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Margaret E. Slade

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Margaret E. Slade*

Bureau of Economics

Federal Trade Commission

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I. INTRODUCTION

The idea that limited natural resources limit economic growth dates at least as far back as the early 19th century, when the British classical economists, particularly Malthus, Ricardo, and Mill, theorized about the steady-state (no-growth) society. Materials shortages related to World War II and the Korean War led to a renewed interest in the subject of natural-resource adequacy. The formation of the U.S. President's Materials Policy Commission (The "Paley" Commission, 1952) as well as the publication of several theoretical and empirical studies (Barnett and Morse, 1963, and Potter and Christy, 1962, for example) are evidence of concern at those times with the role of natural resources in economic growth. In the 1970's, several events and trends, including the Arab oil embargo, the Organization of Petroleum Exporting Countries' (OPEC) price increases, and the United States' growing dependence on imports of many minerals, led to another round of presidential commissions and research efforts attempting to assess natural-resource adequacy.[1] However, there is still no consensus among economists as to whether natural-resource commodities are becoming scarce relative to other factors of production.

One indication of scarcity would be an increase in the real price of natural-resource commodities. In the theoretical literature of exhaustible resources, models are developed that predict an exponential increase in price net of marginal extraction cost over time (Hotelling, 1931, and Solow, 1974, for example). In

contrast, empirical studies, such as that by Barnett and Morse (1963), have found a relative decline in natural-resource commodity prices. Two recent empirical studies by Barnett (1979) and Smith (1979), both updates of Barnett and Morse, reached different conclusions from one another. Barnett maintained that the original Barnett and Morse judgment still holds--there is no sign of an upturn in either real cost or relative price of the output of the extractive industries[2], whereas Smith concluded that the data are insufficient to support the hypothesis of no increase in natural-resource scarcity.

This paper is an attempt to reconcile the theoretical predictions of an increase in prices over time with the empirical findings of falling real prices. A model for long-run price movements of the nonrenewable natural resource commodities (the mineral commodities) when there is exogeneous technical change and endogenous change in the grade of ores mined is derived that suggests a U-shaped time path for relative prices. The relative-price movements implied by this model are tested for all the major metals and fuels, and the model parameters are found to be statistically significant for eleven out of twelve commodities.

The organization of this paper is as follows. In the next section, a theoretical model of price movements for nonrenewable natural-resource commodities is developed. In section III, the data are discussed, and in section IV, the fitted linear and quadratic trends are presented and analyzed. Finally, in the last section, conclusions are drawn.

II. LONG-RUN PRICING MODEL

The most frequently proposed indices of natural-resource-commodity scarcity are relative price (the ratio of an extractive-industry price index to an overall price index), unit cost (labor or labor plus capital inputs per unit of extractive-industry output), and rental rate (the marginal value of the resource in the ground). Several authors (Brown and Field,1978, Fisher,1979, and Smith,1978) have recently addressed the issue of the appropriate choice of scarcity index. Following Fisher and Smith, I choose to focus on relative price as the appropriate measure.[3]

The theoretical model of real-price movements for nonrenewable natural-resource commodities developed here is a modification of a model due to Schultze (1974). In presenting the model, the following notation will be used. Let

- $Q(t)$ be the output of metal in the extractive industry at time t,[4]
- $g(t)$ be the grade of ore mined at time t,[4] where grade is ordered by increasing extraction cost, so that $g < g'$ implies that it is less costly to produce a unit of metal from ore of grade g than from ore of grade g',
- $B(Q)$ be the benefit or willingness to pay for Q (i.e., the area under the demand curve),
- $C(Q,g,t)$ be total extraction and processing cost. Total cost depends on the level of output and the grade extracted as well as on time (a measure of technical change in the industry),
- $f(g)$ be the density of metal for grade g, in the sense that the total amount of metal in the grades between g and $g + \Delta g$ is approximately

$$f(\tilde{g})\Delta g, \qquad g \leq \tilde{g} \leq g + \Delta g,$$

and ρ be the social discount rate.

The problem is to choose a time path for extraction rates that will maximize the discounted stream of current and future benefits minus costs. The extraction rate at time t, Q(t), is equal to the rate of change of grade, $\dot{g}$, (where a dot over a variable denotes its time rate of change) times the density function, f(g) (the metal available at that grade). Therefore, choosing an extraction rate is equivalent to choosing the rate of change of grade. We thus wish to maximize

$$\max_{\dot{g}} \int_0^{\infty} e^{-\rho t} [B(Q)-C(Q,g,t)]dt \tag{1}$$

subject to the production relationship

$$Q(t) = \dot{g}(t)f(g(t)). \tag{2}$$

This optimal-control problem can be solved by introducing the costate variable, $\lambda(t)$, and forming the Hamiltonian, H

$$H = e^{-\rho t}[B(\dot{g}f) - C(\dot{g}f,g,t)] - \lambda\dot{g}. \tag{3}$$

The first-order conditions for an interior maximum of (1) are

$$H_{\dot{g}} = e^{-\rho t}[B'f - C_Q f] - \lambda = 0$$

$$\text{or} \quad B' = P(Q) = C_Q + \lambda e^{\rho t}/f, \tag{4}$$

where P(Q) is the inverse demand function, and

$$\dot{\lambda} = H_g = e^{-\rho t}[B'\dot{g}f' - C_Q\dot{g}f' - C_g]. \tag{5}$$

Differentiating (4) with respect to time we obtain

$$\dot{P} = \dot{C}_Q + \frac{\dot{\lambda}e^{\rho t} + \lambda\rho e^{\rho t}}{f} - \frac{\lambda e^{\rho t} f'\dot{g}}{f^2}. \tag{6}$$

Substituting (5) into (6) we have

$$\begin{aligned}\dot{P} &= \dot{C}_Q + \frac{P\dot{g}f' - C_Q\dot{g}f' - C_g + \lambda\rho e^{\rho t}}{f} - \frac{\lambda e^{\rho t} f'\dot{g}}{f^2} \\ &= \dot{C}_Q + \frac{P\dot{g}f' - P\dot{g}f'}{f} - \frac{C_g}{f} + \frac{\lambda\rho e^{\rho t}}{f} \\ &= \dot{C}_Q - \frac{C_g}{f} + \frac{\lambda\rho e^{\rho t}}{f}. \end{aligned} \tag{7}$$

If we make the simplifying assumption that marginal cost, C_Q, is is constant for a given grade and state of technology and is an additive function of its two arguments, g and t; i.e.,

$$C = [h(g) + k(t)]Q, \tag{8}$$

$$\text{then } \dot{P} = h'\dot{g} + \dot{k} - \frac{h'Q}{f} + \frac{\lambda\rho e^{\rho t}}{f} = \dot{k} + \frac{\lambda\rho e^{\rho t}}{f}. \tag{9}$$

Define $\hat{\lambda}$ by

$$\hat{\lambda} = \lambda e^{\rho t}/f. \tag{10}$$

Then by (4), $\hat{\lambda} = P - C_Q$, so that $\hat{\lambda}$ is the rental rate or the marginal value of the resource in the ground, and (4) and (9) become

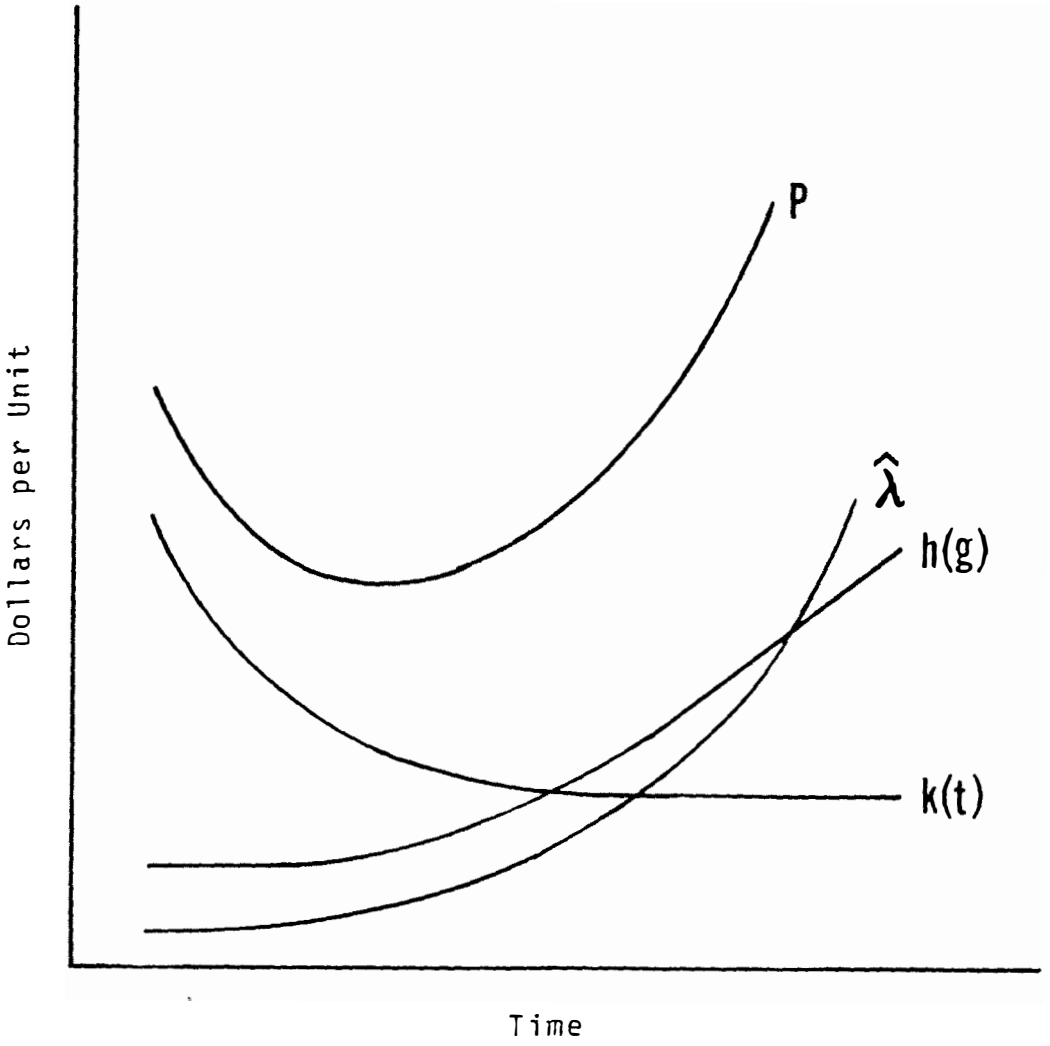
$$\begin{aligned} P &= C_Q + \hat{\lambda} \\ &= h(g) + k(t) + \hat{\lambda} \end{aligned} \tag{11}$$

$$\text{and } \dot{P} = \dot{k} + \rho\hat{\lambda}. \tag{12}$$

Price equals marginal extraction cost plus rent, and the rate of change of price is equal to the rate of change of marginal cost due to changes in technology plus the discount rate times rent. Without technical change, prices increase with time because $\hat{\lambda}$ is always positive. However, if the rate of technical change is sufficiently large so that $\dot{k}$ is sufficiently negative, prices will fall.[5] If k falls with time, but at a decreasing rate, while $\hat{\lambda}$ increases with time, the price path will generally be U-shaped. Figure 1 shows the price path when $\hat{\lambda}$ and h increse with time and k falls.[6]

The copper industry illustrates the historic counterbalancing influences of improvements in technology and deterioration in ore quality in determining production cost. In the period between 1900 and today, the average grade of copper ores mined in the United States declined from about 5 percent to 0.7 percent (U.S. Bureau of Mines, 1975). In spite of this decline in grade, real copper price fell until about 1940. The fall in price was made possible by technological developments in the early part of the century, particularly the advent of large earth-moving equipment, which made possible the strip mining of extremely low-grade ore bodies, and the discovery of froth flotation, which made concentration of low-grade sulfide ores very economical. However, by 1940, the switch to the new technology had reached its natural limits, and, since that time, with no fundamentally new technological development in the industry, the decline in grade has become the dominant factor in determining cost. Peterson and

FIGURE 1
MARGINAL-COST AND PRICE PATHS FOR MINERAL COMMODITIES



Maxwell (1979) document the history of ore grade-technology tradeoffs for other metals (silver, tin, lead, zinc, and iron) and claim that these tradeoffs have been much more important than the discovery of new deposits in the historic determination of metal prices.

Equation 12, although suggesting a U-shaped time path for relative prices, does not specify a specific functional form. Because the slope of the marginal-cost curve is determined by many factors that differ for each commodity, the simplest U-shaped time path, the quadratic, was chosen for estimation purposes.[10] In section IV, fitted quadratic trends for each of eleven metals and fuels will be compared with linear trends for the same commodities.

III. DATA SOURCES

The data consist of annual time series for the period 1870 (or year of earliest available figures) to 1978[8] for all the major metals and fuels with the exception of gold.[9] Prices were deflated by the U.S. wholesale price index (1967 = 1) and are thus in 1967-constant dollars. For some commodities, prices of both ore and metal are available (bauxite and aluminum, for example). However, for consistency, metal prices were always used.[10]

Table I lists the eleven commodities, the units of measurements of their prices, and the data sources. Seven of the eleven price series were taken form Manthy (1978). These series were

TABLE I

DATA SOURCES

Commodity	Units	Source
Aluminum	¢/lb	Schurr, Metal Statistics
Copper	¢/lb	Manthy
Iron	Index (1951-53=100)	Manthy
Lead	¢/lb	Manthy
Nickel	¢/lb	Minerals Yearbook
Silver	¢/oz	Schurr, Metal Statistics
Tin	¢/lb	Metal Statistics
Zinc	¢/lb	Manthy
Coal	$/short ton	Manthy
Natural Gas	¢/1000 ft^3	Manthy
Petroleum	$/bbl	Manthy

updated to 1978 whenever possible by using the sources listed in Manthy (tables MP-3 and MP-6, pp. 211-212).

In addition to the eleven commodities of table I, linear and quadratic trends were fitted to the mineral-aggregate price series constructed by Manthy. However, the commodities included in Manthy's index are not the same as those shown in table I. An aggregate price index could be constructed from the eleven series of table I. However, constructing an aggregate series is a questionable practice because it entails linking the prices of commodities that were not produced in the early years of the period to prices of related commodities (natural gas to petroleum, for example).[11] Therefore, no attempt was made to aggregate the prices of the metals and fuels in table I.

IV. EMPIRICAL RESULTS--QUADRATIC AND LINEAR TRENDS

Tables II and III and figures 2-13 show the fitted linear and quadratic price trends for the major metals and fuels and the aggregate commodity; the trends are based on the equations

$$P_{it} = a_{0i} + a_{1i}t + u_{it} \tag{13}$$

and

$$P_{it} = b_{0i} + b_{1i}t + b_{2i}t^2 + v_{it} \tag{14}$$

where

P_{it} is the deflated price of the ith commodity at time t

t is time measured in years (1800=0)

and u_{it} and v_{it} are random error terms.[12]

In the tables, t-statistics of the estimated coefficients are shown in parentheses under the corresponding coefficients.

In table II, some of the linear-trend coefficients are positive whereas others are negative, but only seven of the twelve are statistically significant at the 90 percent level of confidence. Therefore, no generalization can be made about natural-resource scarcity from the linear model. In contrast, in table III, the trend coefficients for all twelve commodities have the same signs--the coefficients of the linear terms are negative and those of the quadratic terms are positive--implying the predicted convex curvature. In addition, eleven of the twelve quadratic-term coefficients are significant at the 90 percent confidence level (lead is the exception). Figures 2-13 show that the fitted linear models underestimate relative prices in the last years of the 1870-1978 period for all twelve commodities and that even though relative prices of some commodities fell during a considerable part of the last century (note aluminum in particular), prices of every commodity have passed the minimum points on their U-shaped curves and have begun to increase.

With the exception of lead and zinc, the quadratic curvature for the individual commodities is fairly pronounced. However,

TABLE II

FITTED LINEAR TRENDS

Commodity	a_0	Coef. of time	R^2	F	ρ_1 [a]
Aluminum	180	-.95*** (-4.6)	.91	790	.73
Copper	61	-.18** (-2.0)	.70	244	.77
where Iron	85	.13 (.59)	.65	190	.80
Lead	14	.002 (.10)	.51	111	.72
Nickel	-114	1.3** (2.6)	.86	386	.90
Silver	-526	3.7 (.96)	.95	1967	.98
Tin	- 17	.97*** (4.4)	.76	291	.72
Zinc	17	-0.14 (-1.1)	.03	3.6	.14
Coal	-13	.13*** (2.6)	.89	736	.97
Natural Gas	97	-.78** (-1.8)	.84	297	1.0
Petroleum	1.9	.014* (1.5)	.59	151	.73
Aggregate	22	-.021 (-.57)	.64	181	.71

*** Denotes significance at the 99 percent confidence level.

** Denotes significance at the 95 percent confidence level.

* Denotes significance at the 90 percent confidence level.

[a] ρ_1 is the auto correlation coefficient of the error term.

TABLE III

FITTED QUADRATIC TRENDS

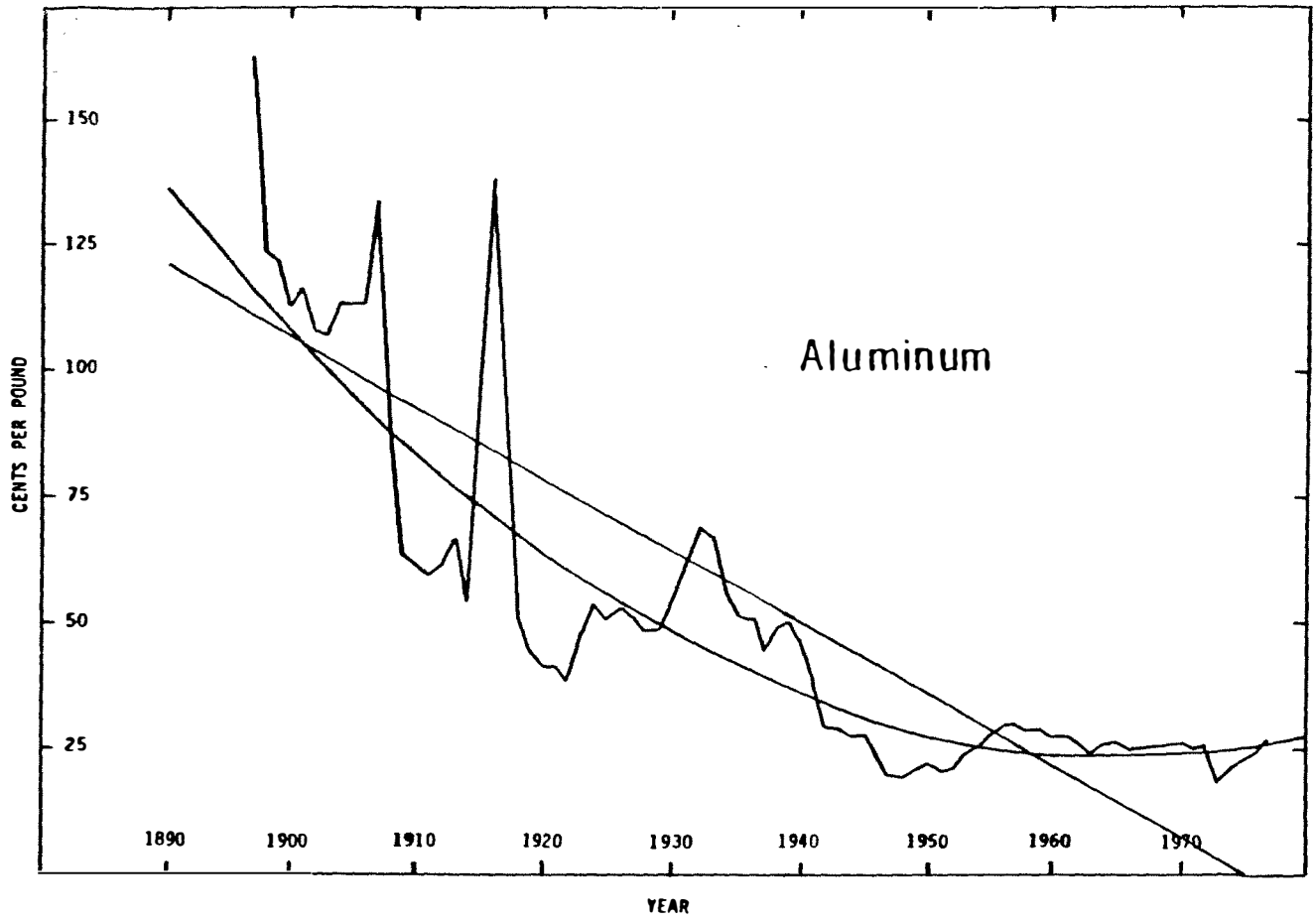
Commodity	b_0	Coef. of time	Coef. of time^2	R^2	F	ρ_2[a]
Aluminum	563	-6.5*** (-3.3)	.020*** (2.8)	.91	413	.64
Copper	165	-1.9*** (-3.5)	.0070*** (3.2)	.72	133	.68
Iron	354	-4.4*** (-4.8)	.018*** (4.8)	.69	110	.60
Lead	22	-.14 (-.9)	.00057 (.9)	.52	56	.71
Nickel	626	-8.4* (-1.3)	.031* (1.5)	.86	194	.85
Silver	1692	-23*** (-7.6)	.083*** (7.1)	.96	1157	.82
Tin	205	-2.6 (-1.2)	.014** (1.7)	.77	149	.69
Zinc	30	-.23** (-2.0)	.00086** (1.9)	.06	3.6	.10
Coal	26	-.40** (-1.9)	.0017** (2.3)	.88	381	.92
Natural Gas	633	-8.3*** (-6.3)	.028*** (6.5)	.88	205	.80
Petroleum	10	-.12** (-1.7)	.00051** (1.8)	.60	78	.68
Aggregate	46	-.42* (-1.4)	.0016* (1.3)	.65	91	.68

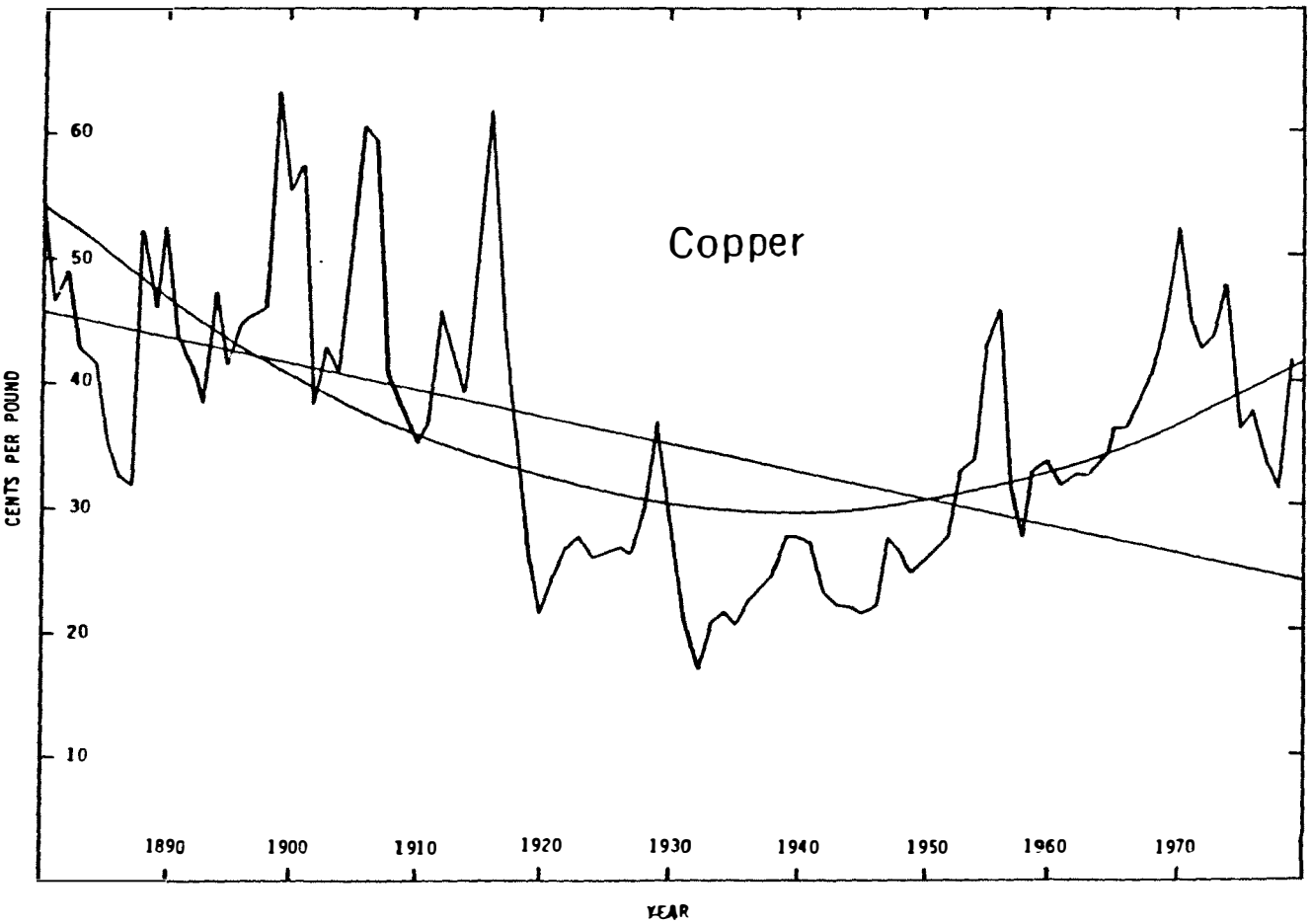
[a] ρ_2 is the auto correlation coefficient of the error term.

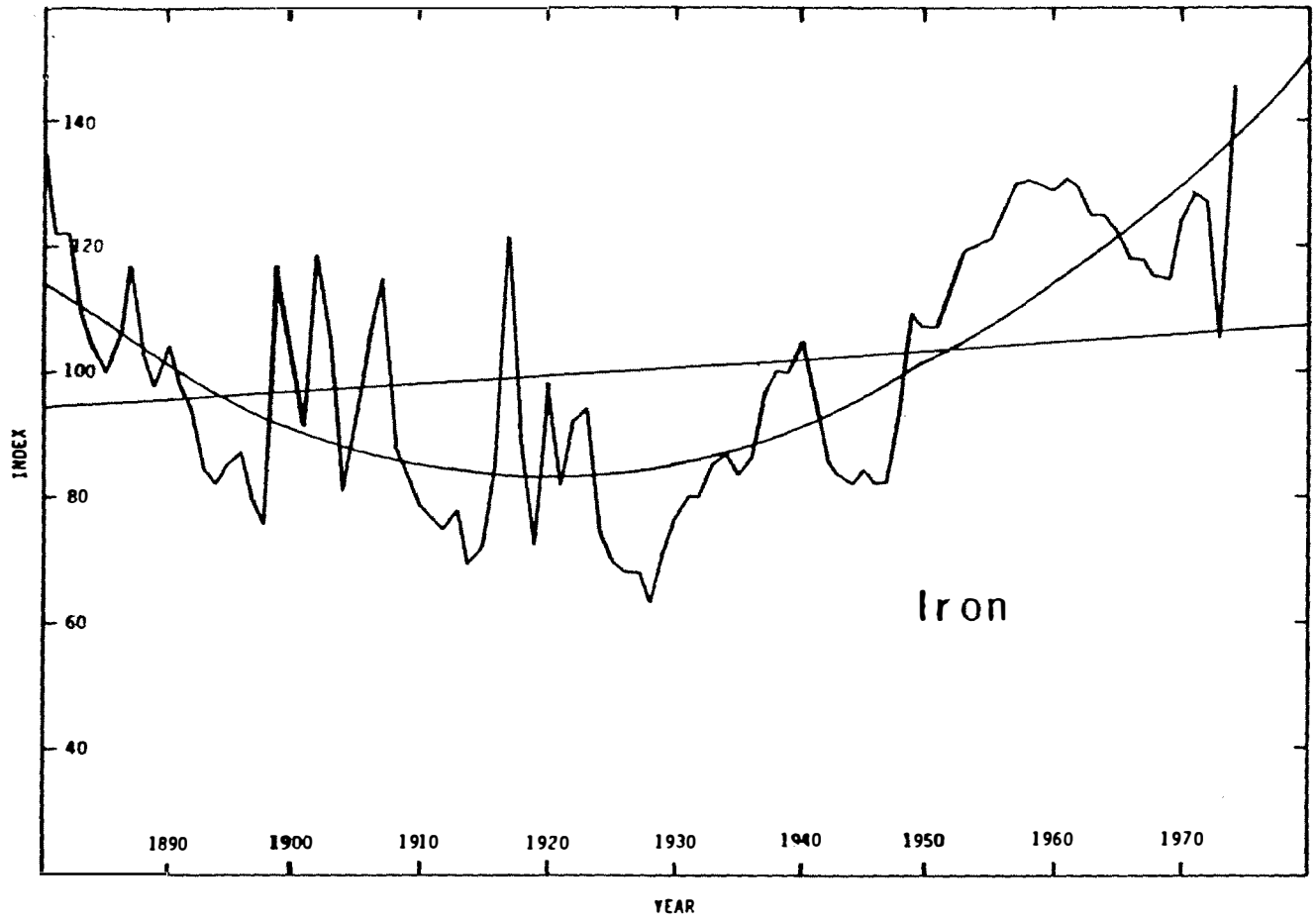
FIGURES 2 - 13

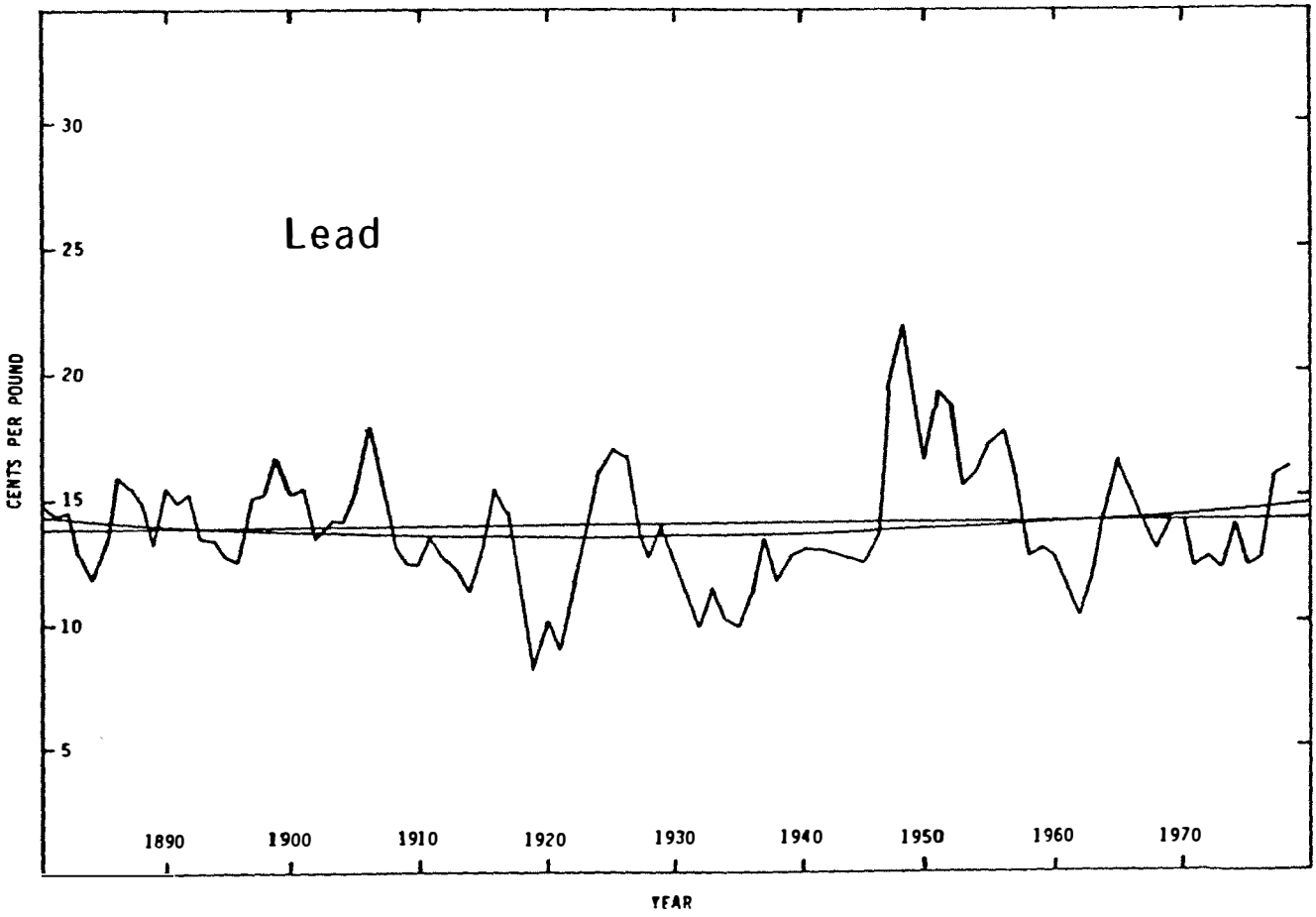
History of Deflated Prices and Fitted Linear and Quadratic Trends

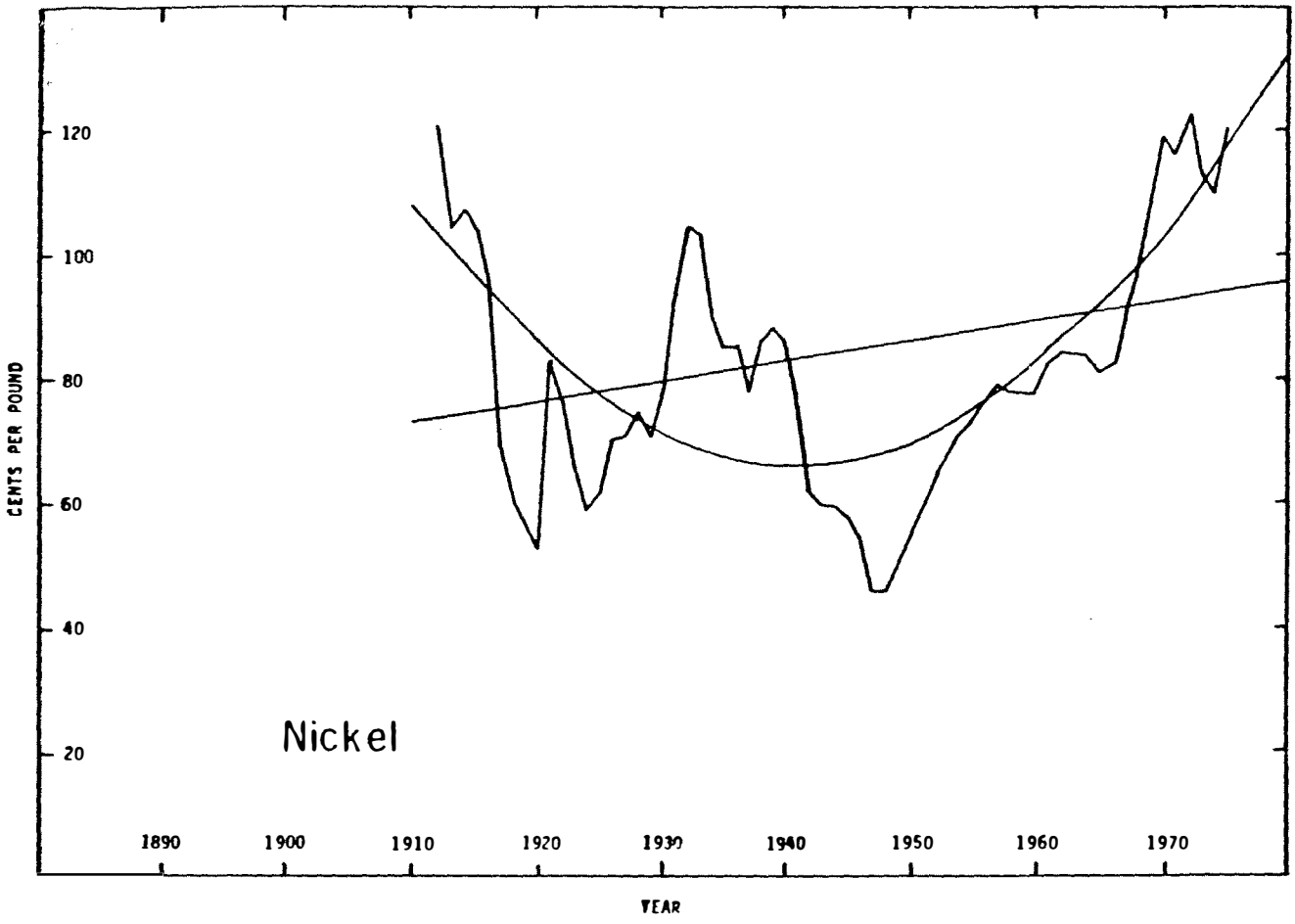
for the Major Metals and Fuels

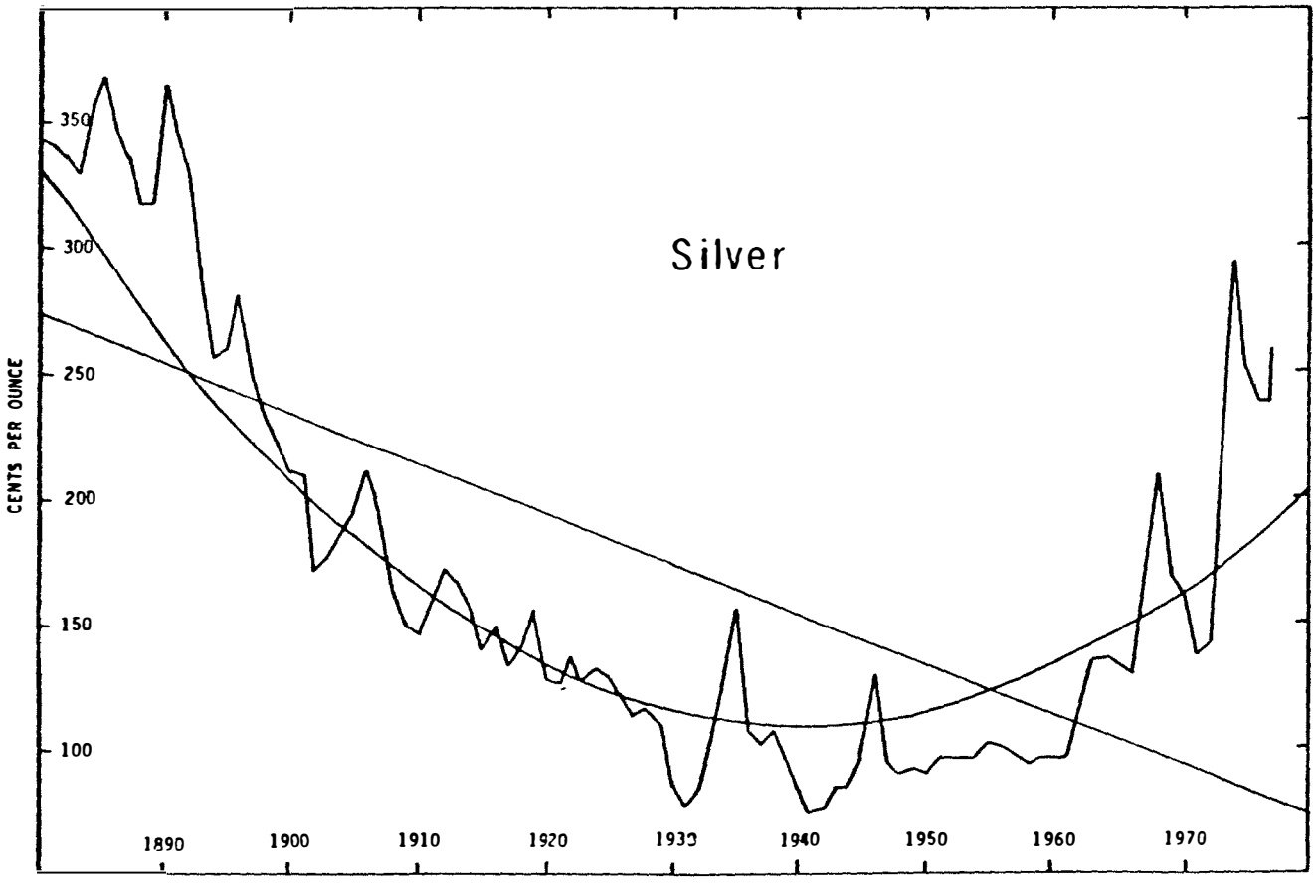

Aluminum
CENTS PER POUND
150
125
100
75
50
25
1890
1900
1910
1920
1930
1940
1950
1960
1970
YEAR

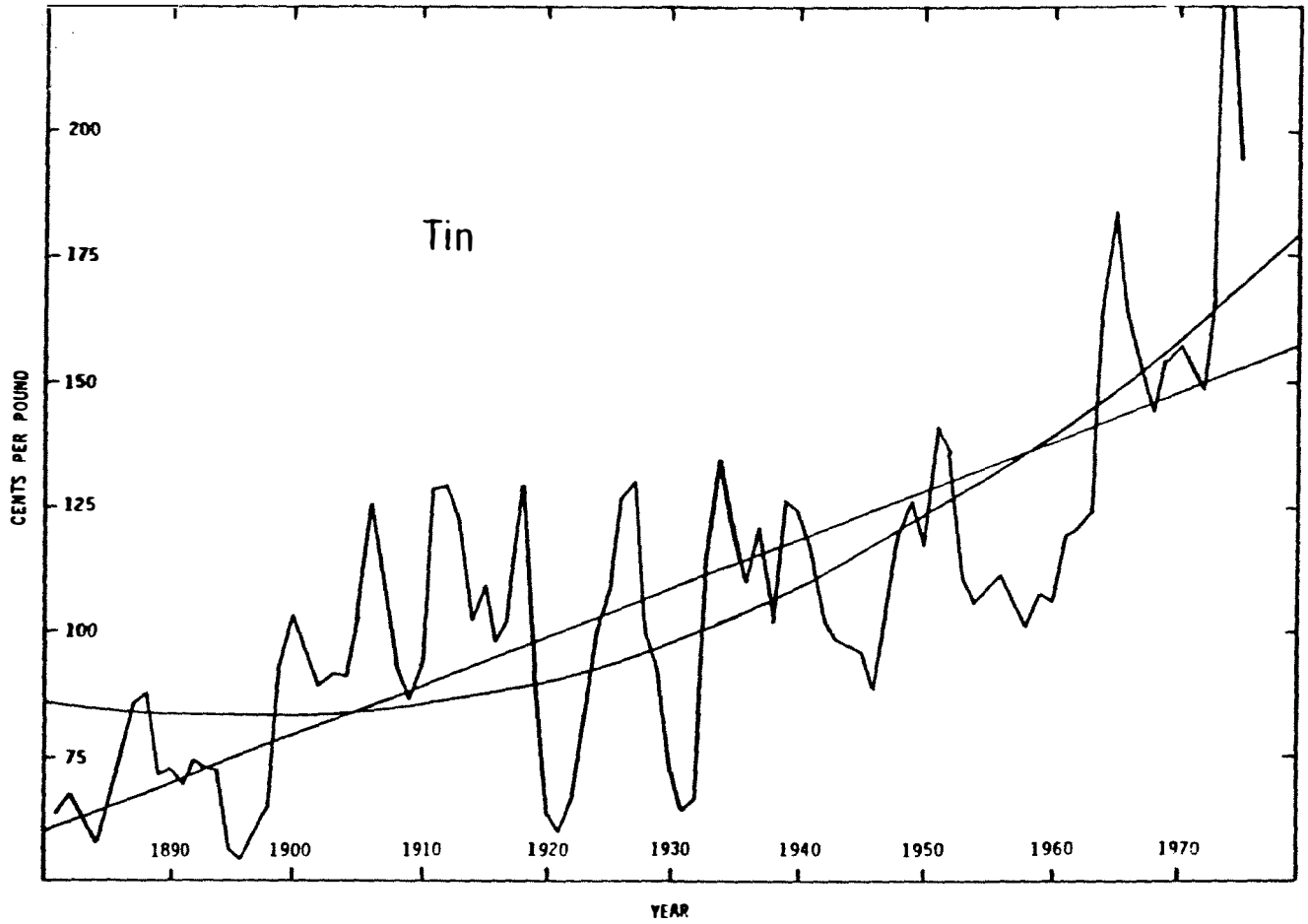

Copper
CENTS PER POUND
60
50
40
30
20
10
1890
1900
1910
1920
1930
1940
1950
1960
1970
YEAR

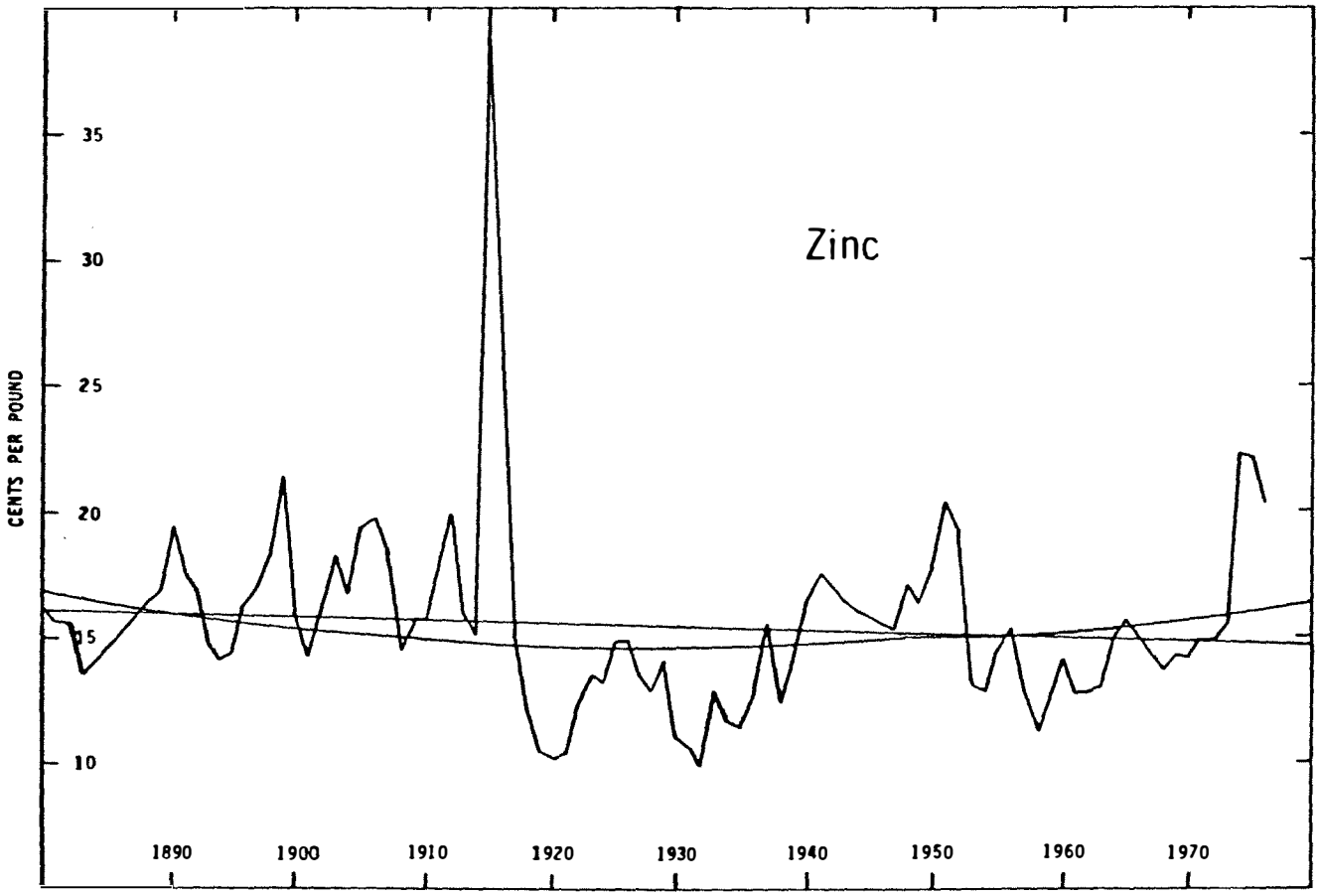

140
120
100
80
60
40
INDEX
Iron
1890
1900
1910
1920
1930
1940
1950
1960
1970
YEAR

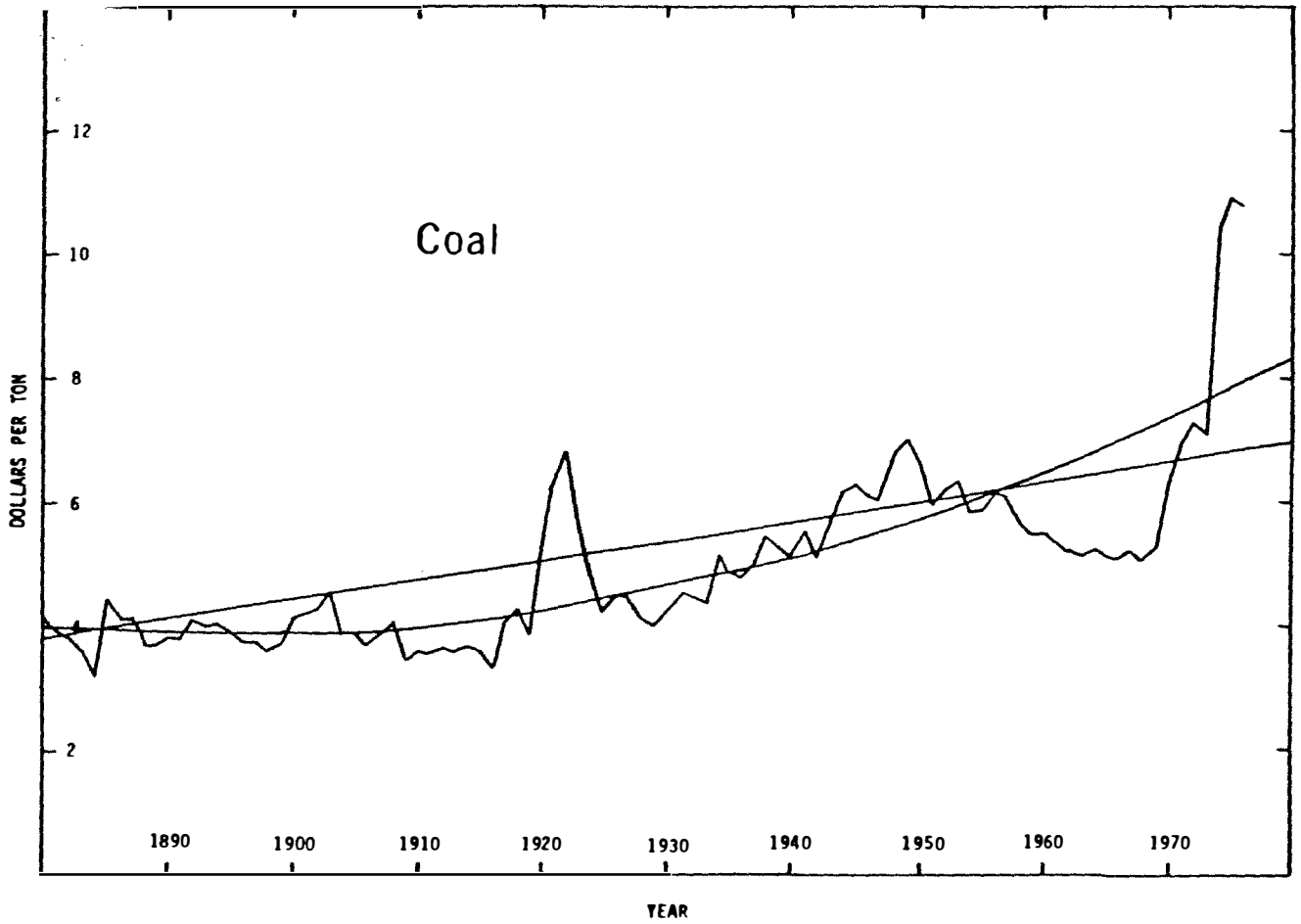

30
25
20
15
10
5
CENTS PER POUND
Lead
1890
1900
1910
1920
1930
1940
1950
1960
1970
YEAR

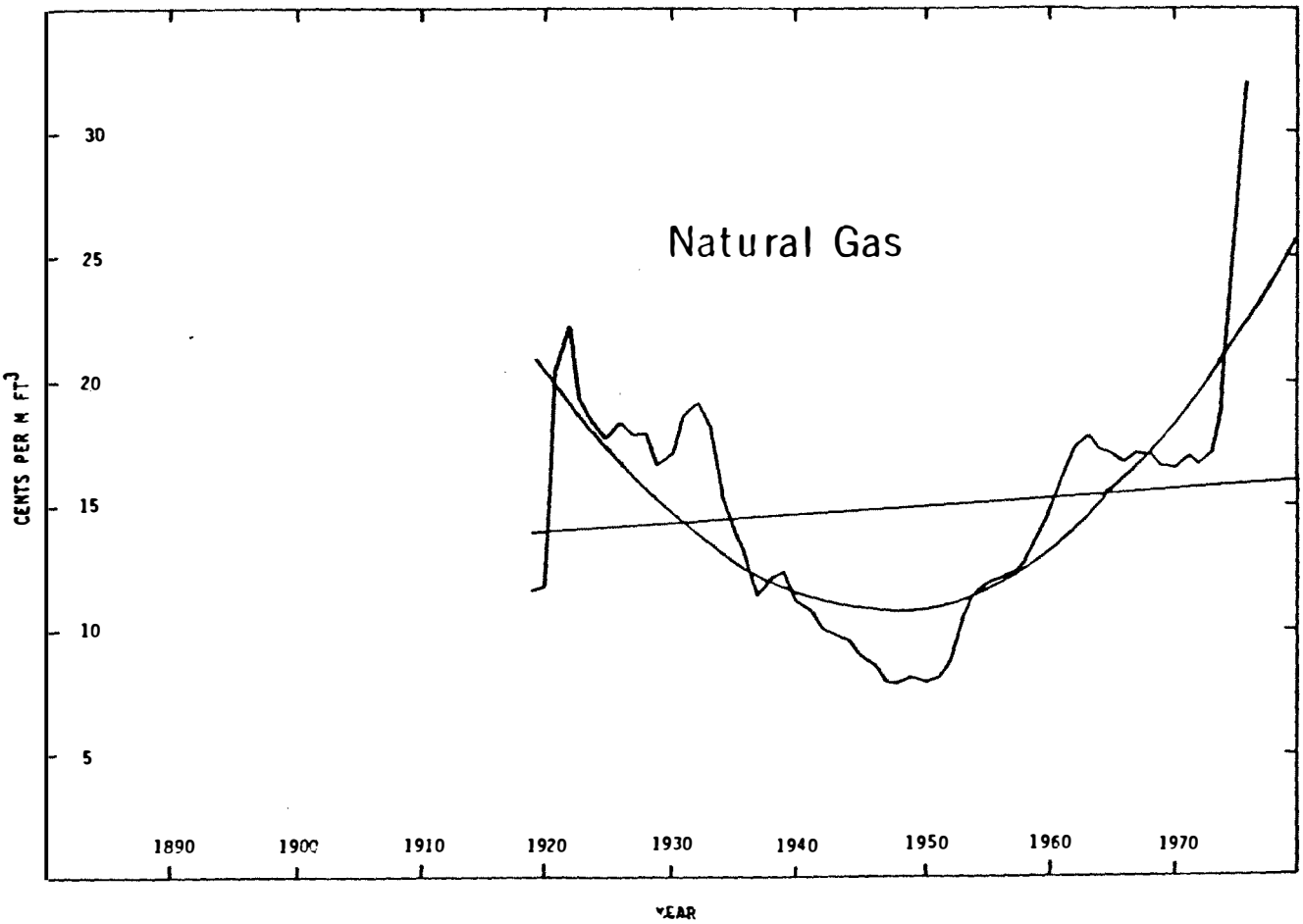

Nickel
CENTS PER POUND
120
100
80
60
40
20
1890
1900
1910
1920
1930
1940
1950
1960
1970
YEAR

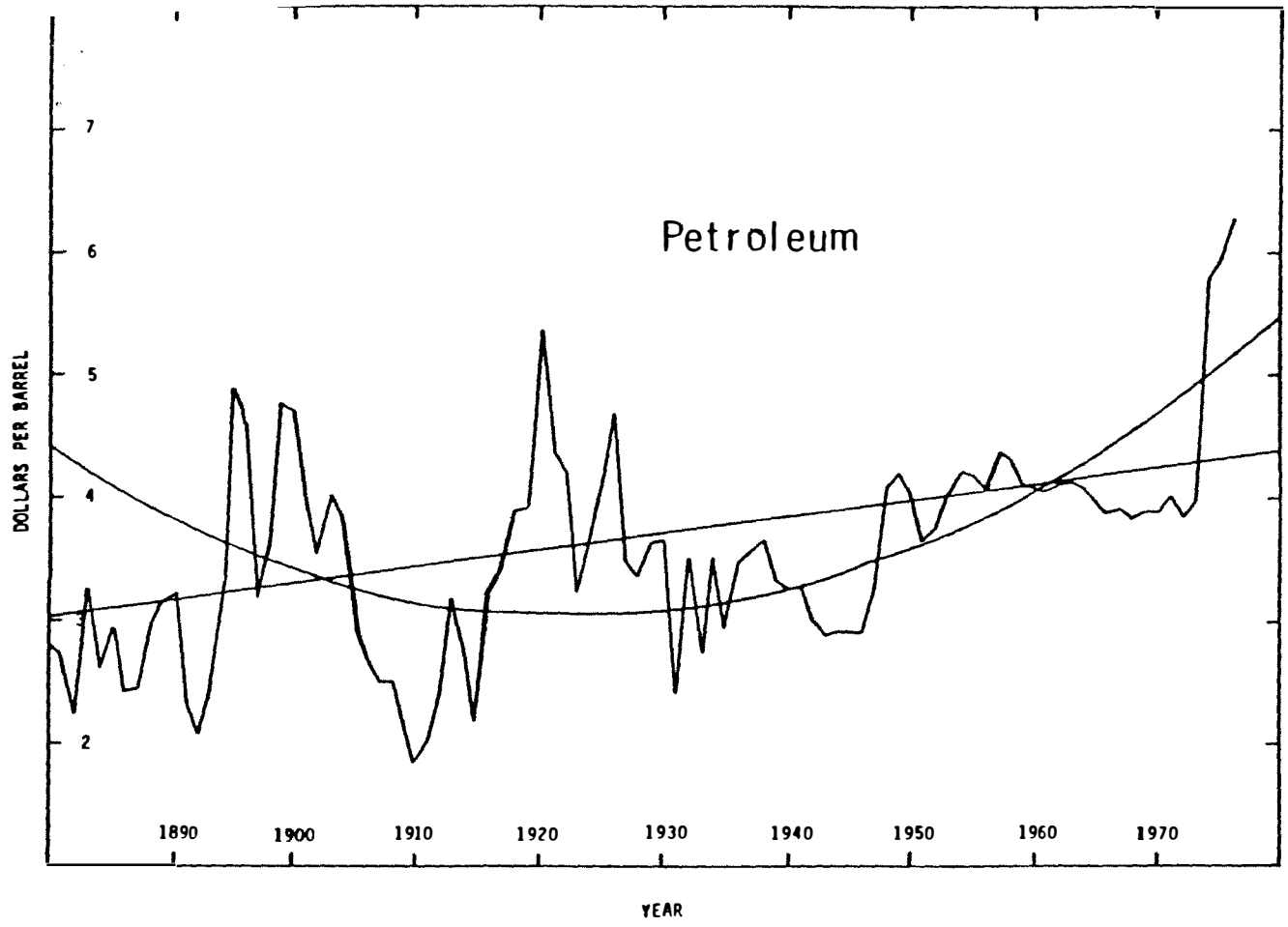

Silver
CENTS PER OUNCE
350
300
250
200
150
100
1890
1900
1910
1920
1930
1940
1950
1960
1970

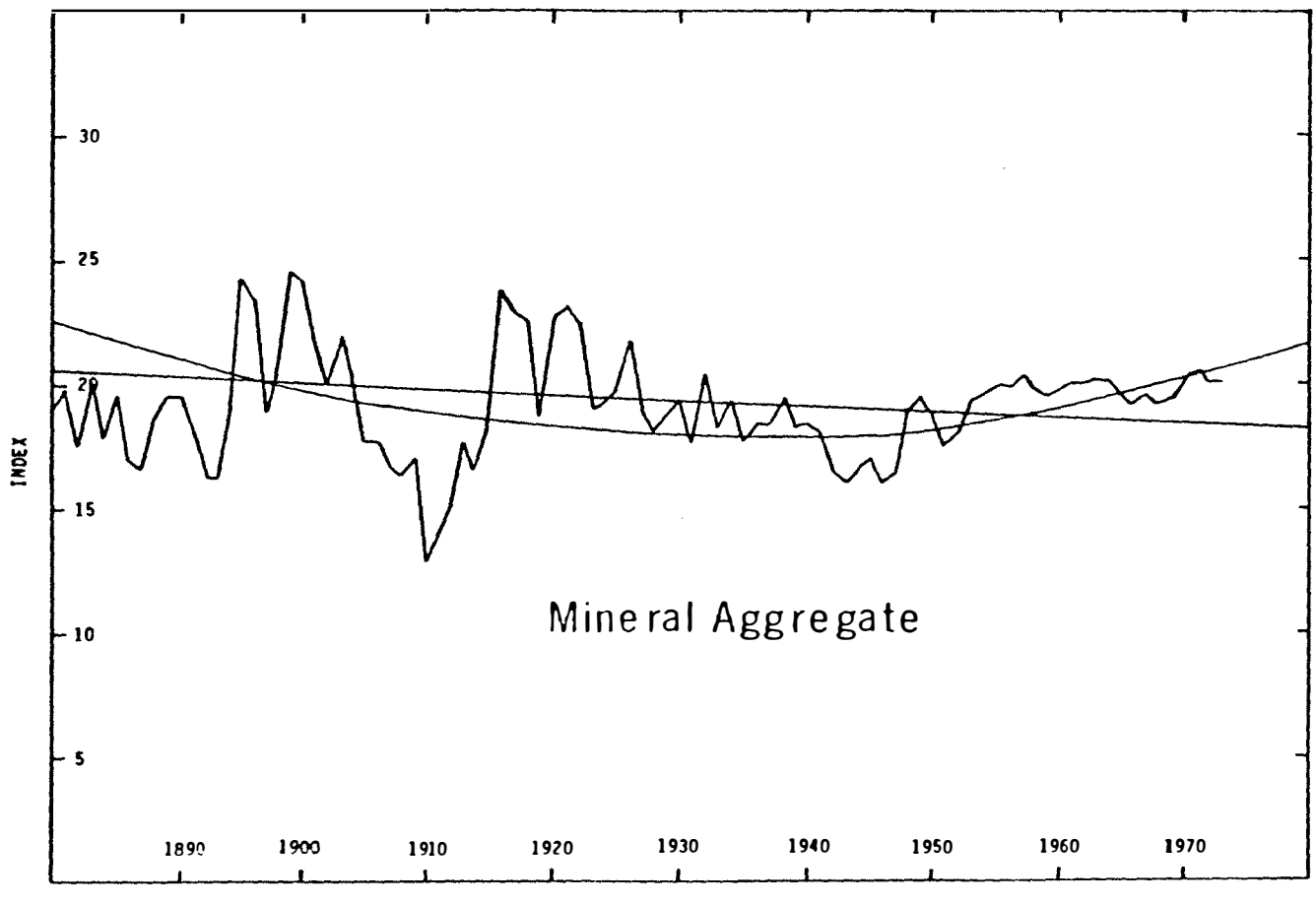

Tin
CENTS PER POUND
200
175
150
125
100
75
1890
1900
1910
1920
1930
1940
1950
1960
1970
YEAR


Zinc
CENTS PER POUND
35
30
25
20
15
10
1890
1900
1910
1920
1930
1940
1950
1960
1970
YEAR


Coal
DOLLARS PER TON
12
10
8
6
4
2
1890
1900
1910
1920
1930
1940
1950
1960
1970
YEAR


Natural Gas
CENTS PER M FT^3
30
25
20
15
10
5
1890
1900
1910
1920
1930
1940
1950
1960
1970
YEAR


Petroleum
DOLLARS PER BARREL
7
6
5
4
2
1890
1900
1910
1920
1930
1940
1950
1960
1970
YEAR


INDEX
30
25
20
15
10
5
Mineral Aggregate
1890
1900
1910
1920
1930
1940
1950
1960
1970
YEAR

prices of the aggregate commodity show no marked trend, either linear or quadratic. Because the minimum point on the price curve occurs early in the period for some commodities (tin and coal) and late in the period for others (aluminum), when the commodities are aggregated, the pronounced curvature disappears. Therefore, general conclusions about natural-resource scarcity cannot be drawn from the aggregate index alone.

In his analysis of natural-resource commodity price aggregates, Smith (1979) found that for all four aggregate price series examined,[13] the linear-trend coefficient was unstable over time. In particular, for the mineral sector, the trend coefficient was initially negative but increased with time until it became zero. If the correct model for price behavior is nonlinear, the slope of a local linear approximation will vary, depending on the time period chosen. And, if the correct model is U-shaped, the trend coefficient will be initially negative and will increase with time, as found for the mineral sector. Therefore, Smith's results are consistent with those reported here.

VII. SUMMARY AND CONCLUSIONS

The analysis of long-run relative-price movements of nonrenewable natural-resource commodities revealed that, with a linear model, estimated trend coefficients were both positive and negative in sign, and were significant at the 90 percent confidence level in only a little over half of the cases examined.

Therefore, no generalization can be made about natural-resource scarcity from a linear model. In contrast, with a quadratic model, fitted trends for prices of all the major metals and fuels showed the predicted convex curvature--initially falling but eventually rising--and all but one of the estimated coefficients of the squared terms were statistically significant at the 90 percent confidence level. The fitted linear-trend models underestimate relative prices of all the major metals and fuels in the last few years of the 1870-1978 period, because prices of all commodities have passed the minimum points on their fitted U-shaped curves and have begun to increase. Therefore, if scarcity is measured by relative prices, the evidence indicates that nonrenewable natural-resource commodities are becoming scarce.[14]

An examination of the fitted quadratic price trends reveals three basic price paths: falling, stable, and rising prices. The first class is best illustrated by aluminum, a modern metal that is very abundant. Growth rates for aluminum consumption have been high as new uses have been found, and technological advances, combined with economies of scale, have lowered prices over most of the period considered. However, even the price of aluminum has begun to rise in recent years. The second class is best illustrated by lead and zinc. The rate of growth of consumption of these metals has not been as high as that of aluminum, and technological advances and grade declines, which have both been

modest, have almost exactly offset each other in determining lead and zinc production costs. Finally, the third class is best illustrated by tin, an ancient metal with slowly increasing or declining consumption rates. Tin ores were not suitable for froth flotation (as were the ores of many sulfide minerals), and the decline in the grade of tin ores mined has been both steady and sizable. Copper and silver in recent years also fall into the third category.[15] We should not think of these categories as three distinct price paths, however, but perhaps should consider them to be different phases in the life cycles of the respective mineral commodities.

The model presented here is very simple and naive. It neglects many important aspects of mineral-industry cost and pricing, such as environmental regulations, tax policy, market structure, and price controls. Nevertheless, a clear pattern of mineral-commodity price movements emerges, and useful generalizations about long-run relative price behavior can be made from this simple model.

FOOTNOTES

1 In the 1970's, three commissions dealt with this problem--the National Commission on Materials Policy (1973), the National Commission on Supplies and Shortages (1976), and the Nonfuel-Minerals Policy Review (1979).

2 Real cost is defined as either labor or labor plus capital inputs per unit of extractive-industry output; relative price is the ratio of a natural-resource-commodity price index to an overall price index.

3 The emphasis here is on metal, not ore, and metal price is a better indicator of metal scarcity than is ore value. Price is preferred to unit cost because it reflects user cost (scarcity rent) as well as extraction and processing cost.

4 The words "metal" and "ore" are used to distinguish the resource in the ground from the extracted and processed resource. "Ore" could be crude petroleum, of which there are different grades.

5 If there is a zero demand point (choke point) then, even when marginal extraction cost is zero, $\hat{\lambda}$ is bounded (because $\hat{\lambda} = P - C_Q$). In this case, there will always be a $\dot{k}$ sufficiently large in magnitude so that $\dot{P}$ will be negative.

6 $h(g)$ will always increase with time (since $\dot{h} = h'\dot{g} \geq 0$) and $k(t)$ will never increase. In contrast, the behavior of $\hat{\lambda}$ is more complicated (see Hanson, 1980, for example).

7 Smith (1978) experimented with a Box-Cox (1964) transformation of the dependent variable (price). The Box-Cox transformation encompasses a linear-trend model, with constant absolute price changes over time, and a log-linear-trend model, with constant rates of price change. However, with the Box-Cox model, the slope of the time path never changes sign, the characteristic feature of the U-shaped curve. Johnson, Bell, and Bennett (1980) looked at percent changes in unit costs over time. They used dummy variables to distinguish between two periods--pre-1957 and post-1957. However, they gave no theoretical justification for the assumption that a change in cost trends might take place in that year rather than in some other year.

8 For some commodities, data for the last 2 or 3 years of the period were not available.

9 The classification of metals as major follows Peterson and Maxwell (1979). Gold was eliminated because its price was linked to the dollar for most of the period under consideration. Its deflated price is thus proportional to the reciprocal of the deflating index.

FOOTNOTES (continued)

10 Prices of most ores are not published, and many metals are produced from several ores, making the use of ore prices extremely complicated.

11 If, following Manthy, weights for each commodity are constructed to be proportional to that commodity's 1967 dollar value of production, natural gas and aluminum would receive weights of .08 and .06, respectively, and for the first 25 years of the period, more than 14 percent of the mineral-aggregate price index would be derived from commodities that were not produced at that time.

12 The error terms u and v were added to allow for short-run fluctuations about the long-run trends. For all commodities, u and v were found to be first-order serially correlated. However, the plotted trends for those commodities with $\rho_1 > .90$ (nickel, silver, coal, and natural gas) are the ordinary-least-squares estimates uncorrected for serial correlation, not those shown in the tables. With a near-explosive model, the correction for serial correlation results in linear-trend coefficients that seem intuitively implausible.

13 The four aggregate price series are aggregates for the agricultural, forestry, and mineral sectors, and an aggregate of the three.

[14] It might be argued that, for the fuels, the upward trend in prices seen in recent years is principally caused by monopoly power in the petroleum market (the OPEC effect) and that, for the metals, it is caused by increased energy costs. However, the minimum points on the fitted quadratic curves occur prior to 1973 (the year of the OPEC price increases) for every commodity tested.

[15] An analysis of silver-price trends is complicated by bimetallism in the nineteenth century and by speculation in recent years that has artificially inflated silver prices. It is nevertheless interesting to note that the silver-price path is similar to that of copper, a result to be expected because the majority of silver is produced as a byproduct of copper production.

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