  $r_t > s_t$ , for all  $t$ ).

Therefore, using standard methods<sup>15</sup>, it can be shown that stability of (44) requires

$$(46) \quad \frac{(2-\gamma)c''g' + c''(1-\gamma)s^*(1-s^*)R_A - 1}{2c''g' + c''s^*(1-s^*)R_A - 1 + c''B} < 1,$$

where  $R_A = u''/u'$ , (the standard measure of absolute risk aversion), and  $B = \lim_{r \rightarrow s^*} (f')^2 u' / A$  (if  $f(a_t) = \kappa\sqrt{a_t}$  and  $u'' = 0$ ,  $B = \kappa^2/2$ ).

The second order conditions corresponding to (43) (a - c) require that the denominator of the left-hand side of (41) be positive, which we will write

$$(47) \quad g' > B/2 + 1/2c'' - s^*(1-s^*)R_A/2.$$

Since the denominator is positive, (41) requires

$$(48) \quad (a) \quad g' > \frac{1}{(2-\gamma)c''} - \frac{(1-\gamma)}{(2-\gamma)} s^*(1-s^*)R_A$$

$$(b) \quad g' > -B/\gamma - s^*(1-s^*)R_A$$

In interpreting (47) and (48) the reader should recall that  $g' = g'(0)$  and  $B = \lim (f')^2 u' / (u_1 - u_2)$ .

Inspection reveals that (47) and (48) are more likely to be satisfied the larger is  $g'c''$ . The reason for this is that  $g'$  can be interpreted as  $\partial r_t / \partial m_t$ , and from (43) (a),  $\partial m_t / \partial s_t = c''$ . Thus,  $r_t$  is positively correlated with  $s_t$  ( $\partial r_t / \partial s_t > 0$ ).



The likelihood of stability also increases with the degree of absolute risk aversion. This is because since consumers are being insured against an absolute loss(L), for any given  $(r_t - s_t) > 0$  the larger is  $R_A$ , the smaller will be  $L - m_t$  (i.e., the larger is  $R_A$ , the more insurance they will buy against a fixed loss). The likelihood of stability is also positively correlated with the speed of adjustment of consumers' perceptions ( $\gamma$ ), which is not surprising since if  $\gamma=0$ , for example, consumers don't learn. Finally, since advertising is misleading, it is not surprising that the stronger is the effect of advertising (the larger is  $-B$ ), the less likely it is that the long run equilibrium will be attained.

If (43) (c) doesn't hold with equality at the stationary equilibrium it is easily shown that the necessary and sufficient conditions for local stability are identical to (42) and (43) with B set equal to zero. Thus the stability of the long run equilibrium is by no means certain, even in the absence of misleading advertising!

## B. Intertemporal Monopoly

In our intertemporal monopoly model we want to capture the possibility that the monopolist will perceive how consumers learn. Since such a model is intrinsically complex we will make some simplifying assumptions. First, we will assume that consumers are risk neutral, so

$$(49) \quad r(y-p) + (1-r)(y-p+m-L) = \hat{U},$$

which means the monopoly price is given by

$$(50) \quad P = \hat{U} + y - (1-r)(L-M)$$

Then the monopolist's problem can be written

$$(51) \quad \max_{\{s, m, a\}} \int_0^{\infty} e^{-it} [y - c(s) - (1-s)m - (1-r)(L-M) - a] dt$$

We will assume that consumers adjust their perceptions according to

$$(52) \quad (a) \quad r = \rho(m, a)\alpha, \quad \rho \text{ increasing and concave}$$

$$\rho_m(0, a) = \infty, \quad \rho(L, a) = 1,$$

in its arguments, and  $0 < M < L$ .

$$(b) \quad \dot{\alpha} = \gamma(s-r).$$

Forming the current valued Hamiltonian, the first order conditions for (51) are

$$(53) \quad (a) \quad -c' + m + \bar{\lambda} \gamma = 0$$

$$(b) \quad s - \rho \alpha + \alpha \rho' (L - m - \bar{\lambda} \gamma) > 0, \quad (m=L \text{ if } > 0)$$

$$(s - \rho \alpha + \alpha \rho' m(L - m - \bar{\lambda} \gamma)) < 0, \quad (m=0 \text{ if } < 0)$$

$$(c) \quad -1 + \alpha \rho a(L - m - \bar{\lambda} \gamma) < 0$$

$$(-1 + \alpha \rho a(L - m - \bar{\lambda} \gamma)) a = 0$$

$$(d) \quad \dot{\alpha} = \gamma(s-r), \quad r = \rho(m, a)\alpha$$

$$(e) \quad \dot{\bar{\lambda}} = -\rho(L-m) + \bar{\lambda} \gamma \rho + \bar{\lambda} i$$

These F.O.C.'s have a stationary solution at

$$(54) (s, m, r, \bar{\lambda}, \rho, \alpha) = (r^*, L, r^*, 0, 1, r^*),$$

where  $c'(r^*) = L$ . Since the Hamiltonian is linear in  $\alpha$  and  $\bar{\lambda}$  and concave in  $(s, m, a)$ , the sufficient conditions can be shown to hold.

We will assume that (53)(b) holds with equality at the stationary equilibrium, but that (53)(c) is an inequality. Therefore, in the neighborhood of the stationary equilibrium we can suppress  $a$ . Differentiating (53)(a) and (b) and evaluating the derivatives at the stationary equilibrium,

$$(55) \begin{bmatrix} -c'' & 1 \\ 1 & -2\alpha\rho \end{bmatrix} \begin{bmatrix} ds \\ dm \end{bmatrix} = \begin{bmatrix} -r d\bar{\lambda} \\ \rho d\alpha \end{bmatrix}$$

The S.O.C.'s require the determinant of the L.H.S. is positive, which requires

$$(56) \quad c''\rho' > 1/2r^*.$$

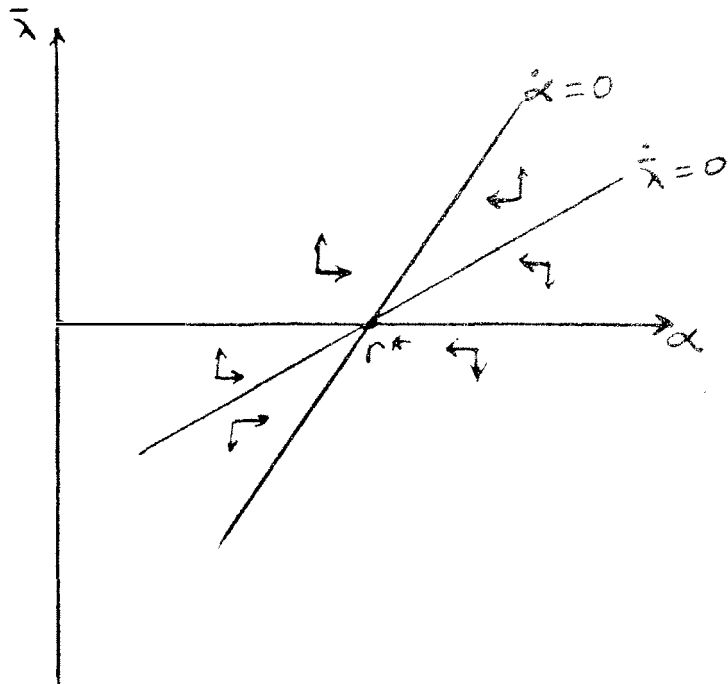
Notice the similarity to the competitive case--the stationary equilibrium again requires that  $c''\rho'$  ( $= \partial r / \partial s$ ) be large.

Using (55) and (53)(d) and (e), the slopes of the stationary curves at the stationary solution can be determined:

$$(57) \quad (a) \quad (d\bar{\lambda}/d\alpha) \Big|_{\alpha=0} = c''/\gamma$$

$$(b) \quad (d\bar{\lambda}/d\alpha) \Big|_{\lambda=0} = 0 = \frac{c''}{\alpha(2c''\rho'r^*) + \gamma} < c''/\gamma,$$

since by the S.O.C.'s,  $2c''\rho'r^* > 1$ . Using (57), the phase plane can be drawn locally:



Now consider the optimal motion in a neighborhood of the stationary solution starting at some  $\alpha > r^*$ . We see from the phase plane that for  $\alpha > r^*$ ,  $\bar{\lambda} > 0$  and  $\dot{\alpha} < 0$ . Therefore,  $s - r < 0$  and so by (53)(b),  $m + \bar{\lambda} \alpha < L$ , or  $m < L - \bar{\lambda} \alpha < L$ . Therefore, starting at  $\alpha > r^*$ , the warranty coverage is less than  $L$ , but increasing over time. Since  $m + \bar{\lambda} \alpha < L$ , by (53)(a),  $s < r^*$  and is increasing over time.

Although the dynamic competitive and monopoly models are somewhat different, a comparison of the results derived from the two models is somewhat provocative. Under similar conditions the long run equilibrium is globally stable in the monopoly model, but can only be shown to be locally stable in the competitive model! Thus the market is more likely to converge to an equilibrium with perfectly informed consumers under monopoly than under a competitive market structure.

### VIII. Summary

In this paper we investigated the implications of the assumption that consumer perceptions of product reliability could be influenced by the terms of warranty offered, and by "advertising." We showed that for products for which safety hazards are not an important adjunct to product failure, misleading advertising is likely to be used by producers, and that such advertising may be designed to influence consumers to either overestimate or underestimate reliability. For such products "informative" advertising will never be used by producers. For products for which safety hazards are an important adjunct to product failure, it was shown that equilibrium may be consistent with either misleading or informative advertising.

Both competitive and monopoly models were constructed, and we showed that the monopoly market equilibrium is qualitatively identical to the competitive market equilibrium.

The market equilibrium was shown to be ex ante efficient. Two concepts of ex post efficiency were considered: (i) a Common Law criterion (equal ex post utility), and (ii) a criterion proposed by Spence (maximization of average ex post utility). The market allocation was shown to be generally ex post inefficient (in either sense). Four policy instruments were evaluated: (i) imposition of producer liability, (ii) device regulation of product reliability, (iii) prohibition of misleading advertising, and (iv) provision of information on product reliability. Imposition of producer liability achieves (first

best) Common Law-efficiency and Spence-efficiency if product failure doesn't involve safety hazards. Use of the other policy instruments is not always justified, even on second best efficiency grounds.

Finally, the positive and normative attributes of a 'long run' equilibrium (in which consumer perceptions of reliability are accurate) were analyzed. We constructed a dynamic model incorporating consumer learning, and derived the conditions under which such a long run equilibrium would exist, be stable, and be efficient.

#### FOOTNOTES

<sup>1</sup> For a review of Federal Trade Commission policies in the U.S. see Post Purchase Consumer Remedies (1980). For a discussion of Canadian policies see Scheffman and Appelbaum.

<sup>2</sup> See Post Purchase Consumer Remedies (1980).

<sup>3</sup> This assumption is made purely for technical convenience. All the results follow without it.

<sup>4</sup> Courville and Hausman (1979) investigate some of the implications of  $\partial r / \partial m \neq 0$ .

<sup>5</sup> Dropping the inelastic demand assumption, the representative consumer's expected utility function can be written  $rV(z, N) + (1-r)V(z, N+(m-L)z)$ , where  $z$  is the number of units of the consumer goods, and  $N$  is the number of units of the composite of other goods consumed. Maximization of this utility function with respect to  $z$  and  $N$ , subject to the budget constraint results in an indirect utility function  $v(p, y, r)$  with  $v_p = -(rV_N^1 + (1-r)V_N^2)z$ ,  $v_r = (v^1 - v^2)$ . All of the results can then be shown to follow.

<sup>6</sup> This would probably be ruled out by moral hazard problems.

<sup>7</sup> We use the Common Law rather than a Rawlsian criterion here because the Common Law criterion would appear to be the basis of tort law.

<sup>8</sup> Suggested in Spence (1977).

<sup>9</sup> Ex post efficiency of course requires  $a=A=0$ .

<sup>10</sup> This allocation is also Rawlsian-efficient.

<sup>11</sup> The Common Law criterion will generally not be (Pareto) efficient for second best policies since the Rawlsian criterion may result in unequal utility levels in second best situations.

<sup>12</sup> This is shown for a simpler model in Courville and Hausman (1979).

13 The results are unaffected by dropping the inelastic demand assumption.

14 As with the competitive case, informative advertising will never be profitable.

15 See Baumol (1959), Chapter 16.



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