Dynamic Price Competition:
Theory and Evidence from Airline Markets

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Dynamic pricing is commonly used in markets with fixed inventory and a sales deadline

- Examples: Airlines, trains, hotels, cruises, entertainment tickets, retailing, etc.
- Capacity drives price dynamics:
  - The opportunity cost of selling changes with scarcity
    - Value of a seat today depends on the ability to sell it in the future
    - Excess inventory $\rightarrow$ expect low prices
  - Demand may change over time
    - If high WTP consumers arrive in future, incentives to save seats

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3. Open questions regarding dynamic price competition in perishable goods markets

   a) Equilibrium prices and profits lack "nice" properties; can we characterize equilibrium outcomes?

   b) Empirical welfare implications unknown

Contributions of the research project

1. We introduce a tractable oligopoly framework for dynamic price competition
   - Provide a differential equation characterization of equilibrium dynamics e.g. Gallego & van Ryzin (1994)
   - Provide insights on existence, uniqueness, competitive dynamics, e.g., the role of "minimum capacity" see Martinez-de-Albeniz & Talluri (2011) for perfect substitutes
2. We estimate the welfare effects of dynamic pricing in the airline industry
   - We find the opposite results compared to studies in the single-firm setting: DP increases output and profits, decreases welfare
     single-firm setting: e.g., Hendel and Nevo (2013), Castillo (2021), Williams (2022)
   - Heuristics similar to airline practices increase surplus relative to DP heuristics differ from, e.g., Calvano et al (2020), Brown and MacKay (2021), Asker et al. (2021)

Oligopoly model

- We consider a set $\mathcal{J} = \{1, \ldots, J\}$ of products and a set $\mathcal{F} := \{1, \ldots, F\}$ of firms
- Firm f owns products in $\mathcal{J}_f \subset \mathcal{J}$
- Initial capacity of each product j is $K_{j,0}$
- Firms must sell all units by time T, in periods $t = \Delta, 2\Delta \ldots, T$
- In every period:
  - each firm f sets prices $\mathbf{p}_t^f := (p_{j,t})_{j \in \mathcal{J}_f}$
  - a consumer arrives with probability $\Delta\lambda_t \in (0, 1)$
  - consumer decides whether to buy a product or not and leave
- Firms observe history of all prices and inventories

Demand model

- Consumers are passive/short-lived $\rightarrow$ demand function (with forward-looking buyers, firm competing with its future self e.g., Board & Skrzypacz (2016); Dilme & Li, (2019); Gershkov, Moldovanu, Strack (2017))

- A consumer who arrives at time t chooses product j with probability:

$$s_{j,t}(\mathbf{p}) = s_j(\mathbf{p}; \boldsymbol{\theta}_t, \mathcal{A}_t) \in [0, 1]$$

  where $\boldsymbol{\theta}_t$ are demand parameters and $\mathcal{A}_t$ is the set of available products

- We impose some regularity assumptions that guarantee that the profit-maximizing price vector of a single firm is unique and satisfies the system of FOCs (see paper).

- Demand assumptions hold beyond IIA, e.g., nested logit

Solution concept: Markov-perfect equilibrium

- We analyze Markov-perfect equilibria
- Payoff-relevant state: vector of inventory $\mathbf{K} := (K_j)_{j \in \mathcal{J}}$ and time t
- Denote the Markov pricing strategy for product j by $p_{j,t}(\mathbf{K})$

Let's start with the single-firm case

- The single-firm case has "nice" properties that mostly do not extend to oligopoly
- Useful to establish notation and motivates solution strategy
- Assume that a single firm "M" owns all products
- "Opportunity cost" or value of a seat j in state $(\mathbf{K}, t)$:

$$\omega_{j,t}(\mathbf{K}) := \underbrace{\Pi_{M,t+\Delta}(\mathbf{K})}_{\text{Continuation Profit}} - \underbrace{\Pi_{M,t+\Delta}(\mathbf{K}-\mathbf{e}_j)}_{\substack{\text{Continuation Profit} \\ \text{if M sells 1 unit of j}}}$$

- When we study oligopolies we will call these differences in value functions *scarcity effects*

Optimal control problem

- The continuation profit of a single firm with capacities $K_j > 0$ for $j \in \mathcal{J}$ at t:

$$\Pi_{M,t}(\mathbf{K};\Delta) = \max_{\mathbf{p}} \ \Delta\lambda_t \sum_{j\in\mathcal{J}} \underbrace{s_{j,t}(\mathbf{p})}_{\substack{\text{Probability}\\ \text{of sale}}} \Big(\underbrace{p_j + \Pi_{M,t+\Delta}(\mathbf{K}-\mathbf{e}_j;\Delta)}_{\substack{\text{Continuation value}\\ \text{after sale}}} \Big) +$$

$$\underbrace{\Big(1 - \Delta\,\lambda_t \sum_{j\in\mathcal{J}} s_{j,t}(\mathbf{p})\Big)}_{\text{Probability of no sale}} \ \underbrace{\Pi_{M,t+\Delta}(\mathbf{K};\Delta)}_{\substack{\text{Continuation value}\\ \text{after no sale}}}$$

- $\Pi_{M,t}(K;\Delta)$ converges as $\Delta \to 0$ uniformly in t and $\mathbf{K}$ to $\Pi_{M,t}(\mathbf{K})$ which solves the differential equation

$$\dot{\Pi}_{M,t}(\mathbf{K}) = -\lambda_t \max_{\mathbf{p}} \sum_{j\in\mathcal{J}} s_{j,t}(\mathbf{p})\Big(p_j - \underbrace{\big(\Pi_{M,t}(\mathbf{K}) - \Pi_{M,t}(\mathbf{K}-\mathbf{e}_j)\big)}_{=:\ \omega_{j,t}(\mathbf{K})}\Big)$$

Properties of the single-firm case

Proposition 1

1. *Value function* $\Pi_{M,t}(\mathbf{K})$ *is decreasing in time t and increasing in capacity*
2. *Opportunity costs* $\omega_{j,t}(\mathbf{K})$ *are decreasing in time t and capacity*
3. *The stochastic process* $\omega_{j,t\wedge\tau}(\mathbf{K}_t)$, $\tau := \inf\{t \geq 0 | K_{j,t} \leq 1\}$ *is a submartingale*

- Insight: *Observed* average price (*conditional on* $K_j > 1$) is increasing $\rightarrow$ demand uncertainty leads to increasing average prices
- *None* of these properties carry over to the oligopoly case!

Now, we consider the duopoly game. A new scarcity force.

- Back to a duopoly where each firm owns one product: $\mathcal{J} = \mathcal{F} = \{1, 2\}$
- Each firm f has its own continuation profit in state $(\mathbf{K}, t)$: $\Pi_{f,t}(\mathbf{K}; \Delta)$

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- Now, there are two scarcity effects for each firm f:
  - "Own-scarcity effect"

$$\omega^{f}_{f,t}(\mathbf{K}) := \Pi_{f,t+\Delta}(\mathbf{K}) - \Pi_{f,t+\Delta}(\mathbf{K} - \mathbf{e}_f)$$

  - "Competitor-scarcity effect"

$$\omega^{f}_{f',t}(\mathbf{K}) := \Pi_{f,t+\Delta}(\mathbf{K}) - \Pi_{f,t+\Delta}(\mathbf{K} - \mathbf{e}_{f'}), \qquad f' \neq f$$

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- This defines a matrix of scarcity effects:
  $$\Omega_t(\mathbf{K}) = \begin{pmatrix} \omega^1_{1,t} & \omega^1_{2,t} \\ \omega^2_{1,t} & \omega^2_{2,t} \end{pmatrix}.$$

Differential equation characterization of equilibrium

Proposition 2 (Continuous-time limit)

Assume sufficient conditions on demand system. For every $\mathbf{K}$*, there exists a* $T_0(\mathbf{K}) > 0$*, non-increasing in* $\mathbf{K}$*, so that the value function* $\Pi_{f,t}(\mathbf{K}; \Delta)$ *converges to a limit* $\Pi_{f,t}(\mathbf{K})$ *as* $\Delta \to 0$ *that solves the differential equation*

$$\dot{\Pi}_{f,t}(\mathbf{K}) = -\lambda_t \Bigg(s_f(\mathbf{p}^*(\Omega_t(\mathbf{K}); \boldsymbol{\theta}_t)) \left(p_f^*(\Omega_t(\mathbf{K}); \boldsymbol{\theta}_t) - \underbrace{(\Pi_{f,t}(\mathbf{K}) - \Pi_{f,t}(\mathbf{K} - \mathbf{e}_j))}_{\text{own-scarcity effect}} \right) - s_{-f}(\mathbf{p}^*(\Omega_t(\mathbf{K}); \boldsymbol{\theta}_t)) \underbrace{(\Pi_{f,t}(\mathbf{K}) - \Pi_{f,t}(\mathbf{K} - \mathbf{e}_{-f}))}_{\text{competitor-scarcity effect}} \Bigg)$$

where $f' \neq f$*, with natural* boundary conditions *and* $p^*(\Omega, \boldsymbol{\theta})$ *is an equilibrium of a stage game parameterized by* $(\Omega; \boldsymbol{\theta})$*.*

Allows us to empirically investigate DPs in oligopoly with large state spaces

The value function

- The Markov structure allows us to summarize the impact of today's price on future revenues into "scarcity effects."
- Given a pricing strategy $\mathbf{p}_t(\mathbf{K}) := (p_{1,t}(\mathbf{K}), p_{2,t}(\mathbf{K}))$, firm f's value function is

$$\Pi_{f,t}(\mathbf{K};\Delta) = \Delta\lambda_t \left(\underbrace{s_{f,t}\left(\mathbf{p}_t(\mathbf{K})\right) \left(p_{f,t}(\mathbf{K}) + \Pi_{f,t+\Delta}(\mathbf{K}-\mathbf{e}_f;\Delta)\right)}_{\text{revenue of own sale}} + \right.$$

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$$\underbrace{s_{f',t}\left(\mathbf{p}_t(\mathbf{K})\right)\ \Pi_{f,t+\Delta}(\mathbf{K}-\mathbf{e}_{f'};\Delta)}_{\substack{\text{continuation value if } f' \text{ is sold} \\ \textbf{new term!}}} \Bigg) + \underbrace{\left(1 - \Delta\lambda_t \sum_{h=\{1,2\}} s_{h,t}\left(\mathbf{p}_t(\mathbf{K})\right)\right)}_{\text{probability of no purchase}} \cdot \Pi_{f,t+\Delta}(\mathbf{K};\Delta),$$

where $f \neq f'$.

The stage game with equilibrium prices $\mathbf{p}^*(\Omega_t(\mathbf{k}); \boldsymbol{\theta}_t)$

- We can write for each firm $f \neq f'$

$$\Pi_{f,t+\Delta}(\mathbf{K};\Delta) - \Pi_{f,t}(\mathbf{K};\Delta) =$$
$$-\Delta\lambda_t \left(\underbrace{s_{f,t}\left(\mathbf{p}_t(\mathbf{K})\right)\left(p_{f,t}(\mathbf{K}) - \omega^f_{f,t}(\mathbf{K})\right) - s_{f',t}\left(\mathbf{p}_t(\mathbf{K})\right)\omega^f_{f',t}(\mathbf{K})}_{\text{stage game payoff of firm } f} \right).$$

  - "Own-scarcity effect"

$$\omega^f_{f,t}(\mathbf{K}) := \Pi_{f,t+\Delta}(\mathbf{K}) - \Pi_{f,t+\Delta}(\mathbf{K} - \mathbf{e}_f)$$

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- This stage game can have multiple equilibria if e.g. $\omega^f_{f',t}$ is very negative
- Typically $\omega^f_{f',t}$ is negative because sale of competitor creates future scarcity

Findings from simulations of this system of differential equations

- Profits are non-monotonic in the own capacity
- Profits are non-monotonic in competitor capacity
- Profits are neither concave nor convex in capacity: Both scarcity effects can be positive or negative
- But the dynamics of scarcity effects close to the deadline depends on which firm has the minimum capacity:
  - competition fiercest when firms have symmetric inventory (independent of symmetry in other dimensions)
  - largest price effects when the firm with min cap sells
- see paper for new markup rule

Data Overview

- Use third-party data provided to us by a large US airline:
  - Daily prices and quantities of *competing airlines*
  - Observe all bookings (counts); including connecting traffic, tickets purchased via travel agents, etc.
  - Daily prices for each flight—we'll use the lowest available economy ticket
  - Data identifies firms, flight numbers, departure dates, etc.
- Think of this as the Nielsen of airline data
- We observe search data, the pricing technology, and all output from the firm's pricing system for one airline
  - Use search data to estimate arrivals, which are then scaled up (robust to scaling factor)

Facts on Routes Studied

- Analysis concentrates on duopoly markets with a large % of local, nonstop traffic
- Distribution of fares similar to all duopoly markets
- Many markets are from large cities to medium-size cities; 58 directional pairs total

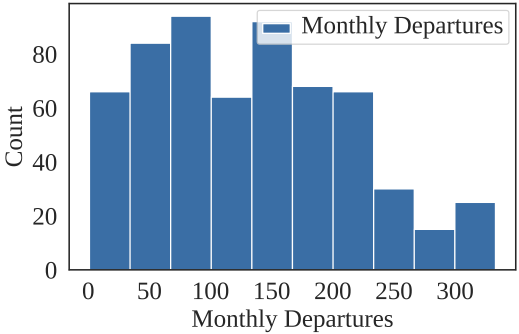

Figure: A few frequencies a day

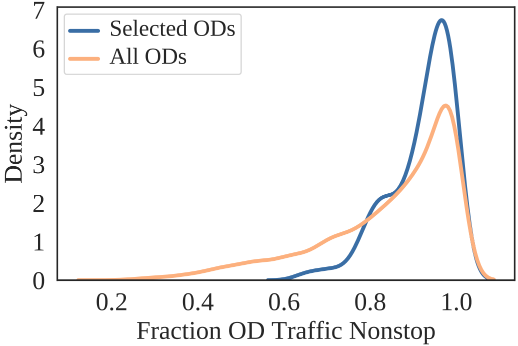

Figure: Non-stop traffic distribution

Summary Statistics—Dynamics

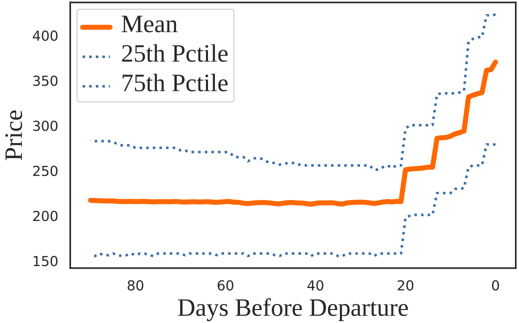

Figure: Prices over Time

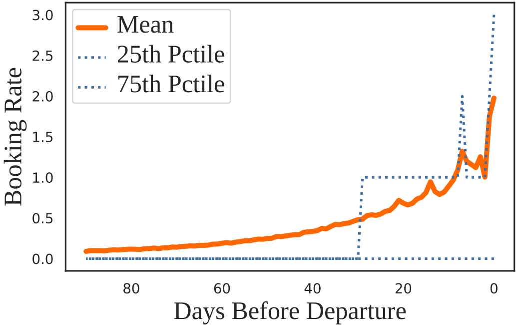

Figure: Booking Rate over Time

- Distribution of fares follow step pattern—AP discounts substantially increase fares
- Booking rates increase, due to both more arrivals (partially observed) and (we will find) higher WTP

Average outcomes across competitors

- No competitor sells consisently a larger fraction of its seats
- Price differences across carriers are small, but one carrier charges relatively lower prices earlier on and higher prices later on (on average)

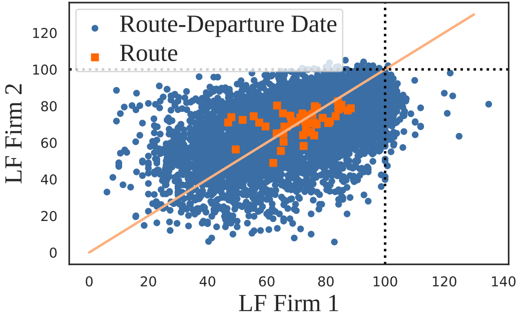

Figure: Average load factors for carriers in duopoly markets

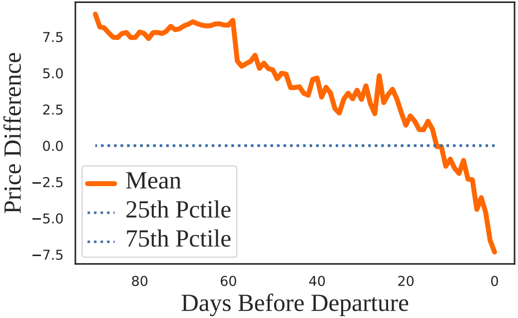

Figure: Difference in prices for markets in which each firm offers exactly one flight

Empirical Model of Demand—Nested Logit with 2 Nests

- Let j be a carrier-flight, d a departure date, t is day before departure, r a route

- Conditional on arrival, we specify consumer utilities as

$$u_{i,j,t,d,r} = \mathbf{x}_{j,t,d,r}\beta - \alpha_t p_{j,t,d,r} + \zeta_{i,J} + (1-\sigma)\varepsilon_{i,j,t,d,r},$$

  where
  - $\zeta_{i,J} + (1-\sigma)\varepsilon_{i,j,t,d,r}$ follows a type-1 extreme value distribution, and $\zeta_{i,J}$ is an idiosyncratic preference for the inside goods;
  - We allow price sensitivity parameters α_t to vary with time
  - Nesting parameter σ captures flight substitutability

- Each arriving consumer solves their utility maximization problem such that consumer i chooses flight j if and only if $u_{i,j,t,d,r} \geq u_{i,j',d,t,r}$, $\forall j' \in \mathcal{J}_{t,d,r} \cup \{0\}$.

- Estimates robust to adding an unobservable ξ, estimated with control function

Empirical Model of Demand—Poisson Arrival

- We assume daily arrivals are distributed Poisson, with rates $\lambda_{t,d,r}$ equal to

$$\lambda_{t,d,r} = \exp\left(\tau_r^{\text{OD}} + \tau_d^{\text{DD}} + \tau_{t,d}^{\text{SD}} + f\left(\text{DFD}\right)_t\right),$$

  where $f(\cdot)$ is a polynomial expansion

- Therefore, $q_{j,t,d,r} = \min\{C_{j,t,d,r}, \lambda_{t,d,r} \cdot s_{j,t,d,r}(p;\theta)\}$, which is censored Poisson

- We scale up arrivals using a factor (1-3.5) to account for unobserved searches, after accounting for the percentage of direct bookings/searches for a single carrier

Demand Estimates Over Time

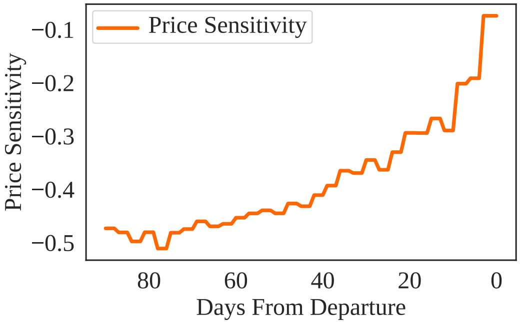

Figure: Price Sensitivity Parameters

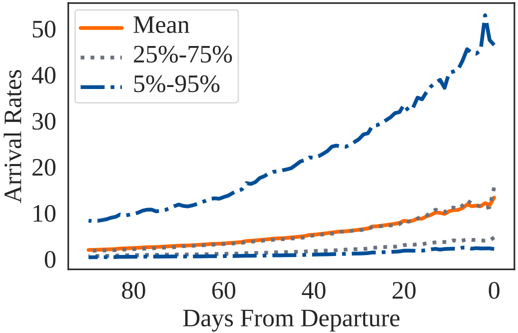

Figure: Arrival Rates

- Estimate nesting parameter $= 0.5$; avg. elasticity of -1.438
- Both the number of arriving customers and the average price sensitivity are increasing towards the deadline

Counterfactual Analysis: Dynamic Pricing

	Price	Firm 1 Rev.	Firm 2 Rev.	CS	Welfare	Q	LF	Sellouts
Benchmark	226.3	5571.5	5759.4	16698.2	28029.0	20.0	70.6	9.2
Uniform	250.8	4629.6	4925.7	19042.4	28597.6	19.2	69.7	7.9
% Diff.	10.8	-16.9	-14.5	14.0	2.0	-3.8	-0.9	-1.3

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2. Consumers are better off with uniform pricing

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4. Fewer units are sold with uniform pricing, and there are fewer sell outs

Counterfactual Results: Dynamic Pricing

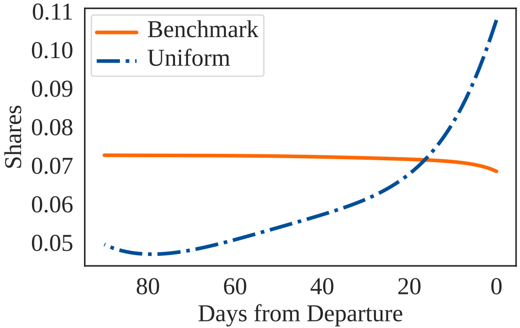

Figure: Shares over Time

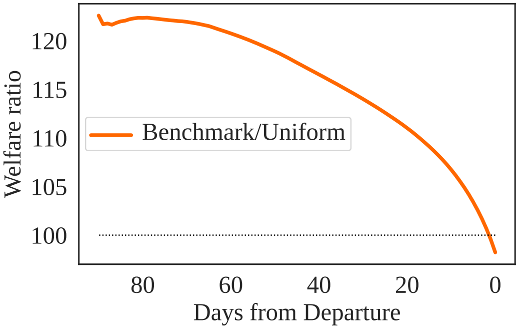

Figure: Cumulative Welfare Comparison

- Early-arriving customers pay a higher price with uniform pricing, late-arriving customers a higher price.
- The firm keeps inefficiently few seats for late-arriving customers under DP to soften competition close to the deadline (competitor-scarcity effect)

Counterfactual Results: Heuristics

1. Lagged-price model
   - Firm assumes last observed price will continue until deadline
2. Deterministic model
   - Firms believe competitors will follow a fixed price path according to the minimum filed price

	Price	Firm 1 Rev.	Firm 2 Rev.	CS	Welfare	Q	LF	Sellouts
Benchmark	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
Lagged	104.6	104.1	105.3	103.3	103.9	100.0	100.1	101.0
Deterministic	98.0	99.4	100.8	108.2	104.9	103.9	101.4	109.2

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Conclusion

- We introduce a framework to study dynamic price competition in perishable goods markets
- We show that competitor scarcity is a key driver of price dynamics and captures the incentive to soften competition in the future
- We apply our framework to airlines and find that DP expands output but decreases welfare in the routes studied
- Open questions remain regarding the use of dynamic versioning, loyalty, and the influence of forward-looking buyers