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Edward L. Golding

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Edward L. Golding *

*/ University of Pennsylvania, Wharton School. Written while a Staff Economist at the Federal Trade Commission. The views expressed here are the author's and do not necessarily reflect those of the Commission, individual Commissioners, or other staff.

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1. Introduction

Previous papers, e.g. Spence (1977) have looked at warranties as a signal of product quality when consumers cannot observe product quality directly. These papers conclude that warranties can signal quality and that the efficient level of quality is often produced. Fundamental to these models is the assumption that all consumers seek performance under the warranty. This paper explores whether warranties can act as signals when this assumption is dropped. Two extensions of the standard model of warranties as a signal of product quality are presented.

In Section 2, I discuss the case where a fixed fraction of consumers, the "returners," seek performance under the warranty. The other consumers, the "nonreturners," never seek warranty performance. I show that only pooled equilibria or equilibria in which both types of consumers buy the same product, exist. Either both returners and nonreturners buy the same product, and only one product is produced, or some returners buy the same product as nonreturners, and other returners buy a product which no nonreturners purchase. However, the warranty, and hence the product quality is not determinant. Instead, there exist many equilibrium combinations of warranty payment and quality with the different equilibria being more favorable to one group than to the other.

In Section 3, the fraction of consumers who seek performance under the warranty is determined endogenously in the market. Firms set a warranty policy which includes a time requirement or delay for getting warranty performance, in addition to the level of the warranty payment. Thus, firms set the hassle level that consumers must endure in order to get warranty performance. Consumers differ in their willingness to spend time seeking warranty performance. I show that firms do not have an incentive to minimize the hassle consumers must endure. Section 4 presents the conclusions and policy implications.

Warranties act as signals of product quality if consumers can use their knowledge of the warranty in order to infer product quality. Warranties can act as different types of signals of product quality depending on the degree of sophistication of the consumer. They acts as "strong" signals when consumers observe prices and warranties in a competitive market and infer product quality. They act as "weak" signals when consumers must know the production technology in order to infer product quality. This paper deals with warranties as weak signals of quality for two reasons. First, by focussing on warranties as weak signals of quality, I show that, even when consumers have a great deal of information, there exist serious problems with using warranties as signals. Second, when warranties are weak signals of quality, quality can be inferred even if only one warranty is observed in the market, and the exposition and modeling are greatly simplified.[1]

2. Fixed Numbers of Returners and Nonreturners

This section discusses warranties as weak signals of product quality in the case where only a fixed fraction of consumers seek warranty performance in the event of product failure. I begin with a model of warranties as a signal. I then show that only pooled equilibria exist, i.e., both returners and nonreturners buy the same product. However, there are many pooled equilibria with different warranties and product qualities.

2.1. The Model

This section lays out the model which is used to derive the equilibrium warranty contracts.

Consumers

(1) Consumers purchase at most one unit of the good.

(2) There are two types of consumers:
Type R (Returners) - return defective products under the warranty.
Type N (Nonreturners) - do not return defective products under the warranty.
The number of consumers of type R is the same as the number of consumers of type N.

(3) Consumers have the following VonNeuman-Morgenstern utility function:

Returners:

$u_R = u(V-p)$ if the product works

$u_R = u(-p-d+W)$ if the product does not work

Nonreturners:

$u_N = u(V-p)$ if the product works

$u_N = u(-p-d)$ if the product does not work

where p is the price and V is the value of the product, d is the damage or loss caused if the product breaks, and W is the value of the warranty.

Also, consumers are not risk loving, i.e., $u'>0$ and $u''\leq 0$.

(4) Consumers purchase products so as to maximize utility.

Firms

(1) Firms set the quality, q, of their product. A product of quality q works with probability q and breaks with probability (1-q).

(2) The average and the marginal cost of producing a product of quality q is c(q) with $c'>0$ and $c''>0$. Thus it is increasingly expensive to increase quality.

(3) Firms set the warranty level, W, and the price, p. Warranties are assumed to be enforcable.

(4) Each firm, i, maximizes expected profits,

$$p_i-c(q_i)-(1-q_i)s_iW_i,$$

by choosing p, q and W, where s_i is the fraction of returners buying from firm i. Thus, $(1-q_i)s_iW_i$ is the expected payout under the warranty. Given p_i, s_i, and W_i,

$$c'(q_iW_j) = s_iW_i.$$

(5) There is free entry and therefore profits are zero in equilibrium, i.e.,

$$p_i = c(q_i) + (1-q_i)s_iW_i.$$

Knowledge of Consumers

Consumers observe p_i and W_i for all firms, but they do not observe q_i. They also know $c(q)$ and that half of the population is made up of returners and the other half of nonreturners.[2]

Knowledge of Firms

Each firm i observes s_i in equilibrium. Firms also know that half of the population is made up of returners and the other half of nonreturners.

Conjectures of Consumers

Consumers purchase from firm i if

$$u(p_i, W_i, s_i^e, q_i^e) > u(p_j, W_j, s_j^e, q_j^e),$$

for all other j, where s_i^e and q_i^e are their expectations about s and q.

Consumers know that firms cost minimize with respect to quality. Therefore:

$$s_i^e W_i = c'(q_i^e).$$

Consumers, seeing a new entrant, k, offering W_K at price p_K, conjecture that:

$$s_K^e = 1/2 \quad \text{if } u_R(k) > u_K(j) \text{ and } u_N(k) > u_N(j)$$

$$s_K^e = 1 \quad \text{if } u_R(k) < u_R(j) \text{ and } u_N(k) < u_N(j)$$

$$s_K^e = 0 \quad \text{if } u_R(k) < u_R(j) \text{ and } u_N(k) > u_N(j)$$

for all j. That is, consumers conjecture that a new entrant will attract either all customers, all returners, or all nonreturners.

Equilibrium

In equilibrium, expectations must be realized, i.e.,

$$s_j^e = s_j .$$

Also, there must be no incentive for entry, so profits must equal zero, i.e.,

$$p = c(q) + (1-q)sW \quad \text{for all products.}$$

Finally, there cannot exist a pair, (p_k, W_k), with which a firm can enter and make nonnegative profits and have expectations realized.

An equilibrium with $s_i=1/2$ for all i is a pooled equilibrium. In this equilibrium, returners and nonreturners purchase from the same firms. If $W_j=W_k$ for all j, k then the pooled equilibrium is symmetric. In this case, all firms are identical. If $s_j=0$, $s_k=1$ characterizes the equilibrium, then the equilibrium is a separating equilibrium. In this case, nonreturners purchase from one set of firms and returners purchase from another. If $s_j=1$ for some firms and $0<s_k<1/2$ for other firms, the equilibrium is a partially pooled equilibrium. In this case, returners purchase from two types of firms and nonreturners purchase from only one type of firm.

Next, I show that if consumers observe quality, the equilibrium is a separating equilibrium. Then, I show that if quality is unobserved, the equilibrium is either a symmetric pooled equilibrium or a partially pooled equilibrium.

If consumers could observe quality, two types of products would be produced, one for returners (s=1) and one for nonreturners (s=0). The product bought by returners would maximize their expected utility,

$$E(u_R) = q\ u(V-p) + (1-q)\ u(-p-d+W)$$

subject to the zero profit constraint,

$$p = c(q) + (1-q)W.$$

Maximizing over p, W and q yields the first order conditions

$$-q\ u'(V-p) - (1-q)u'(-p-d+W) + t = 0$$

$$(1-q)u'(-p-d+W) - t(1-q) = 0$$

$$u(V-p) - u(-p-d+W) - t(c'-W) = 0 ,$$

where t is a LaGrange multiplier. The first order conditions are satisfied at

$$p^* = c(q^*) + (1-q^*)W^*$$

$$c'(q^*) = W^*$$

$$W^* = V+d$$

$$t^* = u'(-p^*-d+W^*) .$$

The product bought by nonreturners would maximize their expected utility,

$$E(u_N) = q\ u(V-p) + (1-q)\ u(-p-d) ,$$

subject to the zero profit constraint,

$$p = c(q) + (1-q)W .$$

The first order conditions are

$$q\ u'(V-p) + (1-q)\ u'(-p-d) - t = 0$$

$$-t(1-q) = 0 \quad \text{or} \quad W = 0$$

$$u(V-p) - u(-p-d) - t(c'-W) = 0 .$$

t cannot equal zero, because $u(V-p) \neq u(-p-d)$. Therefore, either W=0 or (1-q)=0. (If 1-q=0, then W=0 will also maximize utility, since the product never breaks. Therefore

W=0 will always satisfy the first order conditions for nonreturners.)

Since $E(u_N)$ and $E(u_k)$ are both maximized subject to the same constraint, returners will not want to buy the nonreturners' product, and nonreturners will not want to buy the returners' product. Thus, expectations that s=0 and s=1 are realized. Also, no entry can attract customers away from the product they are purchasing, because, for $s^e=1$ or $s^e=0$, both types are already purchasing their most preferred product. Also, nonreturners would never purchase a product with $s^e=1/2$ and W'>0 over the product that maximizes their utility with s=0, because

$$E(u_N(q',W')) \leq E(u_N(q',W=0)) < E\ u_N(q^*,W=0).$$

Returners would never purchase a product with W=0 over the product that maximizes their utility with s=1, because

$$E(u_R(q',W=0) < E[u_R(q^*,W^*)]$$

(since W=0 adds a binding constraint to the maximization of returners' utility).

<u>Proposition 1</u>

Nonreturners would purchase higher quality products than returners when quality is observed, i.e., $q_R^* < q_N^*$.

Proof:

I show that $[dq/da]|_{q_R^* a=0} > 0$,

where a is the fraction of the warranty not collected (a=1 for nonreturners and a=0 for returners). The first order conditions for maximizing utility for a consumer of type a are:

$$-q\ u'(V-p) + (1-q)\ u'[-p-d+(1-a)W] + t = 0$$

$$(1-q)(1-a)\ u'[-p-d+(1-a)W] - t(1-q)(1-a) = 0$$

$$u(V-p) - u[-p-d+(1-a)W] - t[c'-W(1-a)] = 0$$

Totally differentiating the first equation, holding constant everything but a and q, yields

$$-dq\ u'(V-p) - dq\ u'[-p-d+(1-a)W]$$
$$- u''[-p-d+(1-a)W]W(1-q)da = 0 .$$

At a=0,

$$W=V+d$$

and therefore,

$$dq/da = u''W(1-q)/[-2u'] > 0 ,$$

since $u'>0$ and $u''<0$.

When quality is not observed, however, having returners and nonreturners buying separate products (s=1 for some firms and s=0 for others) is no longer an equilibrium. Nonreturners would quickly find that the quality of the products they purchased was approaching zero, as firms "cheated" by lowering quality. An Akerlof (1970) lemons market would result, i.e., quality q=0. The utility to nonreturners would be u[c(0)+d], since price is p=c(0). Nonreturners might find that they could raise their expected utility by buying the same products as the returners, i.e., by paying for warranties they did not plan to use, because firms offering warranties would sell higher quality products. (These firms would have an incentive not to sell zero quality products, because producing at q=0 would drastically increase their payout under the warranty.) The returners would also receive a subsidy from nonreturners, because they would receive a warranty payment whose full value was not payed by them. However, the returners will also find that quality may be distorted because of the presence of nonreturners. Therefore, in order to attract returners, the subsidy must offset the distortion in quality.

Proposition 2

$s_i=0$, i.e., only nonreturners purchasing from firm i, cannot exist in equilibrium. (A separating equilibrium cannot exist.)

Proof:

If $s_i=0$, then $c'(q) = s_i W_i = 0$, which implies that $q_i=0$. However, $u_N(q_i=0) = u[-c(0)-d]$. Therefore, nonreturners would prefer not to buy any product and to receive a utility level of $u(0) > u[-c(0)-d]$.

Since $s_i=0$ and $s_j=1$ is not an equilibrium when quality is not observed, it is necessary to look for other equilibria. To simplify the analysis and to concentrate on the signaling aspect of warranties, consumers are now assumed to be risk neutral, i.e., $u(x)=x$. In this way, all insurance aspects of the warranty are ignored. Below, I will discuss how adding risk aversion might change some results. First, I prove the following propositions about possible equilibria. These propositions will be used in narrowing the class of possible equilibria.

Proposition 3

If an equilibrium exists such that $0<s_j<1$ and $0<s_k<1$, then $u_N(j) = u_N(k)$, and $u_R(j) = u_R(k)$.

Proof:

This result follows directly from the assumption about conjectures of consumers when it is noted that if consumers of each type are purchasing from both j and k, then each consumer weakly prefers the product he is purchasing. Therefore, equal utility must be derived from each product.

Proposition 4

If an equilibrium exists such that $0<s_j<1$ and $0<s_k<1$, then $(1-q_j)W_j = (1-q_k)W_k$, i.e., the expected value of warranty payments is the same for both firms.

Procf:

This result follows directly from proposition 3 and from the assumption that consumers are risk neutral. Since

$$u_N(j) = u_N(k) \quad ,$$

it follows that

$$q_jV - (1-q_j)d - p_j = q_kV - (1-q_k)d - p_k \ ;$$

and since

$$u_R(j) = u_R(k) \ ,$$

it follows that

$$q_jV + (1-q_j)(W_j-d) - p_j = q_kV + (1-q_k)(W_k-d)-p_k \ .$$

Subtracting the first of these equations from the second equation yields

$$(1-q_j)W_j = (1-q_k)W_k \ .$$

<u>Proposition 5</u>

The s-W combinations that satisfy the two equations below and give nonreturners the same level of satisfaction are rectangular hyperbolas in s-W space.

Proof:

The two equations to be satisfied are:

$$c'(q) = sW \ ,$$

and

$$p = c(q) + (1-q)sW.$$

From the first equation, quality is a function of sW. From the second equation, price is a function of quality and sW, and thus a function of sW. Nonreturners only care about p and q. Therefore, if sW is constant p and q are constant, and utility is constant, and rectangular hyperbolas in s-W space represent constant utility. This statement holds in equilibrium. Note, however, that utility is not monotonic in sW. Therefore, higher hyperbolas do not necessarily represent higher or lower utility levels.

Table 1 shows that there are 18 conceivable pooled equilibrium cases for $0<s_j<1$ and $0<s_k<1$. (When s_j and s_k are not equal, it is assumed, for simplicity, that $s_j<s_k$. Whenever there exists a firm j with $s_j > 1/2$, there must exist a firm k with $s_k < 1/2$, because a positive weighted average of the s_i's must equal 1/2, since the numbers of returners and nonreturners are assumed equal.) The cases that are marked "X" are not internally consistent. Eight possible equilibrium cases remain:

(a) pooled symmetric equilibrium:

$$s_j = s_k = 1/2 \text{ and } W_j = W_k \text{ for all j and k.}$$

(b) $s_j = s_k = 1/2$ for all j and k; and there exist j and k such that $s_jW_j > s_kW_k$ and $W_j > W_k$.

(c) $s_j = s_k = 1/2$ for all j and k; and there exist j and k such that $s_jW_j < s_kW_k$ and $W_j < W_k$.

(d) $s_k < 1/2 < s_j$, $s_kW_k = s_jW_j$, and $W_j < W_k$ for some j and k.

(e) $s_k < 1/2 < s_j$, $s_kW_k < s_jW_j$, and $W_j = W_k$ for some j and k.

(f) $s_k < 1/2 < s_j$, $s_kW_k < s_jW_j$, and $W_j > W_k$ for some j and k.

(g) $s_k < 1/2 < s_j$, $s_kW_k < s_jW_j$, and $W_j < W_k$ for some j and k.

(h) $s_k < 1/2 < s_j$, $s_kW_k > s_jW_j$, and $W_j > W_k$ for some j and k.

These eight cases exhaust the possible equilibria for $0<s_i<1$. By proposition 2, $s_i \neq 0$. There are still possible partially pooled equilibria with $s_j=1$ and $s_k<1/2$. For example, if there are 100 returners and 100 nonreturners, 50 returners shopping at firm j and 50 returners and 100 nonreturners shopping at firm k corresponds to $s_j=1$ and $s_k=50/150=1/3$.

Next, I show that the conditions for equilibrium are not met for cases (b) - (h) , but that they are met for case (a), the symmetric pooled equilibrium, and for $s_j=1$ and $s_k<1/2$, the partially pooled equilibrium.

Proposition 6

The only possible equilibria are $s_j=s_k=1/2$ and $W_j=W_k$ for all j and k, or $s_j=1$ and $s_k<1/2$.

Proof:

It is shown below for cases (b) - (h) either that they are inconsistent with the preceding propositions or that entry can break the equilibrium. Case (a) is shown to be consistent with the preceding propositions and impervious to entry. Then I show by example that $s_j=1$ and $s_k<1/2$ can be consistent with the equilibrium conditions and impervious to entry.

TABLE 1

Conceivable Pooled Equilibria

	$s_jW_j=s_kW_k$			$s_jW_j>s_kW_k$			$s_jW_j<s_kW_k$		
	$W_j=W_k$	$W_j<W_k$	$W_j>W_k$	$W_j=W_k$	$W_j<W_k$	$W_j>W_k$	$W_j=W_k$	$W_j<W_k$	$W_j>W_k$
$s_j=s_k=1/2$ for all j,k	a	X	X	X	X	b	X	c	X
$1>s_j>1/2>s_k>0$ for some j,k	X	d	X	e	f	g	X	h	X

One additional assumption is needed for this proof: $(1-q)c'$ must be assumed to be monotonic in q. The reason for this assumption is that without it the set of possible equilibrium qualities is not convex.

For cost functions of the class,

$$c(q) = \sum_{n=0}^{\infty} a_n(1-q)^{-n}$$

with $a_n \geq 0$, the condition that $(1-q)c'$ is monotonic is satisfied, since

$$d[(1-q)c']/dq = \sum_{n=0}^{\infty} n^2 a_n (1-q)^{-n-1} > 0 .$$

Therefore, the above assumption is satisfied for a nontrivial class of cost functions.

<u>Cases (b) and (c)</u> - cannot exist:

In cases (b) and (c) $s_j=s_k=1/2$ and $W_j = W_k$ for some j and k. From above,

$$1/2\ W_i = c'(q_j) \quad i=j,k .$$

Using proposition 4,

$$(1-q_j)2c'(q_j) = (1-q_k)2c'(q_k) .$$

However, since $(1-q)c'$ is assumed to be monotonic, q_j must equal q_k. Therefore, W_j must equal W_k, which contradicts the assumption that $W_j \neq W_k$.

<u>Case (d)</u> - cannot exist

In case (d) $s_k<1/2<s_j$, $s_k W_k = s_j W_k$, and $W_j<W_k$ for some j and k. Since $s_i W_i = c'(q)$ in equilibrium, it follows

that $q_j=q_k$. By proposition 4,

$$(1-q_j)W_j = (1-q_k)W_k$$

in equilibrium. Therefore $W_j=W_k$, since $q_j=q_k$. However, $W_j=W_k$ contradicts the assumption that $W_j<W_k$.

<u>Case (e)</u> - cannot exist

In case (e), $s_jW_j > s_kW_k$ and $W_j=W_k$. By proposition 4,

$$(1-q_j)W_j = (1-q_k)W_k.$$

Therefore, $q_j=q_k$. However,

$$c'(q_j) = s_jW$$

and

$$c'(q_k) = s_kW_k .$$

Since $s_jW_j>s_kW_k$ and $c''>0$, then $q_j>q_k$, which contradicts $q_j=q_k$.

<u>Case (f)</u> - cannot exist

In case (f), $s_k<(1/2)s_j$, $s_kW_k<s_jW_j$, and $W_k<W_j$ for some j and k. I will show that a firm can enter the market with warranty $W'=2s_jW_j>w_j$, quality $q'=q_j$ and price $p'=p_j$ and satisfy the conjecture $s'^e=1/2$. That is, a firm can enter and attract the entire market. The signaled quality will be

$$q' = c'^{-1}[(1/2)2s_jW_j] = c'^{-1}(s_jW_j) = q_j .$$

Also, a firm offering the above warranty will have nonnegative profits,

$$p' - c(q')W' - (1/2)W'(1-q')$$

$$p_j - c(q_j) - s_jW_j(1-q_j) \not< 0.$$

All nonreturners will be indifferent between the new product they were purchasing since, by proposition 5, isoutility curves are a function of sW and $s'W'=s_jW_j$. (This statement includes nonreturners who were not purchasing from either j or k, because, in equilibrium, $u_N(i)=u_N(j)$.) All returners will strictly prefer the new product, because they will be receiving the same quality product at the same price but with a larger warranty payment. ($W'>W_j$ because $2s_j>1$.) Therefore, since returners prefer the new product and nonreturners are indifferent, the new entrant will attract the entire market, and $s^e=s=1/2$ will be realized.

Cases (g) and (h) - cannot exist

The same reasoning applies to cases (g) and (h) as to case (f) and therefore the argument is not repeated.

Case (a) - The symmetric pooled equilibrium can exist.

In case (a), $s_j=s_k=1/2$ and $W_j=W_k$ for all j and k. This case is consistent with proposition 3, because, since the products are identical, the utility is the same for all products. Therefore, this case is also consistent with propositions 4 and 5.

Next, I show that entry cannot occur and break an equilibrium if W_i is between W_{min} and W_{max}, where W_{min} and W_{max} are defined below.

$$1/2\ W_{min} = c'(q_{min})$$

$$1/2\ W_{max} = c'(q_{max})$$

$$q_{min} = \max[q_1, q_2]\ ,$$

where q_1 is the quality below which nonreturners prefer higher quality, and thus $du_N/dq = 0$ at $q=q_1$, or

$$(v+d) = (1-q_1)c''(q_1),$$

and q_2 is the minimum value of q for which returners prefer buying a product with s=1/2 to the product with s=1. With s=1, returners do not receive a subsidy, but they may receive higher quality. q_{max} is the quality for which

$$u_N(q_{max}, W_{max}) = Vq_{max} - (1-q_{max})d - c(q_{max}) - (1-q_{max})c'(q_{max}) = 0.$$

Above q_{max} nonreturners drop out of the market.

<u>Proposition 7</u>

The symmetric pooled equilibrium is not unique. That is, $s_i=1/2$, $W_{min}<W_i<W_{max}$, $q_{min}<q_i<q_{max}$, and $p_i=c(q_i)+(1/2)[(1-q_i)W_i]$, where $(1/2)W_i=c'(q_i)$, are all feasible symmetric pooled equilibria if $(1-q)c''>2c'$. This condition is also satisfied for the class of cost functions described above.[3]

Proof:

No firm can enter and attract only nonreturners, i.e., $s_j^e=0$, since if $s_j^e=0$, the quality signaled would be $q_j=0$. No firm can enter and attract only returners because q_{min}

was chosen so that returners prefered the pooled equilibrium. A firm cannot enter and attract both returners and nonreturners, because returners will prefer only products with higher warranties and quality, while nonreturners will prefer only products with lower warranties and quality. This can be seen by differentiating their utility functions with respect to quality, holding $s=1/2$ and $(1/2)W=c'(q)$.

The utility function for the nonreturners is

$$u_N = qV - (1-q)d - c(q) - (1-q)c'(q).$$

Since $(1/2)W=c'(q)$,

$$du_N/dq = (V+d) - c' + c' - (1-q)c''(q)$$
$$= (V+d) - (1-q)c''(q) \quad ,$$

which is less than zero for $q>q_{min}$. The utility function for the returners is

$$u_R = qV + (1-q)(W-d) - c(q) - (1-q)c'(q).$$

Since $W=2c'(q)$,

$$du_R/dq = V + d - 2c'(q) + (1-q)2c''(q) - c'(q)$$
$$+ c'(q) - (1-q)c''(q)$$
$$= (V+d) - 2c'(q) + (1-q)c''(q) \quad .$$

Thus $du_R/dq>0$ because $(1-q)c''$ was assumed to be greater than $2c'$. Since $du_R/q>0$ and $du_N/dq<0$, no firm can enter with $s^e=1/2$ and attract both returners and nonreturners.

Proposition 8

Besides the symmetric pooled equilibria with $s_i=1/2$ for all i, there are also possible partially pooled equilibria with s=1 and $s_k<1/2$.

Proof:

The proof is given by constructing an example of an equilibrium. The intuition is as follows. For $s_j=1$, returners are receiving no subsidy from nonreturners, but they may be receiving a higher quality product. The returners may be indifferent beween receiving the subsidy with a low quality product and receiving a higher quality product. The nonreturners may strictly prefer the low quality product ($s_k<1/2$) to the high quality product which would carry a larger subsidy. Thus, all nonreturners purchase the low quality product. Entry cannot occur because $s^e=1/2$ cannot attract both returners and nonreturners.

Construct the following equilibrium:

$V=1,\ d=0$

$c(1/2)=1/4$

$c'(1/2)=1$

$c(1/4)=1/8$

$c'(1/4)=1/18$

This cost function satisfies the condition $c''>0$. (A third degree polynomial could be fitted through the points (1/4,1/8) and (1/2,1/4) with slopes of 1/18 and 1.)

$s_j=1,\ s_k=1/4$

$W_j=1$, and therefore $q_j=1/2$, since $s_jW_j=1=c'(1/2)$.

$W_k=2/9$, and therefore $q_k=1/4$, since

$$s_kW_k=1/18=c'(1/4)$$

The utility functions for returners are

$$u_R(j) = q_jV - c(q_j) = 1/2 - 1/4 = 1/4$$

$$u_R(k) = q_kV - c(q_k) + (s-1)(1-q_k)W$$

$$= 1/4 - 1/8 + (3/4)(3/4)(2/9)$$

$$= 1/4$$

Therefore,

$$u_R(j) = u_R(k).$$

The utility functions for nonreturners are

$$u_N(j) = u_R(j) - (1-q_j)W_j = 1/4 - 1/2 = -1/4 \quad < 0$$

$$u_N(k) = u_R(k) - (1-q_k)W_k = 1/4 - (3/4)(2/9) = 1/12$$

$$> 0$$

Therefore, returners are indifferent between j and k, while nonreturners prefer k. A new entrant cannot attract only returners, because $W_j = V$ already maximizes u_R subject to s=1. A new entrant cannot attract an equal number of returners and nonreturners for this cost function. This result can be shown by noting that a nonreturner's utility is

$$u_N = q - c(q) - (1-q)c' ,$$

where sW=c', and

$$du_N/dq = 1 - (1-q)c'' .$$

Thus, since $c''(1/4) = 1/(1-q) = 4/3$, the nonreturners cannot be made better off with $s=1/2$ if signaled quality is higher, because $q=1/4$ is the highest level for nonreturners achievable through a signaled equilibrium. However, with $s'=1/2$, $q'=1/4$, and $W'=W_k/2$, while nonreturners would be indifferent, returners would have lower utility levels, since the warranty level is lower. Thus, both groups cannot be attracted to a new firm, and $s_j=1$ and $s_k<1/2$ can be an equilibrium.

The above propositions have all assumed that consumers are risk neutral. With risk neutrality, an increase in signaled quality has two effects. First, it increases the expected utility derived from the product for both returners and nonreturners by an amount ΔqV. Second, it changes the subsidy going from nonreturners to returners. Therefore, with risk neutrality, it is simple to calculate whether a new entrant could attract both types of consumers. With risk aversion a change in quality also affects the risk borne by both groups. However, each group values the change differently. Thus, it is more difficult to determine whether a new entrant will attract both returners and nonreturners. This problem could be explored by having

$$u_R = qV + (1-q)(W-) - b\ var_R(q)$$

and

$$u_N = qV - (1-q)d - b\ var_N(q) \quad ,$$

where

$$var_R(q) = [V-(W-d)]q(1-q)$$

and

$$var_N(q) = (V+d)q(1-q).$$

In this way it would be possible to tell whether new entrants could attract both groups.

3. Hassle Levels Set by the Firm

In this section, I discuss the case where the acts of returning and not returning defective products are endogenously determined by the market. Consumers vary in their willingness to endure hassle in order to get warranty performance. Firms determine the proportion of defective products returned by setting a hassle level, i.e., a level of warranty performance. Firms can reduce hassle up to T_{min} at zero cost, and then cannot reduce hassle further. I show that in equilibrium firms do not maximize warranty performance, i.e., they do not minimize hassle.

Assume that a firm sets a warranty package (W,T) where W is the reimbursement in the event of product failure and T is the time consumers need to spend to receive performance under the warranty. A firm is free to set $W \geq 0$ and $T \geq T_{min}$, where T_{min} is the minimum time needed to get warranty performance. Since the consumer at least needs to notify the firm, T_{min} is strictly greater than zero. Costs to a firm are not dependent on T.

Individuals vary in their opportunity cost of time, r, or wage rate. The distribution of wage rates in society is assumed to be uniform between zero and one.

When a firm, j, sets a warranty policy of (W_j, T_j) an individual, i, will return a malfunctioning product if the warranty is worth more than the time needed to return the product, i.e., if $W_j > r_i T_j$ or $W_j/T_j > r_i$. Therefore, all individuals with $r_i < W/T$ will be returners. As the distribution of wage rates was assumed uniform, the fraction of the population who are returners, s, equals W/T. Thus the signaled quality is:

$$c'^{-1}(sW) = c'^{-1}[W^2/T].$$

The expected utility for returners is:

$$u_R = qV + (1-q)[W - d - rT] - \text{price},$$

where rT is the cost of seeking warranty performance. The expected utility for nonreturners is:

$$u_N = qV - (1-q)d - \text{price}.$$

Under the assumption of free entry and hence zero profits and assuming d=0, the expected utilities are:

$$u_R = qV + (1-q)(W-rT) - c(q) - (1-q)\ (W^2/T)$$

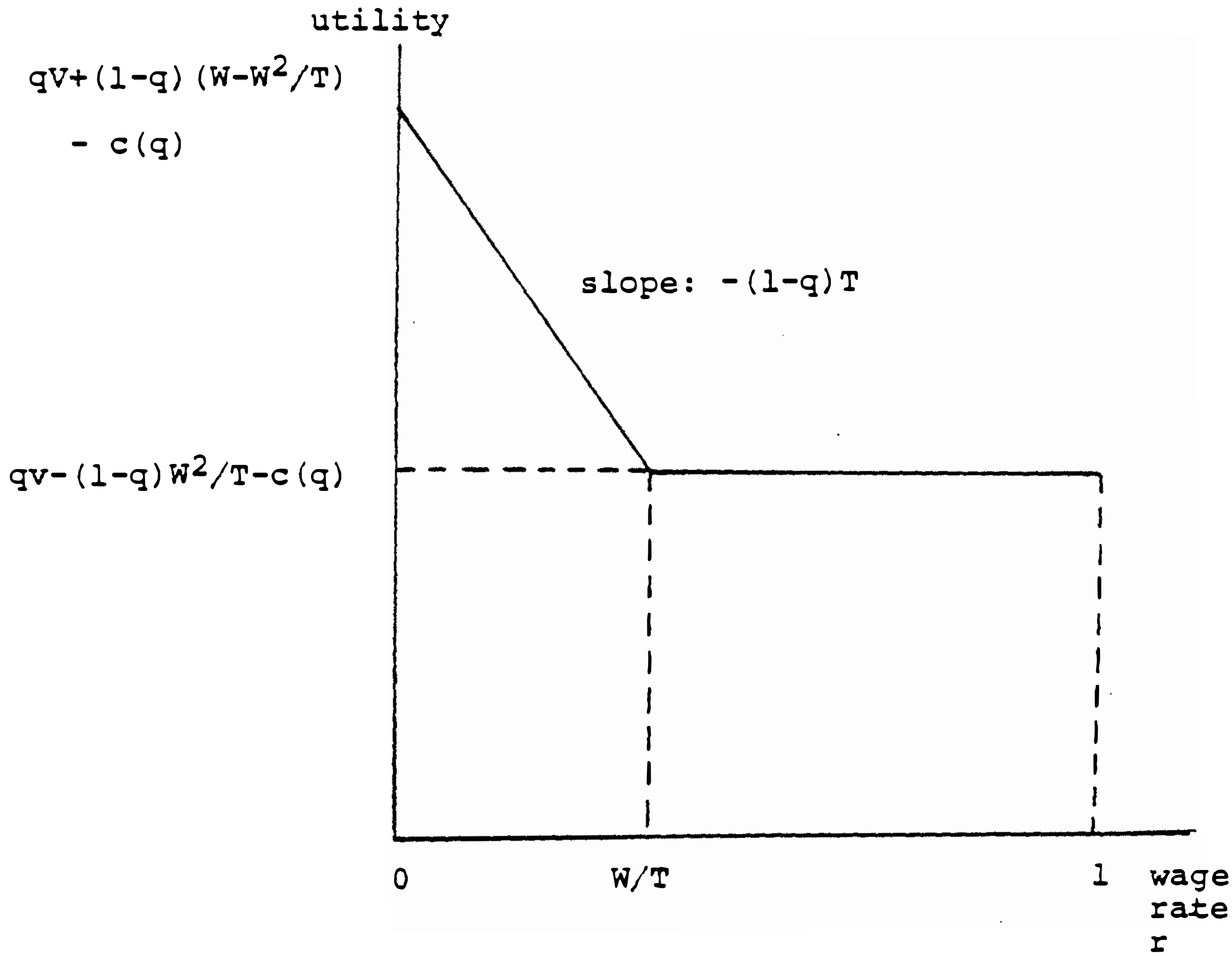
$$u_N = qV - c(q) - (1-q)\ (W^2/T)$$

where $c'(q) = W^2/T$ and $p(q) = C(q) + (1-q)\ W^2/T$.

Figure 1 shows the utility as a function of wage rates:

Figure 1

Utility of Consumers as a Function of Wage Rates



Proposition 9

The signaled quality in equilibrium must be the quality, q^*, that maximizes the expected utility for nonreturners.

Proof:

If the market is producing a product with warranty (W,T), at a competitive price, p, and with the signaled quality different from q^*, another firm can enter with a warranty (W',T') and a price, p', and make nonnegative profits such that their signaled quality is q^* and satisfies:

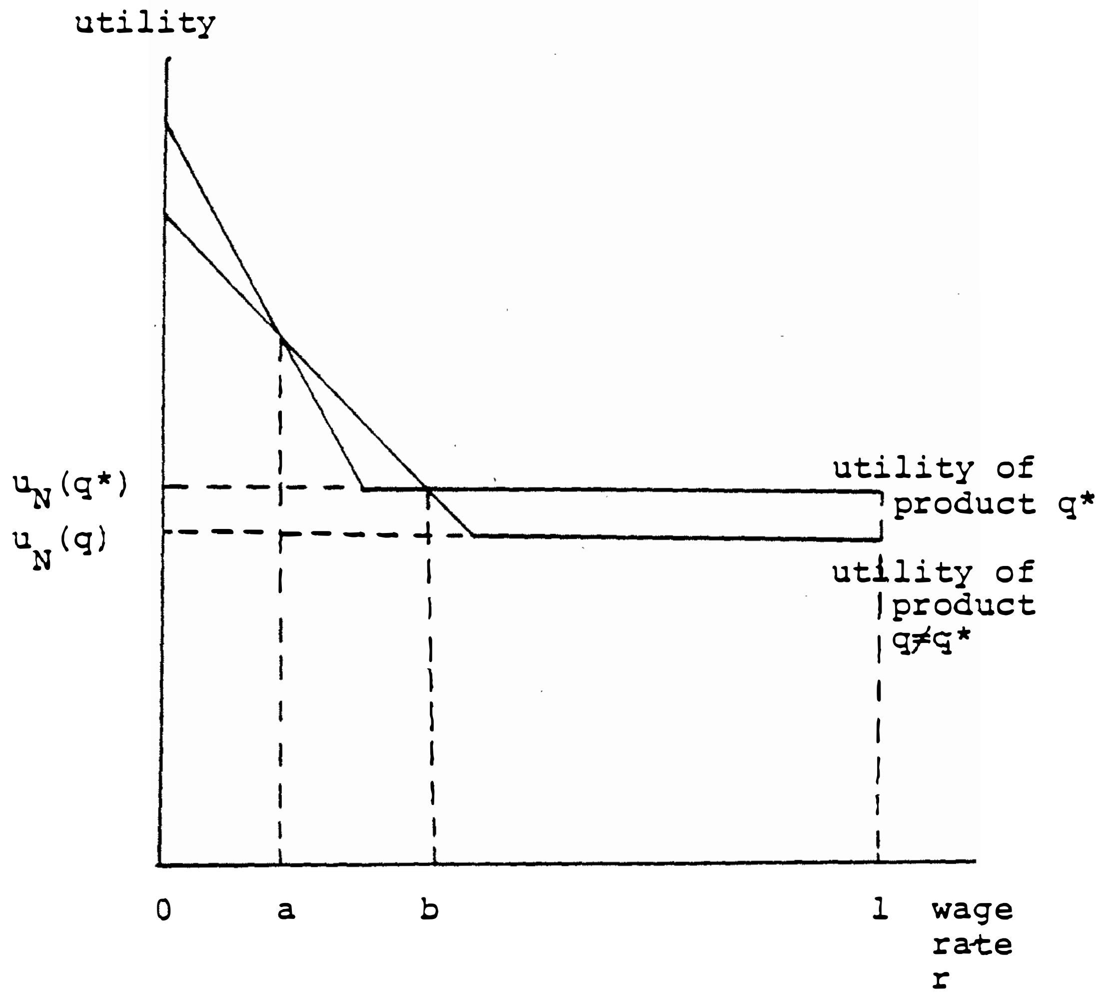
$$V = (1-q^*)\ c''(q^*).$$

(q^* maximizes the utility for nonreturners.) Figure 2 shows how a firm can signal q^*.

By offering (W',T') at price p', the firm attracts all consumers in (0,a) and (b,1) since the utility for these consumers is higher for the new product. Consumers in (0,a) return the product and thus enforce the signal since $[a/((1-b+a)]W=c'(q^*)$. [a/(1-b+a) is the fraction of the customers who are returners.] The consumers in (b,1) are nonreturners. The old firms are left with customers in (a,b) and will begin to earn negative profits because all their nonreturners have left.

Figure 2

Maximum Utility for Nonreturners With q=q*



If q^* is the signaled quality, with $W^2/T=c'(q^*)$, then the nonreturners have maximum utility. There are many combinations of W and T that will produce $W^2/T=c'(q^*)$. If (W,T) are both small then many consumers return the defective products. If (W,T) are both large then most consumers are nonreturners and a few consumers with low wage rates enforce the warranty. As can be seen from figure 3, various (W,T) combinations are prefered by different consumers. Consumers with r near 1/2 may prefer low combinations of warranty and hassle, (W_L,T_L), since they would then act on the warranty. However consumers with wage rates near zero will prefer high warranty payments and hassle levels, (W_H,T_H), as they are not affected by the high hassle levels but enjoy the high warranty payments.

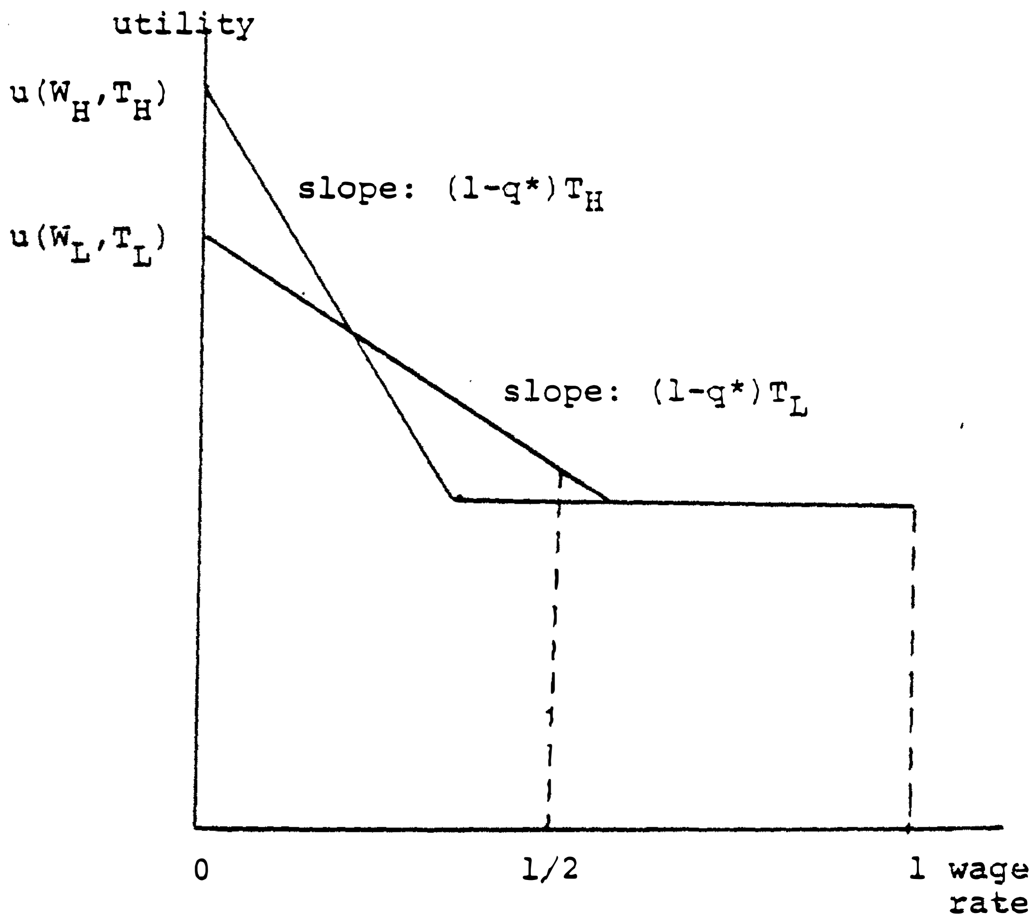
$T = T_{min}$, and $W_{min} = [T_{min}c'(q^*)]^{.5}$ is the smallest warranty package that satisfies $W^2/T = c'(q^*)$. Thus, it is the package with the least hassle and the la-gest number of returners. However, (W_{min},T_{min}) for all firms is not an equilibrium, since firms can enter and attract customers away from a firm with (W_{min},T_{min}).

Proposition 10

All firms producing q^*, W_{min}, T_{min} is not an equilibrium. Firms have the incentive to enter the market with increased hassle levels and increased warranty payments.

Figure 3

Consumers' Preferences on W and T Do Not Agree



Proof:

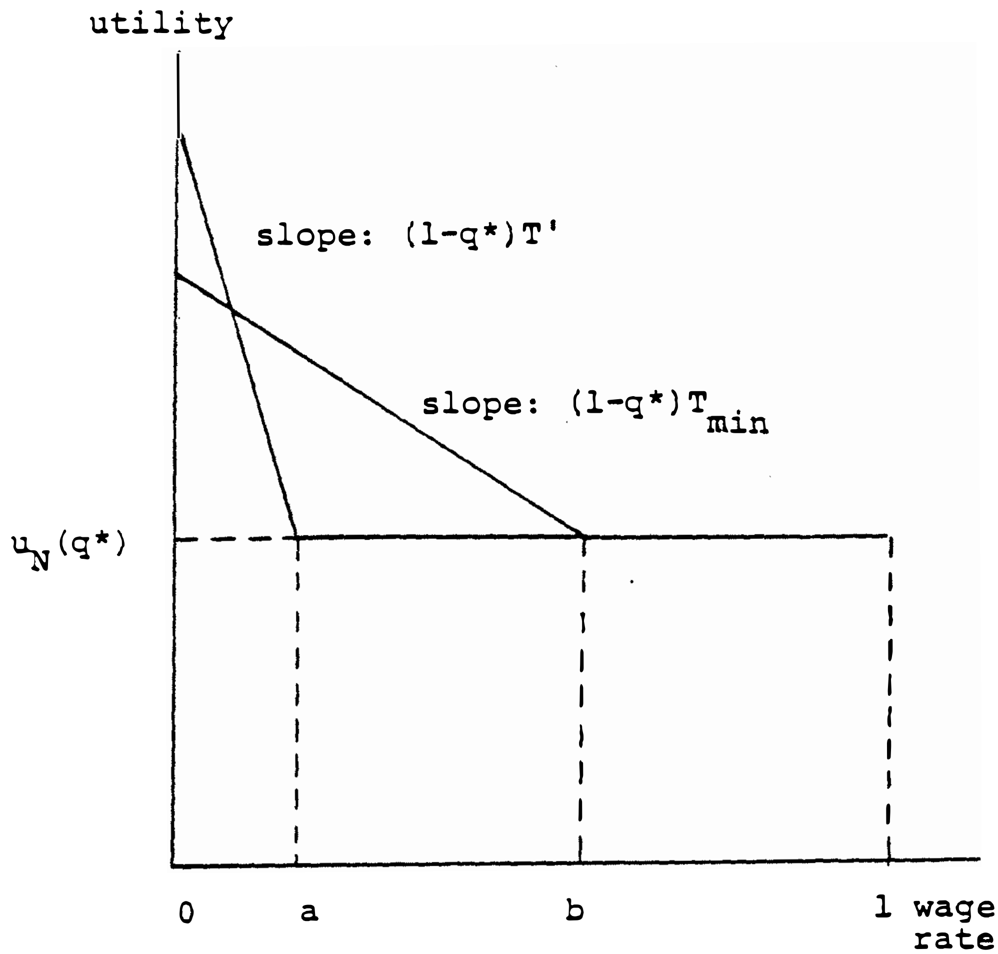
A firm can enter with $T' >> T_{min}$ and $W' >> W_{min}$ and attract consumers in (0,a) and a share of consumers in (b,1). (See Figure 4.) The warranty must be large enough to ensure the signaling of q*, and thus ensure that some nonreturners, (b,1), will also buy the product that has the same competitive price and quality, i.e. $s'W' = [W_{min}]^2/T_{min}$. However, the number of returners must be small enough so that $p' = p_{min}$ still earns the firm nonnegative profits. Since (0,a) can be made arbitrarily small by increasing T, with sW held constant, the firm can signal q* while decreasing the number of returners. Thus, a firm can enter with a very high warranty and hassle level but with q'=q* and $p' = p_{min}$. It attracts those who dislike hassle the least along with a share of nonreturners who are not affected by increased hassle. Therefore, all firms producing at (W_{min}, T_{min}) is not an equilibrium. Firms will enter with increased hassle levels.

In fact, proposition 10 does not apply to W_{min}, T_{min} alone. No matter what warranty, hassle level combination an incumbent provides, an entrant can enter with an increased warranty and hassle level. Therefore, no equilibrium exists if W, T are unbounded. If T is bounded by T_{max}, it must be checked whether T_{max} with the corresponding W_{max} which signals quality q* can be an equilibrium. W_{max}, T_{max}

Figure 4

A Firm with Increased W and T Can Enter

If All Firms are at W_{min} and T_{min}



is an equilibrium if a firm cannot enter with a hassle level, T, such that $T_{min} \leq T < T_{max}$ and a warranty W which signals q* at price p*, and make nonnegative profits.

Proposition 11

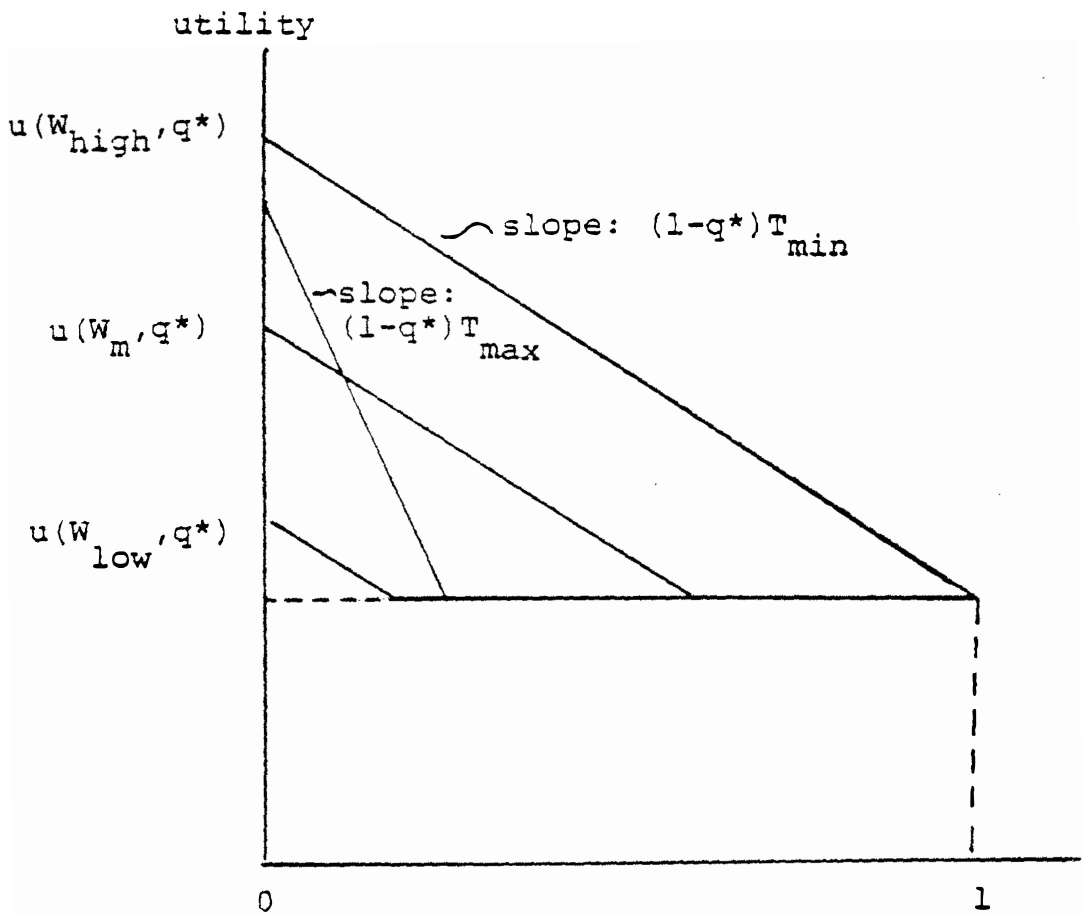
No equilibrium exists, because a firm can enter with a hassle level T_{min} and signal q* at price p*.

Proof:

A firm signals q* if $sW = c'(q^*) = constant = K$. Price is p* if $c(q^*) + sW(1-q^*) = p^*$. Therefore, if $s_i W_i = K$ is feasible, a firm can enter. Figure 5 shows possible W, T_{min} entry combinations. W_{high} will attract $s_{high} = 1$ and will have $s_{high} W_{high} > K$, although profits would be negative. Similarly, W_{low} will attract $s_{low} = 0$ and have $s_{low} W_{low} < K$. Therefore, by continuity, there exists a W_m such that $s_m W_m = K$. At W_m, p*, q* would earn nonnegative profits. Thus, a firm can enter with T_{min}, W_m and break the equilibrium.

Figure 5

A Firm Can Enter with W_m and T_{min}



4. Conclusion

I have shown that consumers who do not return products will still want to purchase a product with a warranty when warranties act as a signal of quality. I have also shown that when warranties act as a signal, firms do not have the incentive to maximize warranty performance (minimize hassle) even though in my model the warranty performance was free to produce.

It is interesting to compare these results with what happens when warranties are used by firms that have established reputations. In a simple reputation model, firms promise some level of satisfaction. The firm has some reputation capital that is forefeited if it cheats on the promised level of satisfaction. The firm relies on repeat purchases to cover the cost of the capital. (See papers by Shapiro (1980) and Klein and Leffler (1979)). Thus, a firm with a reputation, which promises a quality, warranty payment, and warranty performance, has the incentive to promise high levels of warranty performance. Furthermore, the warranty is a good instrument for retaining reputation by compensating owners of malfunctioning products and by decreasing the chance that these people think that the firm has cheated on quality. This is accomplished without the distortions in warranty performance and quality that are found in a pure signaling equilibrium.

Even in a reputation model, warranties may be correlated with quality. After all, the better the product, the cheaper it is to guarantee a certain level of satisfaction. However, price is also correlated with quality in a reputation model. Instead of warranty and price acting as signals in the conventional sense, it is the reputation of the firm that signals the promised quality, and the warranty and price reflect the higher quality. For example, "Sears' best" may sell for more and have a better warranty than "Sears' good." But it is Sears' reputation that actually guarantees the quality. If a no-name firm were to enter with the same or better warranty, it would not be likely to attract customers away from Sears.